R.L.A.M.'S HIGHER SECONDARY SCHOOL
BANGALORE-1.

A TEXT BOOK
OF
ELEMENTARY MATHEMATICS
FOR HIGH SCHOOLS

m/A72

Volume II

BY

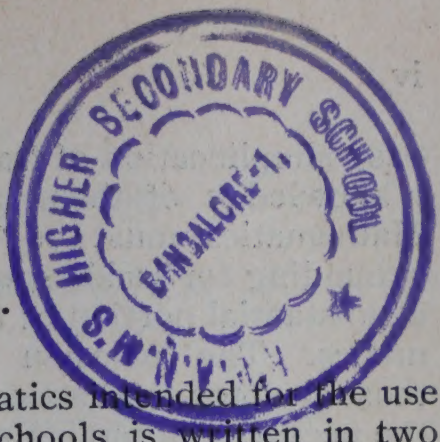
V. VENKATASUBBAYYA, B.A., L.T.
Head Master, Pachaiyappa's College School
and Member of the S.L.C. Board, Madras.

MACMILLAN & CO., LIMITED
MADRAS, BOMBAY, CALCUTTA AND LONDON

1926

2286
857
.13
N26:1

PRINTED AT THE
DIOCESAN PRESS, MADRAS
1926—C8844



PREFACE.

THIS book in Elementary Mathematics intended for the use of Forms IV, V and VI in High Schools is written in two volumes in English, in accordance with the Madras S.S.L.C. Syllabus to consummate the author's series of works compiled in vernaculars for Classes IV and V and Forms I, II and III. But it will also be useful for students preparing for the School Leaving Certificate or the Entrance Examination of Indian Universities. It is complete in itself in the sense that there is no necessity for reference to the lower books of the series. In the chapters with whose principles boys are already familiar, the chief leading points have been put in a nutshell and the questions framed are of greater difficulty than those they were accustomed to work. In the headings of chapters, this differentiation, as to whether they are revision chapters or new ones boys have to learn for the first time, has been indicated. The examples in each chapter assume a knowledge of the elementary portions and take into account the progress in Arithmetic made by the pupils entering upon the High School course. This supposition has given considerable latitude to the author in coining examples of much interest and great variety.

Numerous examples, some of the same type and others of various degrees of difficulty, have been given in each chapter; the former, to drill pupils in the fundamental processes and the latter, to give them sufficient scope for thinking, chiefly involving a combination of the basic principles in various ways. The former are mechanical, the latter intelligent. Every chapter contains an unusually large number of problems, the chief object being to help the teacher in giving different sums of the same kind to different sets of boys for class work, to prevent copying and to furnish them with another fresh stock of questions for home work.

The subject-matter of the book may be divided under four headings: Arithmetic, Algebra, Practical Geometry and Mensuration. Some chapters in Algebra on multiplication, division, H.C.F., L.C.M., factorization, formulæ,

easy simplification of fractions have been added, as, without a knowledge of them, a boy's attainments in Elementary Mathematics must be very poor, lacking the necessary grounding for tackling equations and problems which form a substantial portion of the syllabus in Elementary Mathematics. The inclusion of these chapters in Algebra and those devoted to Practical Geometry does not put boys to the necessity of buying separate text-books in Algebra and Geometry unless they purpose specializing in Group I, i.e. Mathematics and Physics. An attempt has been made to add a chapter on the metric system and to familiarize pupils with questions on areas, weights and volumes and make them face the mathematical conditions of science problems so as to bring about some correlation between Science and Mathematics. Without this correlation, it is not possible for the science teacher to train science pupils in mathematical calculations needed for evaluations in science.

This volume contains chapters XXI–XXXIV of which chapters XXI–XXX may be done in Form V, and the rest in Form VI. Vol. I contains chapters I–XX which may be done mainly in Form IV; some portions however may be relegated to Form V; but this arrangement may be altered if necessary.

These are some of the salient features of the work and it is hoped that it will be found very useful, instructive and adapted to the purpose for which it is intended.

The author takes this opportunity to thank all the Mathematics Assistants of the School Department of Pachaiyappa's College for having prepared answers to the examples given in various exercises which enabled him to compare his answers with theirs. His special thanks are due to M.R.Ry. M. Viraswamayya Garu, B.A., L.T., Senior Mathematics Assistant, for the many useful suggestions made and the constructive criticism offered by him.

Any suggestions for the improvement of the book will be thankfully received.

MADRAS,
April 1926.

V. VENKATASUBBAYYA.

REVISED SYLLABUS IN

ELEMENTARY MATHEMATICS

FOR THE

MADRAS S.S.L.C. PUBLIC EXAMINATION

GROUP A

ELEMENTARY MATHEMATICS

THE four simple rules applied to integers and fractions—Factors—Ratio and Proportion—Square Root.

Indian and English coins, weights and measures of length, capacity and time, in actual use. The metric system. Decimal approximations (not more than three places). Averages.

Percentage and its applications—Simple interest—Direct problems in Compound interest (number of periods for calculation of interest not to exceed two). Mensuration of plane rectilinear figures (a knowledge of the use of the Field Book not required), circles, right prisms, right circular cylinders and cones (frusta excluded).

Symbolic representation of the processes of Arithmetic. Negative quantities and extension of the four rules to them (formal proofs not required).

Solution of linear equations, involving one or two variables, and problems leading thereto.

Plotting statistics. Straight line graphs. Measurement of lengths and angles—Construction of perpendiculars and parallels, of circles touching a given straight line or a circle at a given point, and of angles. Dividing lines in given ratios. Drawing to Scale. Elementary properties of triangles and parallelograms. Simple cases of construction of triangles and quadrilaterals and of regular polygons of 6 and 8 sides. Experimental verification of the theorems required for the previous constructions.

N.B.—The grouping of the items in the above scheme is not intended to indicate the order in which they should be taken for class work.

CONTENTS.

CHAPTER XXI.

Decimal Approximations ; Limits of Error ; Significant Figures ; Contracted Methods. (New).

PAGE

The need for approximations in practice ; error ; significant figures ; limits of error, unit depending upon the object to be measured ; absolute error, relative error ; maximum error ; exercises ; contracted addition and amount of error ; contracted subtraction and amount of error ; exercises ; contracted multiplication and amount of error ; reliability of the product ; exercises ; contracted division, amount of error ; reliability of the quotient ; exercises ; further approximations ; multiplication and division combined ; $\frac{1}{1+x}$, $\frac{1}{1-x}$, $(1+x)(1+y)$, $(1-x)(1-y)$, $(1+x)^n$ $\sqrt[n]{1+x}$; evaluation of a geometrical series to any required degree of accuracy. Decimalization of Indian Money and English Money ; exercises 1

CHAPTER XXII.

Square Root. (New.)

Meaning of square root ; extraction of root by factors ; by evolution ; surds ; geometrical representation of surds, evaluation of surds ; approximations in square root ; exercises 44

CHAPTER XXIII.

Compound Interest. (New.)

Meaning of the term ; model examples ; formulæ ; exercises. 69

CHAPTER XXIV.

Harder Equations of One Unknown Quantity and Problems thereon. (New.)

Equations involving a knowledge of fractions ; many difficult questions in Arithmetic solved algebraically ; model examples ; exercises 88

CHAPTER XXV. PAGE

SOLUTION OF SIMULTANEOUS EQUATIONS OF TWO UNKNOWN QUANTITIES OF THE FIRST DEGREE AND PROBLEMS LEADING TO THEM. (NEW.)

Model examples and exercises 118

CHAPTER XXVI.

THE STRAIGHT LINE GRAPH. (NEW.)

Variables and their dependence ; function ; graph—a locus ; drawing a straight line graph and various methods of drawing it ; exercises ; uses of graphs ; exercises 141

CHAPTER XXVII.

GRAPHS CHIEFLY STATISTICAL. (NEW.)

Model examples ; broken and continuous curves and exercises. 174

CHAPTER XXVIII.

CONSTRUCTION OF QUADRILATERALS. (NEW.) 191

CHAPTER XXIX.

SYMMETRY IN ALGEBRA AND GEOMETRY. (NEW.)

PROPERTIES OF CIRCLES, CHORDS AND DISTANCES FROM CENTRE ; EQUAL CHORDS, EQUAL ARCS, EQUAL ANGLES AT THE CENTRE.

Symmetrical expressions ; axis of symmetry, image points, finding the centre of a circle ; if a diameter cuts a chord at right angles, it must bisect it. *Vice versa* ; locus of the middle points of a system of parallel chords ; equal chords are equally distant from the centre and converse ; equal angles at the centre subtended by equal arcs ; equal arcs cut off by equal chords ; inscribing a regular polygon in a given circle. The area of a regular polygon ; construction of a regular polygon on a given base : exercises 196

CHAPTER XXX.

SOME OTHER PROPERTIES OF CIRCLES NOT BASED ON SYMMETRY AND CONGRUENCE. TANGENTS, ETC. (NEW.)

Angle at the centre, double angle at the circumference on the same arc ; equal angles stand on equal arcs at the circumference ;

PAGE

exercises ; cyclic quadrilaterals ; drawing tangents from a point (1) on the circumference ; (2) without it ; exercises ; potency of a point (1) within a circle ; (2) without it 213

CHAPTER XXXI.

INTERESTING GRAPHICAL SOLUTIONS OF CERTAIN ARITHMETICAL PROBLEMS. (NEW.)

Model examples ; exercises 230

CHAPTER XXXII.

AREAS OF CIRCLES, SECTORS AND SEGMENTS. (NEW.)

π ; exercises ; area of a circle ; exercises ; areas of sectors and segments ; exercises 241

CHAPTER XXXIII.

RECTANGULAR SOLIDS. THE CUBE AND CUBOID. (REVISION.)

The surface of a cube and cuboid ; their volumes ; exercises. 258

CHAPTER XXXIV.

RIGHT PRISM, CYLINDER AND CONE. (NEW.)

Shape of a prism, its volume ; exercises ; cylinder, its shape and generation ; its volume, surface area ; exercises ; a cone, its shape and generation ; its sections, volume, and surface area ; exercises 268

MISCELLANEOUS EXAMPLES ARRANGED ACCORDING TO TOPIC.

Section I.—Simplification of Fractions 293
Section II.—G.C.M. and L.C.M. and Motion Round Circles ... 294
Section III.—Decimals 295
Section IV.—Scale Representation 296
Section V.—Partnership 298
Section VI.—Proportion 303
Section VII.—Percentages 307
Section VIII.—Averages 310
Section IX.—Interest 312
Section X.—Formulæ and Substitution 314
Section XI.—Short Methods 315
Section XII.—Square Root 315
Section XIII.—Equations and Problems 316
Section XIV.—Practical Geometry 318

PAGE

Section XV.—Graphs 320
Section XVI.—Mensuration 322
Section XVII.—Profit and Loss (Percentage) 326
Section XVIII.—Time and Distance 329
Section XIX.—Approximations, Errors, etc. 336
Section XX.—Time and Work 338
Section XXI.—Construction of Quadrilaterals 339
Section XXII.—Bankruptcy 342
Revision Papers. Second Series 344
S.S.L.C. Examination Papers (1911–25) 355
Appendix 403
Answers 413

ELEMENTARY MATHEMATICS.

Volume II.

CHAPTER XXI.

DECIMAL APPROXIMATIONS; LIMITS OF ERROR; SIGNIFICANT FIGURES; CONTRACTED METHODS.

127. The need for approximations in practice. Suppose we have to divide a distance of 10 yds. 2 ft. 3·6 in. into 100 equal parts, then each part by calculation is 3·876 in. If we are to mark these divisions, we should draw a straight line and mark off along it a length of 3·87 in. To do this, we first take 3 inches; then along the next inch which is divided into 10 equal parts we take 8 tenths or small sub-divisions. To get the remaining seven hundredths, we must sub-divide the next tenth of an inch into 10 equal parts and take 7 of these. This is very difficult to do for a tenth of an inch is very small to admit of further sub-divisions; so other devices have been adopted such as the diagonal scale and the vernier for measuring hundredths of an inch. If, however, we wish to get a straight line 3·876 in. long, a hundredth of an inch will have to be sub-divided into 10 equal parts and 6 of these should be taken. This should be a very minute section which cannot be judged by the eye or measured with any ordinary instrument. Thus owing to either the imperfection of our senses or want of very delicate instruments, we are obliged to impose certain limitations on the extent of our measurements. In the calculation of lengths expressed in inches, we should be satisfied with two decimal places and if there is a third decimal place as

in 3·876 in., the figure in the third place, viz., six thousandths may be roughly taken to be equal to one hundredth of an inch. Thus 3·876 in. is approximately (or roughly) equal to 3·88 in.

128. Error. Approximation always involves some error. There must be some difference between the actual value and that roughly taken for our purpose. The error when 3·876 is changed into 3·88 is 3·88—3·876 or ·004 ; but when 3·876 is changed into 3·87, the error is 3·876—3·87 or ·006. Since the error ·004 is less than the error ·006, the former approximation, viz., 3·88 is nearer the truth or the actual value 3·876. 3·88 is said to be correct to the nearest hundredth or correct to the second decimal place.

If we have to express 6·783 correct to two decimal places, we have to consider whether 6·78 or 6·79 will involve the less error. The error in the former case (if 6·78 is taken roughly to be the value), viz. ·003 is less than the error in the latter case (if 6·79 is taken), namely ·007 ; hence 6·78 is the proper approximation.

The same argument may be put in this way :—

(1) 3·876 lies between 3·87 and 3·88 or 3·870 and 3·880.
(2) 6·783 ,, ,, 6·78 and 6·79 or 6·780 and 6·790.

(A)

3·870 ↑
3·871
3·872
3·873
3·874
3·875
3·876 →
3·877
3·878
3·879
3·880 ↓

(B)

6·780 ↑
6·781
6·782
6·783 →
6·784
6·785
6·786
6·787
6·788
6·789
6·790 ↓

It is clear from (A) that 3·876 is nearer to 3·880, i.e. 3·88 than to 3·870 or 3·87.

Similarly it is clear from (B) that 6·783 is nearer to 6·780, i.e. 6·78 than to 6·790, i.e. 6·79.

Hence 3·876 = 3·88 } approximately or correct to the
and 6·783 = 6·78 } 2nd decimal place.

Thus we get the rule **If a quantity is to be expressed correct to n places, increase the digit in the n^{th} place by 1 if the digit in the $\overline{n+1}^{th}$ place is either 5 or greater than 5; and do not alter the digit in the n^{th} place if that in the next place is less than 5.**

129. Significant figures. We have seen that calculations give us more figures than are necessary for practical purposes which compel us to be satisfied with approximations. This is because the difference between the exact value and the approximate value is so small in comparison with the importance of the quantities we are dealing with, that it is practically negligible and will not materially affect our mathematical impressions or calculations. For example, if the income of the Government of India is stated to be 99,00,03,809 rupees, the exact figure may be not only inconvenient for memory but also a hindrance to any mental calculations to be made therefrom, unless the last few figures are discarded. If the diameter of the earth's orbit round the sun is given to be 192,000,125 miles, the last three figures not only belittle but mock the grandeur and immensity of space. What should be noted is that the extra figures are very small when compared with the numbers denoted by the first figure or figures followed by cyphers. The first figure or figures in each case are called the significant figures. In the first example of 99,00,00,000 rupees 99 are the significant figures (two in number) denoting the number of units while the remaining cyphers denote the value of the unit, the unit in this case being a crore. In the second example 192 are the significant figures (three in number) denoting the number of units while the remaining (000,000) cyphers denote the value of the unit, the unit in this case being a million.

Take the example ·075 in. Here the significant part is 75 consisting of two figures denoting the number of units, while the first cypher denotes the value of the unit, the unit in this case being a thousandth.

130. It should also be noted that the unit may vary for different things and for the same thing if a greater degree of approximation is insisted upon. The distance between the sun and the earth or any other planet will be adequately expressed in terms of millions of miles; that between two places on the earth, in thousands or hundreds of miles; or the expansion of a metallic rod when heated, in hundredths of an inch. In weighing heavy things such as iron, we are not so accurate as to take grains of weight into account; but when we measure the weight of gold we are very careful to include even grains. The revenue of a vast country like India may be estimated in crores; that of a corporation in lakhs. Approximation to a given number of significant figures is best done when the unit is known.

Model No. (1) Express 4,867,432 sq. miles correct to (1) one significant figure and (2) correct to two significant figures.

(1) The question means that only the first figure to the left should be retained or modified and the remaining places filled with zeroes. ∵ the second figure from the left, viz. 8 is greater than 5, the preceding figure 4 should be changed into 5. The answer is 5,000,000 or 5 millions.

(2) The second part means that the first two figures to the left should be retained followed by cyphers. ∵ the figure in the third place from the left is 6, 8 should be changed into 9. The answer is 49,00,000 or 49 lakhs.

131. Errors are of two kinds: (1) Absolute error and (2) Relative error. The absolute or actual error is the difference between the actual and the approximate value. For example if 1 metre = 39·3707 in. and if the approximate value of 1 metre is taken to be 40 in., then the absolute error is 40 — 39·3707 or ·6293 in.

$$\textbf{Relative error} = \frac{\textbf{absolute error}}{\textbf{true value}}.$$

In the above example, the relative error

$$= \frac{\cdot 6293 \text{ in.}}{39\cdot 3707 \text{ in.}} = \frac{\cdot 6293}{39\cdot 3707} = \frac{\cdot 06293}{3\cdot 93707} = \cdot 0159.$$

The percentage of error = the relative error × 100 = ·0159 × 100 = 1·59.

The relative error is more important than the absolute error.

If 1004 is taken as 1000, and 64 is taken as 60, the absolute error 4 is the same in both cases; but the relative error in the former case is $\frac{4}{1004} = \frac{1}{251}$; and in the latter case $\frac{4}{64} = \frac{1}{16}$. It is much less in the former case than in the latter.

132. Maximum error. If an estimated length is given to be 2·67 metres correct to the first three significant figures, its actual value must lie between certain limits. Let us see what figures there could be in the actual length after 2·6 so as to approximate to 2·67. The following are possible figures:—

(*a*) 2·665
2·666
2·667
2·668
2·669

(*b*) 2·670
2·671
2·672
2·673
2·674
2·6741
2·6742
...
...
2·67499...
.....
2·675

The *a* list contains 2·66 followed by the figures 5, 6, 7, 8, 9.

The *b* list contains 2·67 followed by the figures 0, 1, 2, 3, 4, etc.

Its actual value must lie between 2·665 and 2·67499... inclusive of both limits or between 2·665 and 2·675 inclusive of the former limit and exclusive of the latter, 2·675 denoting the superior limit up to which the number may range. The actual value of the estimated length lies between 2·665 m. and 2·75 m. If the actual value is 2·665 the absolute error is 2·67—2·665 or + ·005 m.; if the actual value is 2·675, the absolute error is 2·67 − 2·675 = − ·005. The maximum error is ± ·005.

133. If a result is correct to 3 significant figures, it is true within one-hundredth of the real value.

Suppose the first three significant figures are a, b and c. Then the approximate value = $(100a + 10b + c)$ units. Then the actual value may lie between $(100a + 10b + c + \frac{1}{2})$ units and $(100a + 10b + c - \frac{1}{2})$ units.

The absolute maximum error is $\pm \frac{1}{2}$ unit.

$\therefore$ the relative maximum error is $\pm \frac{\frac{1}{2}}{100a + 10b + c}$

The greatest value of this relative error is $\pm \frac{\frac{1}{2}}{100}$ or ± ·005 ($\because$ 100 is the least value of a figure of 3 digits). Note that a fraction with

a constant numerator has its greatest value when its denominator has the least value.

Similarly if a result is correct to four significant figures it is true within $\frac{1}{1,000}$ of the real value and so on.

Exercise XXI (A).

1. Find, correct to 1 significant figure, the value of the following and name the significant figure stating the unit in each case :—

(*a*) Rs. 478205, (*b*) 67830000 miles, (*c*) ·03467 km., (*d*) 896·7 lbs., (*e*) 4864 gallons, (*f*) ·0834 lbs. (Troy.)

2. Write down the following numbers correct to three significant figures :—

(*a*) 64083401, (*b*) 487634, (*c*) 123045, (*d*) 9·06784, (*e*) ·0008346, (*f*) 8·482, (*g*) ·0008092, (*h*) 864324 and (*i*) 470285.

3. Name the significant digits in the following :—

(*a*) The distance from Madras to Calcutta = 1030 miles to the nearest 10 miles.

(*b*) The income of a high school = Rs. 41000 to the nearest thousand rupees.

(*c*) The population of a town = 6200000 correct to the nearest lakh.

(*d*) A gramme = ·0022 of a lb. (avoirdupois) correct to tenths of thousands.

4. Given that 1 cwt. = 2465 grammes, find the equivalent of 1 cwt. in gms. correct to two significant figures.

5. Given that a sq. km. = 246·858 acres, find the equivalent of 1 sq. kilometre in acres correct to four significant figures.

6. Given that a gallon contains 277·274 cub. in., find the equivalent of a gallon in cub. in. correct to two significant figures.

7. The following quantities are correct to three decimal places. Between what limits must the actual quantities lie ?

(*a*) Rs. 767·385, (*b*) £380·309, (*c*) 460·053 miles and (*d*) 851·986 kg.

8. Between what limits must the following quantities lie if they are correct to four significant figures?

(*a*) Rs. 30·04, (*b*) Rs. 300·4, (*c*) Re. ·03004, (*d*) Rs. 3004·000, (*e*) 8·634 kg., (*f*) 86·34 hg., (*g*) 8634 gm. and (*h*) 8634000 mg.

Show that the relative error is the same in *a*, *b*, *c*, *d* and *e* and also in *e*, *f*, *g*, *h*.

9. Between what limits must the following quantities lie if they are, (1) correct to three significant figures and (2) to four significant figures?

(*a*) Rs. 80·30, (*b*) Rs. 8·030, (*c*) Rs. 80,300 and (*d*) Rs. 8,030.

Show that the relative error is the same in *a*, *b*, *c* and *d*.

10. Find the limits to the error in each of the following approximate quantities correct to the last significant digit in each case:—

(*a*) The population of Travancore in 1911 was 4,812,000.

(*b*) The civil, military, and convict population in Port Blair in 1916 was 17,230.

(*c*) The persons with a knowledge of English in India in 1911 numbered 1·7 millions.

(*d*) 11·46 in. was the rainfall at Madras on a certain day.

(*e*) The area of British territory in India in 1911 was 1,033,070 sq. miles.

(*f*) The length of the nautical mile is 2029·07 yds.

(*g*) The equivalent of 1 ton in grammes = 1021·8.

11. (*a*) A metre is taken to be 39·3701 in. If it is taken to be (1) $39\frac{7}{20}$ in. and (2) 40 in., to how many significant figures is it correct?

(*b*) If π is taken to be $\frac{345}{113}$ to how many significant figures is it correct ($\pi = 3{\cdot}14159$)?

(*c*) The population of a town varied from 189,786 to 183,754. To how many significant figures may it be taken to be steady.

12. In question 10, find (1) the relative error and (2) the percentage of error in each case.

134. Contracted addition and subtraction and limits of error. In all approximate additions and subtractions insisting upon a given degree of accuracy, it is enough if we retain in each quantity two more places than the number specified.

Model No. (2) Find, correct to three decimal places, the sum of 16·81984, ·016̇, ·1̇42857̇ and $\frac{5}{6}$.

Method (1)

	16·819	84
	·016	66
	·142	85
$\frac{5}{6}$ =	·833	33
	17·812	68

A vertical line is drawn after the third decimal figure to show that the figures following are unnecessary except for determining the figure to be carried to the third decimal place.

The answer is 17·813 correct to the third decimal place.

Method (2)

Each quantity to be added may be corrected to the fourth decimal place and as before a line may be drawn after the third decimal place.

16·819	8
·016	7
·142	9
·833	3
17·812	7 = 17·813 correct to the third decimal place.

Model No. (3) A five-sided figure is divided into three triangles. The areas of the three triangles are given to be 3·87 sq. in., 4·83 sq. in. and 5·97 sq. in. respectively correct to the nearest hundredth of a square inch. Find the least and the greatest possible area of the figure as deduced from the measurements. How far can the result be relied upon?

The area of the first triangle must lie between 3·865 sq. in. and 3·875 sq. in.

The area of the second triangle must lie between 4·825 sq. in. and 4·835 sq. in.

The area of the third triangle must lie between 5·965 sq. in. and 5·975 sq. in.

∴ the area of the whole figure lies between 14·655 sq. in. and 14·685 sq. in.

∴ the difference = 14·685 − 14·655 = ·030 sq. in.

It is clear that the first three significant figures are the same in the minimum and maximum areas whether the figures stand as they are or they are modified so as to be correct to the first decimal place. The result is reliable to the first decimal place and the second decimal place is not to be trusted.

Or the work may be shown thus :—

$$\begin{array}{l} 3{\cdot}87 \pm {\cdot}005 \\ 4{\cdot}83 \pm {\cdot}005 \\ 5{\cdot}97 \pm {\cdot}005 \\ \hline 14{\cdot}67 \pm {\cdot}015 \end{array}$$

which also shows that the first decimal place may be relied upon.

Model No. (4) Subtract $361\frac{5}{7}$ from $1082{\cdot}8\dot{3}5$. Find the result correct to four decimal places.

$$\begin{array}{lll} 1082{\cdot}8\dot{3}5 = 1082{\cdot}835835 = & 1082{\cdot}8358 & 35 \\ 361\frac{5}{7} \quad = 361{\cdot}\dot{7}14285\dot{} = & 361{\cdot}7142 & 85 \\ & \overline{721{\cdot}1215} & \overline{50} \end{array}$$

The answer = 721·1216 correct to fourth decimal place.

Model No. (5) Two sides of a triangle measure 7·63 in. and 5·38 in. each being true to the nearest hundredth of an inch. Between what limits must the difference of the two sides lie ?

$$\begin{array}{lll} \text{The first side must lie between} & 7{\cdot}625 \text{ and } 7{\cdot}635 \\ \text{The second side must lie between} & 5{\cdot}385 \text{ and } 5{\cdot}375 \\ & \overline{2{\cdot}240} \qquad \overline{2{\cdot}260} \end{array}$$

∴ the difference must lie between 2·240 and 2·260. Here the first two significant figures can be relied upon. Or the work might be arranged thus :—

$$\begin{array}{ll} 7{\cdot}63 + {\cdot}005 & 7{\cdot}63 - {\cdot}005 \\ 5{\cdot}38 - {\cdot}005 & 5{\cdot}38 + {\cdot}005 \\ \hline 2{\cdot}25 + {\cdot}01 & 2{\cdot}25 - {\cdot}01 \end{array}$$

or $2{\cdot}25 \pm {\cdot}01$

The difference of the two sides = 2·25 with a possible excess or defect not exceeding ·01. If the given lengths are subtracted as they stand the last decimal figure cannot be relied upon.

N.B.—In finding the sum of two or more approximate quantities the maximum and minimum results are obtained by taking the greatest and the least possible value of each quantity and adding all such maximum and minimum values respectively.

In finding the difference of two approximate quantities, the maximum result is got by subtracting the least possible value of the smaller quantity from the greatest possible value of the larger quantity; the minimum result is got by subtracting the greatest possible value of the smaller quantity from the least possible value of the larger quantity.

Exercise XXI (B).

Add together the following, giving the sum in each case true to the third decimal figure :—

1. 1063·87567, 563·438, 6·00384, and 18·762584.
2. $1025\frac{1}{4}$, $188\frac{1}{3}$, $729\frac{5}{6}$ and 450·8389.
3. $561\frac{1}{8}$, 4·00625, $8\frac{2}{9}$ and 1·16.
4. 8068·56896 km., 6·00384 km., 115·65938 km. and 4093·48374 km.

Find the result correct to three significant figures in each of the following :—

5. 150·3806 + 64·0098 + ·1009 + 0·8345.
6. 1008·053 — 678·349.
7. $\frac{3}{7} + \frac{5}{6} - \frac{2}{9}$.
8. 818·6025 + 41·3892 — ·008765.
9. The area of Europe is 3,756,970 sq. miles; Asia 17,212,680 sq. miles; Africa 11,514,770 sq. miles; N. America 7,900,350 sq. miles; S. America 6,854,100 sq. miles; Australia 2,964,000 sq. miles. Find the area of all the continents (1) to the nearest million sq. miles, (2) to the nearest hundred thousand sq. miles and (3) to the nearest thousand sq. miles.
10. The following table shows the loans given by the United States to the Allies during the war :—

To	Before the declaration of the war	After the declaration of the war	Total
	in million dollars	in million dollars	in million dollars
Great Britain ...	1131·4	2720·0	
France	736·7	1540·0	
Russia	148·5	187·0	
Italy	26·0	650·0	
Belgium	...	93·4	
Serbia	...	9·0	
Cuba	...	15·0	
Total ...	...	...	...

Find (1) the total sum lent to each country and (2) the total sum lent to the Allies (*a*) before and (*b*) after the war correct to million dollars.

11. In the previous question supposing the figures are approximate (i.e. correct to the first decimal place), find to how many significant figures the totals in 2 (*a*) and (*b*) can be relied upon.

12. Find the difference in area between Europe and Asia in question (9) correct to the nearest thousand sq. miles.

13. The following return shows the nominal capital invested in the United Kingdom in the years 1916 and 1917.

Classification	1916	1917
	£	£
Coal, iron and steel ...	2,385,405	7,233,300
Mining (other than coal and iron)	1,773,640	1,447,509
Textiles	3,843,600	3,359,250
Brewers and distillers ...	5,217,305	420,000
Rubber and oil ...	1,511,225	1,411,100
Transit and transport by land and water ...	4,999,090	5,465,127
Gas, water, etc. ...	72,512	158,500
Land	1,017,276	3,355,950
Finance } Banking }	3,407,000	8,544,565
Others	26,215,818	36,418,625
Total ...	...	...

Find (1) the total nominal value in each of the years 1916 and 1917 to the nearest (*a*) hundred thousand pounds and (*b*) million pounds and (2) the difference of the total nominal values correct (*a*) to the nearest million pounds and (*b*) to the nearest hundred thousand pounds.

14. The land revenue was estimated to be contributed in 1916–17 as follows :—

Administrations	RS.
India, General	21,81,000
Central Provinces and Berar ...	2,00,65,000
Burma	4,71,69,000
Assam	78,57,000
Bengal	3,04,69,000
Bihar and Orissa	1,63,72,000
Agra and Oudh	6,43,51,000
Punjab	2,94,22,000
North-West Frontier Provinces ...	21,01,000
Madras	5,92,58,000
Bombay	5,11,07,000

Supposing these figures are correct to thousands, find how many figures can be relied upon in the total of the above estimates. Find also the total estimate of the land revenue (*a*) to the nearest lakh and (*b*) to the nearest crore.

15. Rs. 22,024,000 and Rs. 22,218,000 were the incomes under the head of Land Revenue in India during the years 1916–17 and 1917–18 respectively correct to thousands of rupees. By how much did the revenue in 1917–18 exceed that in 1916–17 ? (The difference to be stated as correctly as the data justify.)

16. According to the religious statistics of 1911, there were in the Province of Madras 36,807,000 Hindus including Arya Samajists and Brahmos. These two numbered 249,000. These figures are correct to the nearest thousand. Find the number of the other Hindus as correctly as the data justify.

17. Two sides of a triangle measure 8·08 m. and 7·65 m. each length being correct to the nearest cm. Between what two limits should the third side lie ? Find the least and greatest possible values of each limit.

18. The height of the Tibetan Plateau above the sea-level is 12,000 ft.; that of the Arabian Plateau 3,000 ft. (both correct to a ft.). How much higher is Tibet than Arabia? How many first significant digits can be relied upon in the answer?

19. Each of the following measurements is correct to three significant digits only :—1680 gm., 96,300 gm., 8·320 gm. and 0·836 gm. Find their sum as far as the data justify.

20. (*a*) Simplify the following :—

(1) $a+b+c$, (2) $a-b+c$, (3) $b+c-a$ and (4) $a+b-c$ if $a=8{\cdot}325$, $b=6{\cdot}423$ and $c=3{\cdot}489$ each being correct to the third decimal place. Between what limits must each of the results of simplification lie?

(*b*) A man's average annual income and expenditure are Rs. 10,200 and Rs. 8,900 respectively, these values being correct to the nearest hundred. Find the limits between which his savings in ten years should lie. To what degree of accuracy can the answers be relied upon?

135 (*a*). Contracted Multiplication of Decimals.

Model No. (6) Multiply 876·43847 by 25·393 correct to two places of decimals.

1. The multiplier should be put in the standard form, as in the ordinary multiplication of decimals to obtain a rough estimate of the product.

$$876{\cdot}43847 \times 25{\cdot}393 = 8764{\cdot}3847 \times 2{\cdot}5393.$$

The answer should be less than $9{,}000 \times 3$ or 27000.

2. Since the answer is wanted to two places of decimals and the figure in the third will affect that in the second and similarly the figure in the fourth will affect that in the third, let us find the product to two more places, i.e. to four decimal places. Over the figure in the fourth decimal place a mark like $\times$ may be put and a vertical line drawn after the required number of decimal places, i.e. 2 in this case. The multiplier must be so written below the

multiplicand that direct under the mark ×, the figure in the units place of the multiplier comes as shown below :—

Method (1)

D CB	A×
8764·38	47
	2·5393
	ABCD
17528·76	94
4382·19	24
262·93	15
78·87	94
2·62	93
22255·40	20

Method (2)

	×
8764·38	47
3·93	52
17528·76	94
4382·19	24
262·93	15
78·87	94
2·62	93
22255·40	20

3. It is enough if each partial product contains the required number of decimal places and two more for the sake of accuracy and no more. For this purpose* figures equidistant from the mark ×, both in the multiplier and the multiplicand which are signified by the same letters (thus 4 and 5 by A, 8 and 3 by B, 3 and 9 by C, 4 and 3 by D) are multiplied together and form the starting-points of multiplication for

$4 = 4\ (\frac{1}{10000})$, $5 = 5\ (\frac{1}{10})$. $\therefore 4 \times 5 = 20\ (\frac{1}{10000} \times \frac{1}{10}) = 20\ (\frac{1}{100000})$.

Similarly $8 = 8\ (\frac{1}{1000})$, $3 = 3\ (\frac{1}{100})$. $\therefore 8 \times 3 = 24\ (\frac{1}{100} \times \frac{1}{1000}) = 24\ (\frac{1}{100000})$.

Such combinations give us the figures in the fourth decimal place and those beyond the fourth are discarded.

4. However, for ascertaining the figure to be carried, the multiplication is started with a digit to the right of the correspondingly marked digit in the multiplicand ; thus while $5 \times 4 = 20$, we begin the multiplication by 5

---

* To prevent figures equidistant from the × being marked in the same way for purposes of multiplication, the intelligent device is sometimes adopted of writing the digits of the multiplier in reverse order as shown in the working on the right hand side above (Method 2) :

In this arrangement the pairs of figures to be multiplied 7, 2 ; 4, 5 ; 8, 3 ; 3, 9 and 4, 3 come vertically below each other and there is no necessity for marking equidistant figures.

with 7 (to the right of 4) which gives us 35 equivalent to 40 by principles of approximation. Hence 4 is to be carried. The carrying figures will have similarly to be ascertained in other partial products.

The answer = 22255·40, the 0 need not be changed as the succeeding figure is less than 5.

Model No. (7) Multiply 876·43847 by 25·393 correct to the nearest thousand.

Standard form. 876·43847 × 25·393 = 8764·384 × 2·5393.

For accuracy two more places should be taken ; so the mark × may be placed over the figure in the tens' place.

CB	A ×
08	764
	2·5393
	ABC
17	53
4	38
	26
	7
22	24 = 22000 answer.

Model No. (8) Multiply 876·43847 by 25·393 to four significant figures.

Standard form. 876·43847 × 25·393 = 8764·3847 × 2·5393.

8764·3847 lies in value between 8000 and 9000.

2·5393 ,, ,, 2 and 3.

∴ the product should be between 16000 and 27000, i.e. it contains 5 digits in the integral portion. Hence the fourth significant figure is the digit in the tens' place. The question means finding the product to the nearest 10. Two more places should be taken. Hence the mark × should be put over the figure in the first decimal place.

DCB	A ×
876	4·38
	2·5393
	ABCD
1752	88
438	22
26	29
7	88
	26
2225	53 = 22260 answer.

Model No. (9) Multiply ·0008347 by 1·0856 correct to three significant figures.

The multiplier is already in the standard form.

·0008347 lies between ·0008 and ·0009

and 1·0856 ,, ,, 1 and 1·1

∴ the product lies between ·0008 × 1 and ·0009 × 1·1, i.e. ·0008 and ·00099.

It is clear that the third significant figure is in the 6th decimal place. Two more figures should be taken, i.e. the mark × should be put over the figure in the eighth decimal place.

DCB	A×
·000834	70
	1·0856
	ABCD
·000834	70
66	78
4	17
	50
·000906	15

The required answer is ·000906.

Model No. (10) The sides of a rectangle are taken to be 8·9″ and 7·6″ each measurement being correct to a tenth of an inch. Find the difference between the least and the greatest possible value of the area.

The length lies between 8·85″ and 8·95″.

The breadth lies ,, 7·55″ and 7·65″.

∴ the area lies between 8·85 × 7·55 sq. in. and 8·95 × 7·65 sq. in.

8·85	8·95
7·55	7·65
61·95	62·65
4·425	5·370
·4425	·4475
66·8175	68·4675
or 66·82 sq. in. nearly	or 68·47 sq. in. nearly

The difference between the highest and the lowest values is 68·47 − 66·82 = 1·65 sq. in.

∴ the maximum error is $\frac{1·65}{2}$ = ·825 or ·83 sq. in. nearly but the area arising from the given length and breadth is

8·9
7·6
62·3
5·34
67·64

∴ 67·64 sq. in. is liable to an error of ± ·83 sq. in.

The limits of error can be obtained without actually performing the multiplication of the greatest and least values thus :—

$(8{\cdot}9 + {\cdot}05)\ (7{\cdot}6 + {\cdot}05) - 8{\cdot}9 \times 7{\cdot}6$

$= 8{\cdot}9 \times 7{\cdot}6 + {\cdot}05\ (8{\cdot}9 + 7{\cdot}6) + {\cdot}0025 - 8{\cdot}9 \times 7{\cdot}6$

$= {\cdot}05 \times 16{\cdot}5 + {\cdot}0025 = {\cdot}825 + {\cdot}0025 = {\cdot}8275 = {\cdot}83$ nearly.

Model No. (11) If in question (10) 10 similar rectangles are taken and their area computed from any one of them, find the amount of error that will arise.

The error $= 10 \times {\cdot}83$ sq. in. or 8·3 sq. in.

Exercise XXI (C).

Give the following products correct to two decimal figures :—

1. $8{\cdot}34 \times 6{\cdot}25$, (2) $0{\cdot}6208 \times 3{\cdot}453$, (3) $8{\cdot}4376 \times 153{\cdot}2986$, (4) $0{\cdot}003456 \times 62{\cdot}3875$, (5) $708{\cdot}00083 \times {\cdot}0004684$, (6) $8003{\cdot}0057 \times {\cdot}000816$, (7) $101{\cdot}3845 \times 72{\cdot}0086$ and (8) $ {\cdot}005683 \times 101278{\cdot}3$.

Find the products in the following correct to the third decimal place :—

9. $0{\cdot}0007895 \times 108{\cdot}394$, (10) $(1{\cdot}3825)^2$, (11) $(1{\cdot}48496)^2$ and (12) $62{\cdot}843 \times {\cdot}008345$.

Multiply correct to the thousandths' place :—

13. $3087{\cdot}4205 \times {\cdot}00008765$, (14) $78{\cdot}6543 \times {\cdot}0000348$, (15) $6{\cdot}0008375 \times {\cdot}000483$ and (16) $1001{\cdot}3893 \times {\cdot}0000856$.

Find the value correct to four significant digits of :—

17. 86453×176549, (18) $1385{\cdot}46 \times 835{\cdot}43$, (19) $ {\cdot}000836 \times 1{\cdot}00893$, (20) $17685{\cdot}0056 \times {\cdot}00008346$, (21) 183959×68437, (22) 16483005×20038, (23) $146486 \times 3{\cdot}14156$ and (24) $ {\cdot}008956 \times 3{\cdot}4846$.

Find the value of the following :—

25. 89605×38434 to the nearest million.
26. 148305×60498 to the nearest lakh.
27. 6489×4086 to the nearest hundred thousand.
28. 743×604 to the nearest thousand.
29. $106483 \times 98{\cdot}436$ to the nearest million.

30. $108936 \times \cdot00834$ to the nearest hundred.

31. $100836 \times 1\cdot00032$ to the nearest unit.

32. $1\cdot008645 \times \cdot000845$ to the nearest thousandth.

33. Given that 1 metre $=$ 39·37 in., find to the nearest unit the number of inches in 69·23 metres.

34. If 1 m. $=$ 1·0936 yd., find to the nearest unit the number of yds. in 64·25 km.

35. A side of a square measures 43·65 metres. Find its area to the nearest sq. metre.

36. A train runs at the rate of 64·389 km. per hour. Assuming that 1 km. $=$ ·6214 of a mile, find in miles the distance run by the train in 8 hours to the nearest mile.

37. If 1 in. $=$ 2·54 cm., find to the nearest unit the number of sq. decimetres in a sq. yard.

38. The area of the Madras Presidency in sq. miles is 142,330 as surveyed in 1911 and the population per sq. mile was estimated to be 291 per sq. mile. Find the total population of the Madras Presidency correct to the nearest million.

39. The population of the Madras Presidency was estimated to be 41,405,405 in the Census of 1911. The birth rate in 1916 was calculated on the 1911 census population to be 32·54 per thousand. Find the number of births in 1916 correct to the nearest thousand.

40. The railways in India had 35,833 miles open for traffic in 1917. If the traffic receipts amounted to £1203 per mile, find the total gross earnings for the year to the nearest thousand pounds.

41. A gallon of water contains 277·274 c. in. and a c. in. of water weighs ·03616 lb. Find the weight of a gallon in avoirdupois weight to the nearest pound.

42. The number of vessels which entered in the inter-postal trade in 1916-17 was 74,642 of an average weight of 107 tons. Calculate the tonnage of the vessels to the nearest thousand tons.

43. Express a speed of 62·38 miles per hour as one of

metres per minute to the nearest unit assuming 1 yd. to be ·9144 metre.

44. Light travels at the rate of 186·330 miles per second and takes 3 years to reach us from the nearest star. Find the distance of the star from the earth correct to three significant figures.

45. The length of the tropical year is equal to 365·242218 days. Find the total number of days in 3600 tropical years correct to the nearest day.

46. The circumference of a circle is $2\pi r$ where r is the radius of the circle. The radius of the earth is 3949·791 miles. If $\pi = 3{\cdot}14159625$, find the length of the equator to the nearest thousand of a mile.

47. 1 ton = 1·3611 candies. Express 98462 tons in candies to the first three significant figures.

48. The total acreage under rice in the year 1916–17 in British India was 79,700,000 and the average yield per acre was ·4276 of a ton. Find the number of tons of rice produced that year to the nearest hundred tons.

49. If Re. 1 amounts at the end of a certain period to Re. 1·889065, find what Rs. 870-10-0 will amount to in the same period. Give the answer to the first three significant figures.

50. Two numbers 4·5 and 1·8 are correct to the first decimal place. Find their product and the limits of error therein.

51. Two numbers 3·4 and 1·8 are correct to the tenths. Find their product and limits of error.

52. The sides of a rectangle are 5·4″ and 3·2″ correct to the last significant figure. Find the difference between the least and the greatest possible area.

53. The sides of a rectangle are 6·8 cm. and 4·3 cm. correct to the nearest millimetre. Find the difference between the least and the greatest possible area.

54. The sides of the following squares are measured with a plain scale graduated to tenths of an inch. Find the

least and the greatest values of the areas :—(*a*) 2 ft. 5·5 in., (*b*) 4 ft. 0 in., (*c*) 3 ft. 0·1 in. and (*d*) 1 ft. 4·8 in.

55. Find the weight of a block of copper 364·32 c.c. in volume if each c.c. weighs 7·482 gms. as accurately as the data justify (the figures being approximate).

56. The length of a steel bar is 12·161 . . . ft. When it is heated each foot expands to a length of 1·000416 ft. What will be its length after it is heated? The figures given are approximate only.

57. The area of the horizontal surface of a sheet of water is 618·34 sq. inches. Find the total atmospheric pressure on the sheet of water if the pressure of the atmosphere is 14·706 lbs. per sq. inch. What is the degree of accuracy of your answer if the results are approximate?

58. It is estimated that 500 men will be required to construct a building and the work will last for 180 days. If the daily wages of each man are Re. 1-4-0, find the total money required for the construction. Supposing the number of men is correct to the nearest ten and there may be an error of 30 days either way in the period required for construction, find how far the answer is trustworthy.

135 (*b*). **Contracted Division.**

Model No. (12) Divide 186·483058 by 48·3205 to two places of decimals.

Standard form.

$$\frac{186{\cdot}483058}{48{\cdot}3205} = \frac{18{\cdot}6483058}{4{\cdot}83205}.$$

It is clear that the integral part of the quotient contains one digit.

∵ 2 decimal places are wanted, we should find three digits in the quotient and for correcting the third digit we require the fourth digit in the quotient. Hence four digits in the quotient should be obtained.

To get the first digit in the quotient, we proceed to take as many figures in the dividend as are necessary to make the first step in division. To get 3 more digits in the

quotient we bring down 3 figures from the dividend successively, but in the contracted division, instead of bringing down figures from the dividend we omit 3 figures successively from the right in the divisor. The reason for this process may be seen from the following :—

365)48135 We cannot divide 116 hundreds into 365 equal
365 parts.
116 Hence we convert them into tens by affixing a cypher. Then we have to divide 1160 tens into 365 equal parts and the quotient is a certain number of tens.

$$\frac{116 \times 10}{365}\text{ (tens)} = \frac{116}{\frac{365}{10}} = \frac{116}{36\text{ (roughly)}}\text{ tens.}$$

Thus it is clear that the same answer is got by cutting off 1 figure from the divisor as by bringing down a digit from the dividend which is really reducing it to the next lower denomination. Hence it is enough if 4 digits are retained in the divisor to carry on the division with the least number of digits in it in the light of the new process of curtailment of a figure at every step in division explained above.

```
        3·859
4832)18648
     14496
     ----
      4152
      3866
      ----
       286
       242
       ---
        44
        43
        --
         1
```

The quotient = 3·859. = 3·86 correct to two places.

Note.—As in contracted multiplication, we begin the multiplication with the figure struck off or rejected in the divisor to ascertain the carrying figure.

Model No. (13) Divide 3·486789 by 346·1 to five significant figures.

Standard form.

$$\frac{3{\cdot}486789}{346{\cdot}1} = \frac{{\cdot}03486789}{3{\cdot}461}.$$

It is clear that there is no integral portion in the quotient and the decimal portion begins with one zero (we cannot get another 0 in the quotient for the digits in the divisor 346 are less than 348 in the dividend). We must get 5 more places and for correcting the fifth significant figure we require another place. Thus we should get 6 decimal places excluding the zero. Consequently we have to retain 6 places in the divisor. But we have only 4 places in the divisor. To get over this difficulty (1) we may add two cyphers to the divisor thus getting 6 places needed for the work or (2) we may carry on the division as usual (i.e. without rejecting figures in the divisor) until the number of decimal places to be obtained in the quotient is one less than the number of digits in the divisor and then commence contracted division rejecting one digit from the right at each step of division.

Method (1)

$$
\begin{array}{r}
100744 \\
346\not{0}\not{0}\not{0}\overline{)348678} \\
346100 \\
\hline
2578 \\
2423 \\
\hline
155 \\
138 \\
\hline
17 \\
14 \\
\hline
3
\end{array}
$$

∴ the answer is ·010074.

Method (2)

$$
\begin{array}{r}
100744 \\
346\not{0}\overline{)348678} \\
3461 \\
\hline
2578 \\
2423 \\
\hline
155 \\
138 \\
\hline
17 \\
14 \\
\hline
3
\end{array}
$$

∴ the answer is ·010074.

Explanation for Method (2).—In the divisor there are 4 digits. One less than the number of digits in the divisor is 3. The rejection should commence when three figures have yet to be got, i.e. when 6 — 3 figures have been obtained. After the digits 1,0,0 are got in the quotient, we cut off 1 in the divisor.

Model No. (14) Divide 165·3 by 4·18 each number being correct to the last significant figure. Find the limits between which the answer should lie and how far the answer can be relied upon.

$$\text{Quotient} = \frac{\text{Dividend}}{\text{Divisor}}.$$

Maximum value of the quotient

$$= \frac{\text{Maximum value of the Dividend}}{\text{Minimum value of the Divisor}} = \frac{165{\cdot}35}{4{\cdot}175}.$$

Minimum value of the quotient

$$= \frac{\text{Minimum value of the Dividend}}{\text{Maximum value of the Divisor}} = \frac{165{\cdot}25}{4{\cdot}185}.$$

The ordinary quotient $= \frac{165{\cdot}3}{4{\cdot}18}$.

4·175) 165·35 (39·60	4·185) 165·25 (39·48	4·18) 165·3 (39·54
125 25	125 55	125 4
40 100	39 700	39 90
37 575	37 665	37 62
2 5250	2 0350	2 280
2 5050	1 6740	2 090
2000	36100	1900
	33480	1672
	2620	228

The quotient = 39·54 ± ·06.
Answer lies betwen 39·48 and 39·60.

Exercise XXI (D).

1. Divide 865·345 by 12·438 to three significant figures.
2. ,, 7·8934 by 16·342 ,, ,, ,,
3. ,, ·834952 by 156·24 ,, ,, ,,
4. ,, ·056789 by 139·28 ,, ,, ,,
5. ,, 382·4896 by 42·9876 correct to the third decimal place.
6. Divide 28·0678953 by 163·0876 to the second decimal place.
7. Divide 12·384698 by 168·0876 to the second decimal place.
8. Divide 1·0896543 by ·689763 to the second decimal place.
9. Divide 0·38465 by 0·48796 to the second decimal place.

10. Divide 1·08765 by 24·30046 to the second decimal place.

11. Divide 3 by 2·83475 to four places of decimals.

12. Divide 1 by 3·141569 to four places of decimals.

13. Find the reciprocal of 6·8438 to three places of decimals.

14. Simplify $\frac{1461}{83479}$ correct to three places of decimals.

15. Solve $x \times 8{\cdot}3476 = 6{\cdot}5379$ correct to the nearest tenth.

16. Divide 86743008 by 36428 correct to the nearest unit.

17. Divide 186786789 by 4684 correct to the nearest ten thousand.

18. Divide 489000386 by 86,439,001 to four significant figures.

19. A six-inch line measures 15·24 cm. Find the number of inches in 1 cm. correct to four places of decimals and hence find the equivalent of 1 metre in inches.

20. The circumference of a race course is two miles and a half. Calculate its radius given that the circumference of a circle = 3·141569 times the diameter to the nearest yard.

21. The velocity of electricity is $1{\cdot}864 \times 10^5$ miles per second and the distance of the sun from the earth is 92×10^6 miles. Supposing it possible to send a message to the sun, find the number of seconds it will take to reach the sun. (Answer to the nearest second.)

22. Find the density of 5065 c.c. of tin weighing 36975 gms. (Answer to the nearest tenth.)

23. If 1 metre = 39·3709 in. express a yard in metres correct to four places of decimals.

24. If 1 oz. avoirdupois = 28·35 gms. calculate the number of lbs. in 1 kg. correct to the nearest unit.

25. The net profits of a fund Rs. 78,412 are equally divided among 11,450 share-holders. How much does each share-holder get? Read off the result correct to the nearest

pie, obtaining the answer to the third decimal place by contracted division.

26. From the following statistics, find the average number of persons per sq. mile in the British territory in India in each of the years 1891, 1901 and 1911.

Year	Area in sq. miles	Population in Millions
1891	964,993	221·38
1901	1,097,901	231·61
1911	1,093,074	244·27

27. If 1 metre = 39·37 inches, find the number of metres in a mile correct to the nearest integer.

28. A metre = 39·37 inches, express a sq. ft. as a decimal of a sq. metre to the first two significant figures.

29. If 1 m. = 39·3709 inches, express a sq. km. in acres to two places of decimals.

30. 1 m. = 39·37 inches. Express a c. ft. in. litres or cubic decimetres correct to three decimal places.

31. 1 m. = 39·37 inches. A gramme is the weight of a c. cm. of water. How many grammes does a cubic foot of water weigh ?

32. If a cubic foot of water weighs 62·5 lbs., express a gramme as a decimal of a lb. correct to three decimal places using the answer in the preceding question.

33. A metre = 39·3709 inches. A gallon contains 277·274 c. inches. Find the number of litres in a gallon.

34. Prove, by contracted division that $1{\cdot}732 \times 1{\cdot}732 = 3$.

35. In the year 1916, the number of paid messages was 16,596,697 and the revenue receipts of the Telegraph

Department were £1,238,517. Find to the nearest penny the average cost of a message.

36. Assuming $\pi = 3{\cdot}14159$, find the value of $\frac{1}{\pi}$ to four significant figures by the method of contracted division.

37. Using the result obtained in question (36) for $\frac{1}{\pi}$, find the diameter of the earth if the equatorial circumference is 2,500 miles.

38. If $e = \frac{1}{2{\cdot}7182818}$, find the value of $\frac{1}{e}$, hence find the value of $e + \frac{1}{e}$ and $e - \frac{1}{e}$ to four decimal places.

39. The area of Ceylon is 16,212,000 acres. The approximate area under paddy in 1916 was 41,753 acres. Find to the nearest tenth what per cent. of the total area of Ceylon was the acreage of paddy.

40. The value of silk goods of all sorts imported from Hongkong into the United Kingdom in 1915 and 1916 was £67,876 and £58,629. Find to the nearest tenth the decrease per cent. in the exports during the latter year when compared with those in the former year.

41. The weight of a kilometre of wire is 784 gm. true to the nearest gramme. To what unit may the weight of (1) a metre and (2) a centimetre of the wire be assumed to be correct.

42. (*a*) If $x = £85{\cdot}387694$, by what should x be multiplied if all the figures are to be used to express the result correct to the nearest farthing?

(*b*) By what power of 10 should x be multiplied if the last significant figure is to be rendered useless in determining the result to the nearest farthing?

(*c*) By what power of 10 should x be divided if the last five figures are to be of no use in evaluating the result to the nearest farthing?

43. 1 cubic foot of iron weighs 556·25 lbs. correct to the last significant figure. Find the weight of 1 c. in. of the substance as correctly as you can.

44. If the cost of 1 ton of iron is Rs. 1,060 correct to the

last significant figure, find the cost of 1 lb. as correctly as the data justify.

45. If 111,000 looms produced 377,350,000 lbs. of cloth, find the average production of a loom in lbs. as correctly as the data justify, supposing that each of the figures given is correct to the last significant figure.

46. Given that 1 km. = ·6214 mile correct to the last significant figure, find with such accuracy as the data admit, the value of 1 metre in inches.

47. Given that 1 c. ft. of water weighs 62·5 lbs. and 1 c. ft. of lead weighs ·32 ton, find the specific gravity (i.e. the number of times lead weighs its own bulk of water) of lead as nearly as the data justify supposing the weights are correct to the last significant figure.

48. In Question (21), supposing each quantity to be true to the last significant figure, indicate the error to which your result is liable.

49. In Question (22) supposing each quantity to be true to the last significant figure given, indicate the error to which your result is liable.

50. A sidereal year = 365·2564 solar days correct to the last significant figure. A solar year = 365 solar days. Express a solar year as a decimal of a sidereal year not attempting at greater accuracy than the data justify.

51. A sidereal day = 86164·09967 solar seconds correct to the last significant figure. A solar day = 86400 solar seconds. Express a sidereal day in terms of a solar day as accurately as the data justify.

52. In the following questions the quantities stated are true only to the last significant digit. Obtain the quotients not proceeding beyond the first doubtful figure.

$$(a)\ \frac{26{\cdot}38}{4{\cdot}79},\quad (b)\ \frac{864{\cdot}08}{41{\cdot}03},\quad (c)\ \frac{1625{\cdot}3}{0{\cdot}0384}\ \text{and}\ (d)\ \frac{9463000}{8260}.$$

136. Repetition of the processes.

Model No. (15) Evaluate 2·384 × 48·67 × ·03698 correct to two places of decimals.

The sum = 2·384 × 48·67 × ·03698.

Standard form of 2·384 × 48·67 × ·03698

= 23·84 × 4·867 × ·03698 = ·2384 × 4·867 × 3·698.

Rough estimate = ·2 × 5 × 4 = 4 roughly.

Method I.—All the multipliers should be in the standard form. The multiplicand may be in any form. There are two multiplications involved. In the final multiplication the answer must be correct to two decimal places. So, in the first multiplication the product should be correct to three places of decimals so that, by multiplying it by a number of the standard form lying between 1 and 10, the final result may be true to two places. In the first multiplication for the product to be correct to three places we must take one or two more decimal places. Hence the working takes the following form :—

First multiplication

	×
·238	40
	4·867
·953	60
190	72
14	30
1	67
1·160	29

Final multiplication

	×
1·16	02
	3·698
3·48	06
69	61
10	44
	93
4·29	04

4·29 is the required answer.

Method II.—Since ·2384 is to be eventually multiplied by a number (12 and odd in this case) lying in value between 1 and 100 generally speaking, two more places will have to be retained beyond the required number of places, i.e. 2 + 2, i.e. 4. The multiplication may be carried on throughout to 4 decimal places and then the approximation made to two places.

	×
·23	84
	4·867
·95	36
19	07
1	43
	16
1·16	02
	3·698
3·48	06
·69	61
10	44
	93
4·29	04

∴ 4·29 is the answer.

N.B.—If three multiplications are to be made then according to Method (I) the product of the third multiplication should be correct to 2 places ; so, that of the second multiplication to 3 places ; so, that of the first to 4 places ; and for accuracy two more figures are wanted. Hence 6 places should be taken to start the operation. According to Method (II) the multiplicand is to be multiplied virtually by a number lying between 1 and 1000. In the multiplicand 3 more places should be retained beyond the number required. The multiplication may be carried on throughout to 5 places of decimals.

Model No. (16) Evaluate $1008{\cdot}962 \times 56{\cdot}43 \times {\cdot}0084$ correct to the first three significant figures.

Standard form $1008{\cdot}962 \times 56{\cdot}43 \times {\cdot}0084 = 10089{\cdot}62 \times 5{\cdot}643 \times {\cdot}0084 = 10{\cdot}08962 \times 5{\cdot}643 \times 8{\cdot}4$.

Rough estimate $= 10 \times 6 \times 8 = 480$ roughly.

Third significant figure is the units' figure. The question means the evaluation of the answer to the nearest unit.

Method I.—In order that the product of the second multiplication may be correct to the nearest unit, that of the first multiplication should be correct to the nearest tenth and for accuracy two more figures should be taken. Hence in the multiplicand retain 3 decimal places.

First multiplication.

	×
10·0	89 6
	5·643
50·4	48
6·0	53
4	03
	30
56·9	34

Second multiplication.

	×
56	·93 4
	8·4
455	·47
22	77
478	24

Hence 478 is the answer required.

Method II.—

	×
10	·089
	5·643
50	45
6	05
	40
	3
56	93
	8·4
455	44
22	77
478	21

Here 2 more places will have to be retained beyond the units' place throughout as 10·08962 is to be eventually multiplied by a number (40 and odd) lying in value between 1 and 100.

Hence 478 is the answer.

Model No. (17) Evaluate $\dfrac{81{\cdot}384 \times {\cdot}00746}{450{\cdot}381}$ correct to two significant figures.

Method I. Standard form of $\dfrac{81{\cdot}384 \times {\cdot}00746}{450{\cdot}381} = \dfrac{{\cdot}081384 \times 7{\cdot}46}{450{\cdot}381}$

$$= \frac{{\cdot}00081384 \times 7{\cdot}46}{4{\cdot}50381}.$$

Rough estimate $= \dfrac{{\cdot}0008 \times 7}{5} = \dfrac{{\cdot}0056}{5} = {\cdot}001 \ldots$

The quotient is to be evaluated correct to two significant figures. To be able to do this we should find the first three significant figures of the quotient which means the retention of three significant figures in the divisor and the evaluation of the dividend to three or four significant figures. For accuracy two more places are wanted. So the numerator should be correct to five or six significant figures. Hence the working is as follows :—

The numerator is roughly equal to $\cdot 0008 \times 7 = \cdot 0056$. ∵ there are two cyphers after the decimal point, the product will have to be worked out to seven or eight places of decimals.

	×
·00081	38 4
	7·46
·00569	69
32	55
4	88
·00607	12

134
45 ∅)607
)450
157
135
22
18
4

∴ the answer is ·00134 or ·0013 correct to two significant figures.

Method II.—As before, since there are two zeroes between the decimal point and the first significant figure, and as the answer is to be given to the first two significant figures, the result is to be found correct to four decimal places. The multiplicand ·008134 is to be eventually multiplied by (roughly say) $\frac{7}{5}$ or 1 and odd, i.e. a number between 1 and 10, so as to get the final result correct to four places. Hence five places should be retained in the multiplicand.

	×
·0008	1 38
	7·46
0056	9
3	2
	5
0060	6

134
45 ∅)606
)450
156
135
21
18
3

∴ the answer is ·00134 or ·0013.

137. 1. If x is a very small fraction then x^2, x^3, etc., are still smaller fractions and can be neglected.

e.g. $x = \frac{1}{100}$, $x^2 = \frac{1}{10000}$, $x^3 = \frac{1}{1000000}$.
i.e. $x = \cdot 01$, $x^2 = \cdot 0001$, $x^3 = \cdot 000001$.

2. If x and y are two very small fractions, then xy is smaller still and therefore xy can be neglected.

e.g. $x = \frac{1}{100}$, $y = \frac{1}{1000}$, then $xy = \frac{1}{100000}$
or $x = \cdot 01$, $y = \cdot 001$, then $xy = \cdot 00001$.

3. $\dfrac{1}{1+x}$ by ordinary division $= 1 - x + x^2 - x^3 +$ . . . .

Now by (1) and (2) x^2, x^3 . . . are negligible when x is a very small quantity.

$\therefore \dfrac{1}{1+x} = 1 - x$ when x is a very small quantity compared with 1,

e.g. $\dfrac{1}{1 \cdot 001} = \dfrac{1}{1 + \cdot 001} = 1 - \cdot 001 = \cdot 999$ roughly.

4. $\dfrac{1}{1-x}$ by ordinary division $= 1 + x + x^2 + x^3 +$ . . . .

Now x^2, x^3 . . . are negligible when x is a very small quantity compared with 1.

$\therefore \dfrac{1}{1-x} = 1 + x$ when x is a very small quantity.

e.g. $\dfrac{1}{\cdot 999} = \dfrac{1}{1 - \cdot 001} = 1 + \cdot 001 = 1 \cdot 001$ roughly.

5. $(1 + x)(1 + y) = 1 + x + y + xy = 1 + x + y$ when x and y are very small and xy may be neglected.

e.g. $(1 + \cdot 001)(1 + \cdot 002) = 1 + \cdot 001 + \cdot 002 = 1 \cdot 003$ roughly.

6. $(1 - x)(1 - y) = 1 - x - y + xy$
$= 1 - x - y$ when x and y are very small quantities compared with 1 for xy may be neglected.

e.g. $\cdot 999 \times \cdot 998 = (1 - \cdot 001)(1 - \cdot 002)$
$= 1 - \cdot 001 - \cdot 002 = 1 - \cdot 003$ roughly $= \cdot 997$ roughly.

7. $(1 + x)^n = 1 + nx = 1 + nx$ higher powers being negligible when x is a very small quantity.

Similarly $\left.\begin{aligned} (1+x)^2 &= 1 + 2x \\ (1+x)^3 &= 1 + 3x \\ (1+x)^4 &= 1 + 4x \\ (1+x)^5 &= 1 + 5x \end{aligned}\right\}$ when x is a very small quantity.

n may be fractional, then $(1 + x)^{\frac{1}{2}} = 1 + \frac{1}{2}x$.
or $(1 + x)^{\frac{1}{3}} = 1 + \frac{1}{3}x$.

e.g. $(1 \cdot 001)^2 = (1 + \cdot 001)^2 = 1 + 2 \times \cdot 001 = 1 \cdot 002$ roughly.
e.g. $(1 \cdot 001)^3 = (1 + \cdot 001)^3 = 1 + 3 \times \cdot 001 = 1 \cdot 003$ roughly.

Model No. (18) Evaluate 257×192.

Roughly $250 \times 192 = \frac{192000}{4} = 48000$ as a first approximation.

$$257 \times 192 = (250 + 7)\ 192 = 250 \times 192 + 7 \times 192$$
$$= 250 \times 192 + \frac{7 \times 192}{250} \times 250$$
$$= 250 \times 192 + \frac{28}{1000} \text{ of } 250 \times 192$$
$$= 48000 + 2{\cdot}8\% \text{ of it}$$
$$= 48000 + 48 \times 28$$
$$= 48000 + 1344$$
$$= 49344.$$

Model No. (19) Evaluate $37{\cdot}6 \times 24{\cdot}1$.

$$37{\cdot}6 \times 24{\cdot}1 = (37{\cdot}5 + {\cdot}1)\ (24 + {\cdot}1)$$
$$= 37{\cdot}5\ (1 + \tfrac{1}{375})\ 24\ (1 + \tfrac{1}{240})$$
$$= 37{\cdot}5 \times 24\ (1 + \tfrac{1}{375} + \tfrac{1}{240}) \text{ roughly.}$$
$$= 900\ (1 + {\cdot}25\% \text{ roughly} + {\cdot}4\% \text{ roughly}).$$
$$= 900 + {\cdot}65\% \text{ of } 900$$
$$= 900 + 5{\cdot}85 = 906 \text{ roughly.}$$

By actual Multiplication

$$\begin{array}{r} 376 \\ 2{\cdot}41 \\ \hline 752 \\ 150{\cdot}4 \\ 3{\cdot}76 \\ \hline 906{\cdot}16 \end{array}$$

Model No. (20) Evaluate 405×975.

$$405 \times 975 = (400 + 5)\ (1000 - 25).$$
$$= 400\ (1 + \tfrac{5}{400})\ 1000\ (1 - \tfrac{25}{1000}).$$
$$= 400000\ (1 + \tfrac{5}{400} - \tfrac{25}{1000}) \text{ roughly.}$$
$$= 400000\ (1 + 1{\cdot}25\% - 2{\cdot}5\%).$$
$$= 400000\ (1 - 1{\cdot}25\%).$$
$$= 400000 - 5000.$$
$$= 395000 \text{ roughly.}$$

$$\begin{array}{r} 975 \\ 405 \\ \hline 3900 \\ 4875 \\ \hline 394875 \end{array}$$

Model No. (21) Multiply $126 \times 88 \times 97 \times 62$.

First approximation $125 \times 90 \times 100 \times 60 = 67500000$.

$$126 \times 88 \times 97 \times 62 = 125\ (1 + \tfrac{1}{125})\ 90\ (1 - \tfrac{2}{90})\ 100\ (1 - \tfrac{3}{100})\ 60\ (1 + \tfrac{2}{60}).$$
$$= 67500000\ (1 + \tfrac{1}{125} - \tfrac{2}{90} - \tfrac{3}{100} + \tfrac{2}{60}) \text{ roughly.}$$
$$= 67500000\ (1 + {\cdot}8\% - 2{\cdot}2\% - 3\% + 3{\cdot}3\%).$$
$$= 67500000\ (1 - 1{\cdot}1\%).$$
$$= 67500000 - 742500 = 66757500.$$

Model No. (22) Evaluate approximately $\frac{684{\cdot}568}{999}$.

$$\frac{684{\cdot}568}{999} = \frac{684{\cdot}568}{1000\ (1 - \frac{1}{1000})} = \frac{684{\cdot}568}{1000}\ (1 + \tfrac{1}{1000})$$
$$= {\cdot}684568\ (1 + {\cdot}1\%) = {\cdot}684568 + {\cdot}0006 = {\cdot}6851.$$

Model No. (23) Evaluate $\frac{865 \times 763}{203 \times 387}$.

$$\frac{865 \times 763}{203 \times 387} = \frac{900 \times 800}{200 \times 400} = 9 \text{ roughly.}$$

$$\frac{865 \times 763}{203 \times 387} = \frac{900\,(1 - \frac{35}{900}) \times 800\,(1 - \frac{37}{800})}{200\,(1 + \frac{3}{200}) \times 400\,(1 - \frac{13}{400})}$$

$$= \frac{900 \times 800}{200 \times 400}\,(1 - \tfrac{35}{900})\,(1 - \tfrac{37}{800})\,(1 - \tfrac{3}{200})\,(1 + \tfrac{13}{400})$$

$= 9\,(1 - \frac{35}{900} - \frac{37}{800} - \frac{3}{200} + \frac{13}{400})$ roughly.
$= 9\,(1 - 4\% - 4\frac{1}{2}\% - 1\frac{1}{2}\% + 3\frac{1}{4}\%)$ roughly.
$= 9\,(1 - 6\frac{3}{4}\%)$.
$= 9 - 6\frac{3}{4}\%$ of $9 = 9 - \cdot6075 = 8\cdot3925$ Ans.

Model No. (24) Evaluate $\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \ldots$ correct to three decimal places.

$\frac{1}{4} = \cdot25$.

$\frac{1}{4^2} = \frac{1}{4} \div 4 = \cdot25 \div 4 = \cdot0625$.

$\frac{1}{4^3} = \frac{1}{4^2} \div 4 = \cdot0625 \div 4 = \cdot015625$.

$\frac{1}{4^4} =$ similarly $= \cdot00390625$.

$\frac{1}{4^5} =$,, $= \cdot0009765625$.

$\frac{1}{4^6} =$,, $= \cdot000244140625$.

$\frac{1}{4^7} =$,, $= \cdot00006103515625$.

$\frac{1}{4^8} =$,, $= \cdot000015258789$.

$\frac{1}{4^9} =$,, $= \cdot00000381469$.

$\cdot33333$ ∴ the answer is $\cdot333$.

R'B.A.N.M'S HIGHER SECONDARY SCHOOL BANGALORE-1.

We have to find the figure in the fourth decimal place to correct that in the third. Hence we must proceed with our work until we get zero in the fifth place as we are sure we may be getting zeroes thereafter in the fifth and sixth decimal places which will not affect the result wanted.

138. Decimalization of English and Indian money.

$1s. = £\frac{1}{20} = £\cdot 05$ (1) $\therefore x\ s. = £\ x \times \cdot 05$.

$1\ q. = \frac{1}{4}\ d. = \frac{1}{48}\ s = £\frac{1}{960} = £\frac{1}{1000} \times \frac{1000}{960} = £\cdot 001 \times (1 + \frac{40}{960}) = £\cdot 001\ (1 + \frac{1}{24})$ (2).

$$\therefore\ y \text{ farthings} = £\cdot 001\left(y + \frac{y}{24}\right).$$

From (1) and (2) any given sum consisting of shillings and pence can be expressed as a decimal of a £ without going through the ordinary process of converting a vulgar fraction into a decimal.

Model No. (25) Express $9s.\ 7\frac{1}{2}d.$ as a decimal of a £.

$9s. = £9 \times \cdot 05 = £\cdot 45$ from (1).

$7\frac{1}{2}\ d. = 30\ q. = £\cdot 001\ (30 + \frac{30}{24}) = £\cdot 001 \times 31 = £\cdot 031$ roughly.

$\therefore 9s.\ 7\frac{1}{2}d. = £\cdot 45 + £\cdot 031 = £\cdot 481.$

By actual conversion $9s.\ 7\frac{1}{2}d. = 9\frac{5}{8}s. = £\frac{77}{160} = £\cdot 48125.$

To express a decimal of a £ as shillings and pence

$$£1 = 960\ q. = 1000\left(1 - \frac{40}{1000}\right)q = 1000\left(1 - \frac{1}{25}\right)q.$$

$$\therefore\ £\ x = 1000\left(x - \frac{x}{25}\right)q = \text{(3)}.$$

Model No. (26) Convert £·876 into shillings and pence.

Take the first two decimals and see how many times ·05 is contained in the first two figures. In ·87 there $\frac{\cdot 87}{\cdot 05}$ i.e. $17s.$ and £·02 left over.

$\therefore £\cdot 876 = 17s. + £\cdot 026.$

$$= 17s. + 1000\left(\cdot 026 - \frac{\cdot 026}{25}\right)q.$$

$$= 17s. + \left(26 - \frac{26}{25}\right)q.$$

$= 17s. + 25q.$ roughly $= 17s.\ 6\frac{1}{4}d.$ nearly.

By ordinary method $£\cdot 876 = \cdot 876 \times 20s. = 17\cdot 52s. = 17s.$ and $\cdot 52s. = \cdot 52 \times 12d. = 6\cdot 24d.$

Thus we get two rules :—

(1) To express pence as a decimal of a £, convert the pence into farthings and add $\frac{1}{24}$ of itself to it. Divide this sum by 1000.

(2) To express a decimal of a £ as farthings, multiply the portion of the decimal excluding that representing an integral number of shillings by 1000 and from the product take away $\frac{1}{25}$ of itself correct to the nearest integer.

INDIAN MONEY.

8 as. = Re. ·5; 4 as. = Re. ·25; 2 as. = Re. ·125; 1 a. = Re. ·0625. . . . . (1).

1 pie = Re. $\frac{1}{192}$ = Re. $\frac{1}{200} \times \frac{200}{192}$ = Re. ·005 $(1 + \frac{8}{192})$ = Re. ·005 $(1 + \frac{1}{24})$.

$\therefore$ x pies = Re. ·005 $\left(x + \frac{x}{24}\right)$ = Re. ·001 $\left(5x + \frac{5x}{24}\right)$. . . . . (2).

From (1) and (2) a sum of money consisting of annas and pies may be expressed as a decimal of a rupee.

Model No. (27) Express 13 as. 7 ps. as a decimal of a rupee.

12 as. = 8 as. + 4 as. = Re. ·5 + Re. ·25 = Re. ·75

The remainder = 1 a. 7 ps. = 19 ps.

But x ps. = Re. ·001 $\left(5x + \frac{5x}{24}\right)$.

$\therefore$ 19 ps. = Re. ·001 $\left(5 \times 19 + \frac{5 \times 19}{24}\right)$.

= Re. ·001 $\left(95 + \frac{95}{24}\right)$ = Re. ·001 × 99 = Re. ·099.

$\therefore$ 13 as. 7 ps. = Re. ·75 + ·099 = Re. ·849.

By ordinary method 13 as. 7 ps. = $13\frac{7}{12}$ as. = Re. $\frac{163}{12 \times 16}$

(1) *By long division*

= Re. $\frac{163}{192}$.

```
1·92 ) 1·630 ( ·8489
       1536
       ----
        940
        768
        ---
        1720
        1536
        ----
         1840
         1728
         ----
```

(2) *By factors*

```
12 | 163
16 | 13·5833
     ·8489.
```

= ·8489 or ·849 correct to the third decimal place.

RULE.

To express pies as the decimal of a rupee, multiply the number of pies after deducting the greatest possible even number of annas from the sum, by 5 and add $\frac{1}{24}$ of itself to the product. Divide this sum by 1000.

Model No. (28) The atmospheric pressure on a sq. inch is the weight of a column of mercury standing on that area to a height of 29·2 inches. Given that 1 cub. ft. of water weighs 62·3 lbs. and the specific gravity of mercury is 13·6, find the atmospheric pressure in lbs. per sq. inch. to the nearest tenth of a lb.

The volume of the mercury $= 29{\cdot}2 \times 1$ sq. in. $= 29{\cdot}2$ cub. in.

The weight of 29·2 cub. in. of mercury $= \frac{29{\cdot}2}{1728} \times 62{\cdot}3 \times 13{\cdot}6$ lbs.

Standard form of $\frac{29{\cdot}2}{1728} \times 62{\cdot}3 \times 13{\cdot}6 = \frac{2920 \times 6{\cdot}23 \times 1{\cdot}36}{1728}$

$= \frac{2{\cdot}92 \times 6{\cdot}23 \times 1{\cdot}36}{1{\cdot}728}$

Rough estimate $= \frac{3 \times 6 \times 1{\cdot}4}{1{\cdot}8} = 14$ roughly.

There are two digits in the integral portion of the answer which should be correct to the first decimal place. To correct the first, we require the second. Therefore the quotient should contain 2 decimal places or four significant figures. Hence in the divisor there should be four significant figures and in the dividend four or five significant figures. The dividend $= 3 \times 6 \times 1$ or 18 which shows there are two significant digits in the integral portion of the dividend. The numerator, i.e. the product of the three numbers should be worked out correct to five significant figures, i.e. four decimal places should be retained in the multiplicand.

	×
2·920	0
	6 23
17·520	0
584	0
87	6
18·191	6
	1·36
18·191	6
5·457	5
1·091	5
24·740	6

Correct to the fifth significant figure the dividend $= 24{\cdot}741$

$$1728 \overline{)2474(} \quad 14{\cdot}32$$

1728

746

691

55

52

3

3

14·32 or 14·3 lbs.

Model No. (29) In the previous question, convert the pressure of lbs. per sq. inch into one of grammes per sq. cm. (Answer to the nearest gramme) 1 in. = 2·54 cm.; 1 c.c. of mercury weighs 13·6 gm.

The atmospheric pressure on 1 sq. inch = the weight of 29·2 c. in. of mercury.

i.e. the atmospheric pressure on $(2{\cdot}54)^2$ sq. cm. = the weight of $29{\cdot}2 \times (2{\cdot}54)^3$ c.c. of mercury.

$= 29{\cdot}2 \times (2{\cdot}54)^3 \times 13{\cdot}6$ gm.

∴ the atmospheric pressure on 1 sq. cm.

$$= \frac{29{\cdot}2 \times (2{\cdot}54)^3 \times 13{\cdot}6}{(2{\cdot}54)^2} \text{ gm.}$$

$= 29{\cdot}2 \times 2{\cdot}54 \times 13{\cdot}6$ gms.

Standard form. $[29{\cdot}2 \times 2{\cdot}54 \times 13{\cdot}6 = 292 \times 2{\cdot}54 \times 1{\cdot}36.]$

Rough estimate $= 300 \times 2{\cdot}5 \times 1{\cdot}4 = 1050$ gm.

It is clear that the integral portion of the answer contains four digits. To express the answer correct to the nearest unit, the final product should be worked out correct to the first decimal place. Hence in the multiplicand two decimal places may be retained.

	×
292	·00
	2·54
584	·00
146	00
11	68
741	68
	1·36
741	68
222	50
44	50
1008	68

∴ the answer is 1009 gm

Exercise XXI (E).

Find the following products to the first decimal figure:—

1. 6·8975 × 13·454 × ·086.
2. 18·072 × 6·453 × 0·008.
3. 48·3674 × 0·0834 × 100·48.
4. 642·3815 × ·00735 × 16·438.

Evaluate the following to two decimal places :—

5. $\dfrac{8{\cdot}785 \times 16{\cdot}403}{48{\cdot}349}$.

6. $\dfrac{1008{\cdot}346 \times {\cdot}000896}{24{\cdot}489}$.

7. $\dfrac{10{\cdot}896 \times 383{\cdot}46}{4860{\cdot}35}$.

8. $\dfrac{{\cdot}000567 \times 26{\cdot}893}{{\cdot}4183}$.

Evaluate the following correct to three significant figures :—

9. $8475 \times 1376 \times 9088$.

10. $1612 \times 1418 \times 3576$.

11. $\dfrac{16{\cdot}3843}{24{\cdot}3008 \times {\cdot}00089}$.

12. $\dfrac{484{\cdot}3856}{26{\cdot}34 \times 108{\cdot}456}$.

13. $\dfrac{148{\cdot}245 \times 98{\cdot}349}{8{\cdot}6439 \times 4{\cdot}035}$.

14. $\dfrac{16{\cdot}598 \times 49{\cdot}008}{2{\cdot}346 \times 3{\cdot}489}$.

Solve the following equations giving the root to two places of decimals :—

15. $\dfrac{x}{99{\cdot}085} = 6{\cdot}857 \times 7{\cdot}487$.

16. $\dfrac{\dfrac{x}{16{\cdot}483}}{1{\cdot}0084} = {\cdot}0053$.

17. $x \times 100{\cdot}846 = 46783 \times 5{\cdot}6743$.

18. $\dfrac{x}{1{\cdot}0086} \times 3{\cdot}485 = \dfrac{1}{6{\cdot}487}$.

Evaluate :—

19. $\dfrac{843}{346} \times 21 \times 10^6$ (to the first four significant figures).

20. $\dfrac{{\cdot}00875 \times 48 \times 10^8}{64 \times {\cdot}008647}$ (to the first three significant figures).

21. $\dfrac{1}{2} + \dfrac{1}{2^2} + \dfrac{1}{2^3} + \ldots$ to three places of decimals.

22. $\dfrac{1}{3} + \dfrac{1}{3^2} + \dfrac{1}{3^3} + \ldots$ to four places of decimals.

23. $1 + \dfrac{1}{4} + \dfrac{1}{4{\cdot}6} + \dfrac{1}{4{\cdot}6{\cdot}8} + \dfrac{1}{4{\cdot}6{\cdot}8{\cdot}10} + \ldots$ to three places of decimals.

24. $\dfrac{1}{7} + \dfrac{1}{7^2} + \dfrac{1}{7^3} + \dfrac{1}{7^4} + \ldots$ to four decimal places.

Express, correct to the nearest pie or penny as the case may be, as a compound quantity :—

25. Rs. 3·496, (26) Rs. 4·708, (27) Rs. 16·256, (28) Rs. 39·024, (29) £5·705, (30) £6·098, (31) £4·768 and (32) £864·034.

Express the following as decimals of a £ or a rupee as the case may be :—

33. £19-17-11, (34) Rs. 530-15-6, (35) £20-13-10, (36) Rs. 438-14-11, (37) £4-11-10½, (38) Rs. 16-7-10, (39) £0-8-11¼, (40) Rs. 8-5-10.

41. Convert a velocity of 218600 yds. per 3840 minutes into one of miles per hour correct to two significant figures.

42. If $\pi = 3{\cdot}1416$, find the value of (1) π^2 and (2) π^3 correct to four significant figures.

43. The radius of the earth is 3957 miles. Assuming the earth is a sphere and the surface of a sphere $= \frac{4}{3}\pi r^2$ where $\pi = 3{\cdot}14159$, find the number of square miles on the earth correct to the nearest thousand.

44. The mass of the earth is $5{\cdot}87 \times 10^{21}$ tons. The mass of the moon is $\frac{1}{81{\cdot}5}$ of that of the earth. Express the mass of the moon in tons correct to two significant figures.

45. The mass of the sun is $3{\cdot}3 \times 10^5$ times that of the earth. Find the mass of the sun correct to three significant figures if the mass of the earth is the same as in question (44).

46. The distance of the earth from the sun $= 9{\cdot}28 \times 10^7$ miles. The radius of the earth $=$ 3960 miles. These figures are correct to the last significant figures stated. Find the distance of the earth in terms of the radius of the earth as accurately as the data justify.

47. The distance of the earth from the sun is $9{\cdot}28 \times 10^7$ miles. Assuming that the orbit of the earth is a circle and that the earth takes 365·25 days to make a revolution round the sun, find how many miles it moves per second in its orbit to the nearest unit. $\pi = 3{\cdot}1416$.

48. The distance of the moon from the earth $= 2{\cdot}39 \times 10^5$ miles. If the moon takes $27\frac{1}{4}$ days to revolve round the earth in its orbit, find its velocity in miles per second, to two decimal places, assuming the orbit to be a circle and that $\pi = 3{\cdot}14159$.

49. Find the approximate values of the following :—

(a) $1{\cdot}002 \times 1{\cdot}003$; (b) $1{\cdot}002 \times 1{\cdot}003 \times 1{\cdot}004$; (c) $1{\cdot}006 \times 1{\cdot}007$; (d) $6{\cdot}03 \times 1{\cdot}006$; (e) $\frac{1}{999} \times 1{\cdot}007 \times 1{\cdot}009$; (f) $\frac{1}{1{\cdot}001} \times 1{\cdot}008 \times 1{\cdot}006$; (g) $(1{\cdot}003)^2 \times 1{\cdot}02$; (h) $\frac{1{\cdot}008 \times (1{\cdot}007)^6}{1{\cdot}002}$; (k) $(1{\cdot}013)^4 \times (1{\cdot}036)^3$; (l) $\frac{1}{998} \times \frac{1}{1002} \times 997 \times 999$; (m) $\cdot 9998 \times 1{\cdot}993 \times 1{\cdot}93$; (n) $\cdot 999 \times \cdot 998 \times \cdot 997 \times 1{\cdot}002$.

50. Decimalize approximately :—

(a) $\frac{768}{102}$; (b) $\frac{864}{104}$; (c) $\frac{5478}{202}$; (d) $\frac{685}{97}$; (e) $\frac{1768}{98}$; (f) $\frac{1620}{998}$; (g) $\frac{1620}{1002}$; (h) $\frac{968}{692}$; (i) $\frac{856}{248}$; (j) $\frac{980}{(104)\ (101)}$; (k) $\frac{768 \times 961}{304 \times 407}$; (l) $\frac{4{\cdot}8}{(8{\cdot}9)^3}$; (m) $\frac{39{\cdot}8}{34{\cdot}7}$; (n) $\frac{1}{(1{\cdot}03)^4}$.

(p) Show that $\frac{1}{999} = \frac{1}{1000} + \frac{1}{1000^2}$ and hence find the value of $\frac{1}{999}$ of Rs. 12,345 correct to the nearest pie.

(q) Show that $\frac{1}{1003} = \frac{1}{1000} - \frac{3}{(1000)^2}$ and hence find the value of $\frac{1}{1003}$ of £57897 to the nearest penny.

51. $\pi = h\ \rho g$. Evaluate π to four significant figures if $h = 76$, $\rho = 13{\cdot}596$ and $g = 981$.

52. $W = h\ \rho g dv$. Evalute W to three significant figures if $h = 76{\cdot}5$, $\rho = 13{\cdot}596$, $g = 981$ and $dv = 44600$.

53. $d' = d(1 + \alpha)(1 + \beta)$. Evaluate d' if $d = 15{\cdot}24$, $\alpha = {\cdot}00057$, $\beta = {\cdot}00018$ by approximate methods.

54. $d' = d\dfrac{1 + \alpha}{1 + \beta}$. Evaluate d' if $d = 15{\cdot}24$, $\alpha = {\cdot}00057$, $\beta = {\cdot}00018$ by approximate methods.

55. The total number of officers in Government service drawing Rs. 200 and over in 1913 was 11064 and consisted of Europeans, Anglo-Indians, Indians and Burmans. The percentage of Europeans was 44 per cent., that of Anglo-Indians 14 per cent., to the nearest integer in each case. Find the number of Indians and Burmans to as many figures as are reliable.

56. Calculate the values of tea plucked in India during 1915–16 from the table :—

	Average value per lb. to the nearest anna.
Tea from Assam 359,000 lbs. ...	Re. 1-2-0.
,, from Madras 24,000 lbs. ...	Re. 0-15-0.

57. Work out question 56 assuming the lbs. are given to the nearest thousand pounds.

58. Work the following questions as shortly as you can :—

(*a*) Find the cost of 64,487 pencils at 1 a. 4 ps. each.

(*b*) ,, ,, 4896 dozens at $7\frac{1}{2}$ pies each.

(*c*) Evaluate $88{\cdot}29 \times 2{\cdot}125 + 1{\cdot}71 \times 2\frac{1}{8}$.

(*d*) ,, $8{\cdot}43598 \times 5$ as. 4 ps. correct to the nearest anna.

(*e*) Find the total cost of 96 lbs. of tea at Re. 0-10-3 per lb. and 48 lbs. of coffee at Re. 1-11-6 per lb.

59. Do not work the following questions but give a reason for knowing that the answers proposed are wrong :—

(*a*) Multiply 687 by 485. Ans. 333095.

(*b*) ,, 927 by 386. Ans. 357722.

(*c*) ,, $15\frac{3}{16}$ by $2\frac{2}{3}\frac{3}{6}$. Ans. 36.

(*d*) ,, 4·75 by 8·33. Ans. 3·95675.

(*e*) The product of two numbers 268 and 364 = the product of x and 468. Ans. 260 and odd.

(*f*) Which is the greater 8 sq. ft. or a sq. metre? Ans. 8 sq. ft.

60. The area of an ellipse is πab sq. miles where a and b are semi-major and semi-minor axes. Evaluate πab when $\pi = 3{\cdot}1416$, $a = 9{\cdot}28 \times 10^7$ miles and $b = 8{\cdot}63 \times 10^7$ miles, to the first four significant figures. π, a and b are each correct to the last significant figure stated.

61. Taking Re. 1 = 1*s.* 5*d.* express the value of (*a*) Rs. 1,018-5-4; (*b*) Rs. 10,018-5-4; (*c*) Rs. 1,00,018-5-4; (*d*) Rs. 1,00,18,500-5-4 to the nearest penny.

62. Find the limits to the possible weights in gm. of a block of lead: Length 15·2 cm., breadth 15·2 cm., thickness 6·1 cm. and sp. gr. 11·2 stating the answer to the units' place only (each is correct to the last significant figure stated).

63. A mass of metal is weighed and found to be 133·26 gm. Its length measures 22·2 cm. and breadth 11·1 cm. Its specific gravity is 7·4. Find the limits to its thickness in cm.

64. Use the formula $\text{s.g.} = \frac{W}{W - w}$ where W is the wt. in air, w = wt. in water of a substance, to find the s.g. when W = 719·483 . . . . gm. and w = 592·876 . . . . gm., attempting no more degree of accuracy than is warranted by the data.

65. If a barometer reads r inches at a place, the altitude of the place above sea-level is roughly equal to $\frac{30 - r}{30 + r} \times$ 52428 ft. For small elevations it is said a fall of one inch in the height of the barometer corresponds to a rise of 900 ft. in the altitude of the barometer. Find the percentage of error in the latter empirical rule.

66. If π radians = 180°, where $\pi = 3{\cdot}1416$ true to the last significant figure, find the value of a radian in degrees, minutes, and seconds as accurately as the data justify.

67. Show that a sum of money has an error less than a farthing if it is given correct to three places of decimals and has no error if given in rupees correct to three places of decimals.

68. Show that if a length is given in miles correct to four places of decimals, the maximum error is less than a foot and if given correct to five places of decimals the maximum error is less than an inch.

69. To how many places of decimals of a ton should a weight be expressed that the maximum error may be (1) less than a lb., (2) an oz. and (3) a grain?

70. Correct to how many places of decimals of a lb. (apothecary's) must a weight be expressed so that the maximum error may be less than a dram?

71. Correct to how many places of decimals of a year must a length of time be expressed so that the maximum error may be less than (1) a day, (2) an hour and (3) a minute?

72. The weights 6897 gm., 68·97 gm., 6·897 are each correct to four significant figures. Show that the maximum relative error is the same in each case.

73. A certain quantity lies in value between $\frac{8678}{2596\frac{1}{3}}$ and $\frac{9873}{3614\frac{3}{4}}$. To how many significant figures can you find the true value of the quantity from these values?

74. A heap consists of worn out rupees. A rupee may not weigh more than 179·5 grains and may not weigh less than 178·5 grains. The heap weighs 18 lbs. troy to the nearest grain. Find the greatest and the least possible number of rupees in the heap.

75. A distance is measured with a chain supposed to be 22 yds. long. If the chain is an inch too short, find in inches the error in measuring 3·5 miles with this chain. To how many significant figures will the result given in yds. be correct?

76. Assuming the radius of the earth is 3960 miles (correct to tens of miles) and that of the moon is 1080 miles (correct to tens of miles), compare their volumes and give the answer to five significant figures. Suppose the volume of a sphere varies as its cube.

77. The undermentioned formula gives the cub. contents of round timber in the trunk of a tree : $\frac{1}{144} \times L \times \left\{ \frac{1}{4} \times \pi \frac{(d_1 + d_2)}{2} \right\}^2$ cub ft. where L is the length of the timber in ft., d_1 and d_2 are the diameters of the ends of the trunk in ft. Evaluate it when $L = 79{\cdot}3$ ft., $d_1 = 43{\cdot}3$ ft., $d_2 = 18{\cdot}2$ ft. and $\pi = 3{\cdot}1416$. Answer correct to the nearest unit.

78. Find the value of g from the formula $g = \pi^2 l$. If $\pi = 3{\cdot}14159$ and $l = 99{\cdot}610$ cm., find the value of g correct to two places of decimals.

79. Find l in the above formula if $g = 978{\cdot}29$ and $\pi = 3{\cdot}14159$ correct to three places of decimals.

80. A big mango garden owned by A is to be leased to B for three years. The amount of the annual payment is to be settled by statistical information. Both are satisfied that the garden yields 75,000 mangoes and they are sold at Rs. 25 per thousand. Each figure is true to the last significant figure stated. They agree that the annual sum may be taken as the average of the two limits between which the sale proceeds lie. What difference does it make if the estimated income is calculated on the above figures and prescribed as the annual amount of the lease ?

CHAPTER XXII.

Square Root.

139. When a number is multiplied by itself the product is the **square** of that number and the number itself is said

to be the **square root** of the product. $2 \times 2 = 4$; $3 \times 3 = 9$; $5 \times 5 = 25$. The square root of 4 is 2, of 9 is 3 and of 25 is 5.

The square root of 4 is 2 Symbolically $\sqrt{4} = 2$.
,, ,, 9 is 3. ,, $\sqrt{9} = 3$.
,, ,, 25 is 5. ,, $\sqrt{25} = 5$.
,, ,, a^2 is a. ,, $\sqrt{a^2} = a$.
Similarly $\sqrt{a} = a^{\frac{1}{2}}$.

The square root of a number which is a perfect square can be found if it has factors, by breaking it up into its primary factors. **This method is called the extraction of square root by the method of factors.**

Model No. (1) Extract the square root of 441 by means of factors.

$441 = 3^2 \times 7^2$.
$\therefore \sqrt{441} = \sqrt{3^2 \times 7^2} = \sqrt{3^2} \times \sqrt{7^2}$.
$\therefore \sqrt{441} = 3 \times 7 = 21$.
Thus 21 is the square root of 441.

3|441
3|147
7|49
7

Model No. (2) Extract the square root of 625 by means of factors.

$625 = 5^4$
$\therefore \sqrt{625} = \sqrt{5^4} = 5^2 = 25$

5|625
5|125
5|25
5

Model No. (3) Find the square root of $a^4b^2c^2$.

$\sqrt{a^4b^2c^2} = \sqrt{a^4} \times \sqrt{b^2} \times \sqrt{c^2} = a^2.b.c. = a^2\,bc$.

140. When a product consists of three factors each equal to one another, each factor is called the cube root of the number; similarly when a product consists of four factors each equal to one another, each factor is called the fourth root and so on.

$2 \times 2 \times 2 = 8$ $\therefore$ 2 is the cube root of 8 or $2 = \sqrt[3]{8}$.
$3 \times 3 \times 3 = 27$ $\therefore$ 3 ,, ,, of 27 or $3 = \sqrt[3]{27}$.
$a \times a \times a = a^3$ $\therefore$ a ,, ,, of a^3 or $a = \sqrt[3]{a^3}$.

Similarly $2 \times 2 \times 2 \times 2 = 16$ $\therefore$ 2 is the fourth root of 16 or $2 = \sqrt[4]{16}$.

Similarly $a \times a \times a \times a = a^4$ $\therefore$ a is the fourth root of a^4 or $a = \sqrt[4]{a^4}$.

Exercise XXII (A).

Read off the square roots of :—

1. 4, 9, 16, 25, 36, 49, 64, 81, 100, 144, 169, 196.
2. $2^2 \times 3^2$.
3. $3^2 \times 4^2$.
4. $4^2 \times 5^2$.
5. $5^2 \times 6^2$.
6. $2^2 \times 4^2 \times 3^2$.
7. $2^2 \times 6^2 \times 8^2$.
8. $2^2 \times 9^2 \times 10^2$.
9. $4^2 \times 6^4$.
10. $3^4 \times 8^4$.
11. $3^4 \times 6^4 \times 8^2$.
12. $25a^4 \times b^4 \times c^2$.
13. $36a^6b^4c^2$.
14. $49x^2y^4z^2$.
15. $100x^6y^6z^4$.
16. $25m^4p^4q^4r^2$.

Break up into prime factors, determine which of the following are perfect squares and find their square roots :—

17. 225.
18. 289.
19. 324.
20. 361.
21. 484.
22. 576.
23. 625.
24. 676.
25. 1428.
26. 2304.
27. 2916.
28. 3136.
29. 4096.
30. 5184.
31. 7056.
32. 14404.
33. 9216.

Find the least number by which the following numbers must be multiplied so that the product in each case may be a square number :—

34. 412.
35. 640.
36. 848.
37. 436.
38. 1028.
39. 1064.
40. 6080.
41. 12120.
42. 1976.

43. Read off the cube root of :—

8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728, 2197.

44. Find cube roots of the following numbers by means of factors :—

15625, 262144, 373248, 884736.

45. Show that any square number ending in 1, 4, 9, 6, 5, 00 must have a square root ending in 1 or 9, 2 or 8, 3 or 7, 4 or 6, 5, 0 respectively.

141. The method of evolution or extracting the square root of large numbers whose square root cannot be easily

got by the method of factors will be taught here and the following preliminary principles should first be learnt :—

(*a*) $1^2 = 1$.
$10^2 = 100$.
$100^2 = 10000$.
$1000^2 = 1000000$.

(*b*) Conversely $\sqrt{1} = 1$.
$\sqrt{100} = 10$.
$\sqrt{10000} = 100$.
$\sqrt{1000000} = 1000$.

It is clear (1) that the square of any number lying between 1 and 10 (i.e. containing 1 digit) lies between 1 and 100 (i.e. contains 1 or 2 digits for it lies between 1 and 100);

(2) that the square of any number lying between 10 and 100 (containing 2 digits) lies between 100 and 10000 (contains 3 or 4 digits for it lies between 100 and 10000);

(3) that the square of any number lying between 100 and 1000 (i.e. containing 3 digits) lies between 10000 and 1000000 (contains 5 or 6 digits for it lies between 10000 and 1000000) and so on. In general the square of a number of n digits contains $2n - 1$ or $2n$ digits.

The converse of the above principles is also true, i.e. the square root of a number of 1 or 2 digits consists of 1 digit.
,, ,, ,, 3 or 4 ,, ,, 2 digits.
,, ,, ,, 5 or 6 ,, ,, 3 digits.

In general, the square root of a number of $2n - 1$ or $2n$ digits consists of n digits. From these principles it can be easily judged by merely looking at the number how many digits there are in its square root, e.g., 16859 consists of 5 digits. A number of 5 or 6 digits has 3 digits in its square root according to the law enunciated. Hence divide the given number into periods placing a dot over the figure in the units' place and over every alternate figure to the left if it is purely an integer, to the right if it is purely a decimal, to both left and right if it contains integral and decimal portions.

Model No. (4) Extract the square root of 625.

$625 = \dot{6}\dot{2}\dot{5}$.

It is clear that there are two digits in the square root.

625 lies between two square numbers 400 and 900.

$\therefore \sqrt{625}$ lies between 20 and 30.

$\therefore \sqrt{625} = 20 +$ a number of 1 digit.

$= (a + b)$ say, where a is 20, and b is to be found.

$\therefore 625 = (a + b)^2 = a^2 + 2ab + b^2$.

$= 20^2 + 2.20.b + b^2$.

$\therefore 625 - 20^2 = 40b + b^2$.

Or $225 = 40b + b^2$.....(1).

b is a number of 1 digit and should be 9 or less than 9. The value of b satisfying the equation (1) can be seen to be 5 or got by dividing 225 by 40.

In general if $N = (a + b)^2 = a^2 + 2ab + b^2$.

$N - a^2 = 2ab + b^2$.

If b is small when compared with a, $N - a^2 = 2ab$, b^2 being neglected. $\therefore \dfrac{N - a^2}{2a} = b$. Thus b is found by subtracting a^2 from N and dividing it by $2a$; but since $N - a^2 = 2ab + b^2 = b(2a + b)$, b should be added to $2a$ and the sum multiplied by b.

Graphical illustration—

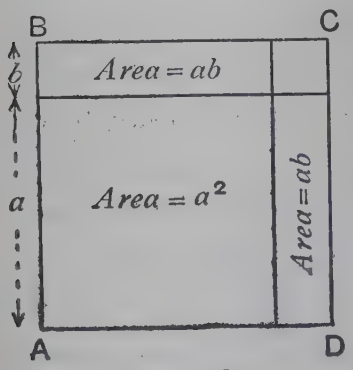


FIG. 139.

Suppose the square ABCD represents $(a + b)^2$ and the figure or square marked a^2 is very nearly equal to it so that a is a sort of conjecture to the square root of $(a + b)^2$. We have therefore to find another small quantity b which has to be added to a to obtain the required square root. From the figure it is clear that what is required is a number or quantity b such that twice the product of a and b together with the square of b is equal to the difference between the square of $(a + b)$ and the square of a.

Further $(a + b)^2 - a^2$ is very nearly $2ab$ and therefore b is got

by dividing $\frac{(a+b)^2 - a^2}{2a}$. This leads to the arithmetical method of extracting square root.

Contracted form.

$$\begin{array}{l|l} a^2 + 2ab + b^2 & a + b \\ a^2 & \\ \hline 2a + b \mid 2ab + b^2 & \\ \quad 2ab + b^2 & \\ \hline \end{array}$$

$$\begin{array}{l|l} \dot{6}25 & 20 + 5 \\ 400 & \\ \hline 40 + 5 \mid 200 + 25 & \\ \quad 200 + 25 & \\ \hline \end{array}$$

$$\begin{array}{l|l} 625 & 25 \\ 4 & \\ \hline 45 \mid 225 & \\ \quad 225 & \\ \hline \end{array}$$

The pupil can easily see that the following steps of procedure are adopted in the process of the extraction of square root :—

i. Divide the number into periods commencing with a dot over the units' place.

ii. Take the first period (consisting of one figure with a dot over it in some cases or of two figures the dot being placed over the second of these) and find the greatest digit whose square does not exceed the first period. Write that digit in the square root (say a). Put down the square of that digit (a^2) under the first period and subtract ; bring down the next period and tack it on to the remainder.

iii. Multiply the digit in the square root by 2 ($2a$) and put it in the left hand column which will be the trial divisor. Omit the last figure in the remainder (corresponding to neglecting b^2) and put the quotient ($+ b$) in the square root and next to the trial divisor also which is now ($2a + b$).

iv. $2a + b$ now becomes the real divisor and is to be multiplied by b. The product should be entered under the remainder and subtracted. If there is no remainder the process has come to an end. If there is a remainder, we bring down the next period and continue the process until all the periods are used.

Model No. (5) Extract the square root of 389376.

Contracted form.

$$\begin{array}{l|l} \dot{3}8\dot{9}3\dot{7}6 & 6(00) + 2(0) + 4 \\ 360000 & \\ \hline 12(00) + 2(0) \mid 29376 & \\ \quad 24000 + 400 & \\ \hline 12(00) + 4(0) + 4 \mid 4976 & \\ \quad 4800 + 16(0) + 16 & \\ \hline \end{array}$$

$$\begin{array}{l|l} 389376 & 624 \\ 36 & \\ \hline 122 \mid 293 & \\ \quad 244 & \\ \hline 1244 \mid 4976 & \\ \quad 4976 & \\ \hline \end{array}$$

Model No. (6) Extract the square root of 104·6529.

$$
\begin{array}{r|l}
\dot{1}0\dot{4}\cdot\dot{6}5\dot{2}\dot{9} & 10\cdot23 \\
1 & \\
\hline
202 \;|\; 465 & \\
\quad\;\; 404 & \\
\hline
2043 \;|\; 6129 & \\
\quad\;\; 6129 & \\
\hline
\end{array}
$$

Model No. (7) Extract the square root of 5 correct to three places of decimals.

It is clear that there is only one digit in the integral portion of the square root. To evaluate the result to three decimal places, a fourth decimal place is also required. Thus we should get five digits in the square root. We must therefore have five periods, one in the integral portion and four in the decimal. Hence we affix eight cyphers to get four periods.

$$
\begin{array}{r|l}
5\cdot0\,\dot{0}\dot{0}\dot{0}\dot{0}\dot{0}\dot{0}\dot{0} & 2\cdot2360 \\
4 & \\
\hline
42 \;|\; 1\cdot00 & \\
\quad\;\; 84 & \\
\hline
443 \;|\; 1600 & \\
\quad\;\; 1329 & \\
\hline
4466 \;|\; 27100 & \\
\quad\;\; 26796 & \\
\hline
4472 \;|\; 30400 & \\
\hline
\end{array}
$$

$\therefore$ the required square root is 2·236.

Model No. (8) Find the square root of (*a*) $\frac{169}{225}$ (*b*) $4\frac{483}{4800}$ (*c*) $5\frac{1}{6}$.

(*a*) Reduce the vulgar fraction to its lowest terms. Convert it into an improper fraction if it is a mixed number and extract the square root of the numerator and denominator if both are perfect squares; if not convert it into a decimal and then extract the root.

$$(a)\quad \sqrt{\frac{169}{225}} = \frac{\sqrt{169}}{\sqrt{225}} = \frac{13}{15}\,; \quad (b)\quad 4\frac{483}{4800} = 4\frac{161}{1600} = \frac{6561}{1600}.$$

$$\therefore \sqrt{4\frac{483}{4800}} = \sqrt{\frac{6561}{1600}} = \frac{\sqrt{6561}}{\sqrt{1600}} = \frac{81}{40} = 2\frac{1}{40}.$$

(*c*) $5\frac{1}{6}$ is in its lowest terms and it is not a complete square.

$5\frac{1}{6} = 5\cdot\dot{1}\dot{6}$

$$
\begin{array}{r|l}
\dot{5}\cdot\dot{1}6666666 & 2\cdot2730 \\
4 & \\
\hline
4\cdot2 \;|\; 1\cdot16 & \\
\quad\;\; |\;\; 84 & \\
\hline
447 \;|\; 3266 & \\
\quad\;\; |\;\; 3129 & \\
\hline
4543 \;|\; 13766 & \\
\quad\;\; |\;\; 13629 & \\
\hline
4546 \;|\; 13766 &
\end{array}
$$

The answer correct to three places of decimals is 2·273.

142. Surds. If the root of a number cannot be exactly obtained, the root is called a **surd or an irrational number.** $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{8}$ are surds and said to be incommensurable because the decimal part of such quantities cannot be accurately expressed as a fraction of the unit but can be approximately evaluated to any required degree of accuracy (as has been done in Model No. 7); but it may be represented by a straight line of finite length.

Model No. (9) Find the value of $\frac{4}{\sqrt{2}}$.

$\sqrt{2}$ can be found to be equal to 1·414 by the ordinary process of extraction of square root. If we substitute the value of $\sqrt{2}$ in $\frac{4}{\sqrt{2}}$ we get $\frac{4}{1\cdot414}$. We have to divide either employing long or contracted division.

$$
\begin{array}{r|l}
2\cdot829 & \\
1\cdot\not4\not1\not4\;)\,4000\,(\; & \\
\quad 2828\;(& \\
\hline
\quad 1172 & \\
\quad 1131 & \\
\hline
\quad\;\; 41 & \\
\quad\;\; 28 & \\
\hline
\quad\;\; 13 & \\
\quad\;\; 13 & \\
\hline
\end{array}
$$

To avoid this division we proceed thus :

$$\frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2}$$

$$= \frac{4 \times 1\cdot414}{2} = \frac{5\cdot656}{2} = 2\cdot828 = 2\cdot83.$$

We see that the result got by division is not so accurate as that got by the other method.

$2\cdot829 = 2\cdot83$ to two places of decimals.

Model No. (10) Evaluate $\frac{1}{2-\sqrt{3}}$.

$$\frac{1}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{2+\sqrt{3}}{2^2-(\sqrt{3})^2} = \frac{2+\sqrt{3}}{4-3} =$$
$$\frac{2+\sqrt{3}}{1} = 2 + 1{\cdot}732 = 3{\cdot}732.$$

Model No. (11) Evaluate $\frac{\sqrt{5}+1}{\sqrt{5}-1}$.

$$\frac{\sqrt{5}+1}{\sqrt{5}-1} = \frac{(\sqrt{5}+1)(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)} = \frac{(\sqrt{5}+1)^2}{(\sqrt{5})^2-1^2} = \frac{5+1+2.\sqrt{5}}{5-1} =$$
$$\frac{6+2\sqrt{5}}{4} = \frac{6+2(2{\cdot}236\ldots)}{4} = \frac{6+4{\cdot}472}{4} = \frac{10{\cdot}472}{4} = 2{\cdot}618\ldots$$

When the denominator of a fraction is freed from surds, we are said to **rationalize the denominator.** Thus in each of the Models 9, 10 and 11 we have rationalized the denominator. If the denominator is of the form $a+b$, or $a-b$ we multiply both the numerator and denominator by its conjugate, viz. $a-b$, or $a+b$ as the case may be. In the process of rationalization and simplification, the following formulæ will be very useful:

1. $\sqrt{a^2} = a$.

2. $\sqrt{a}.\sqrt{m}.\sqrt{n} = \sqrt{amn}$; also $\sqrt{a^2b} = \sqrt{a^2} \times \sqrt{b} = a\sqrt{b}$.

3. $(\sqrt{a}+\sqrt{b})^2 = (\sqrt{a})^2 + (\sqrt{b})^2 + 2.\sqrt{a}.\sqrt{b} = a+b+2\sqrt{ab}$.

4. $(\sqrt{a}-\sqrt{b})^2 = (\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{a}.\sqrt{b} = a+b-2\sqrt{ab}$.

5. $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a-b$.

Model No. (12).

Evaluate $\sqrt{45}+\sqrt{20}-\sqrt{245}+\sqrt{320}$.

The sum $= \sqrt{9\times5}+\sqrt{4\times5}-\sqrt{49\times5}+\sqrt{64\times5}$.

$= \sqrt{9} \times \sqrt{5} + \sqrt{4} \times \sqrt{5} - \sqrt{49} \times \sqrt{5} + \sqrt{64} \times \sqrt{5}.$

$= 3\sqrt{5} + 2\sqrt{5} - 7\sqrt{5} + 8\sqrt{5} = 6\sqrt{5} = 6 \times 2{\cdot}236 = 13{\cdot}416.$

143. Geometrical representation of surds.

Draw lines to represent $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{8}$, $\sqrt{10}$, $\sqrt{11}$, $\sqrt{12}$, etc.

Method (1) By making use of the Theorem of Pythagoras as in the adjoining figure.

Let OA = 1 unit, let OB = 1 unit drawn at right angles to OB; BC is perpendicular to AB and equal to 1 unit. CD is perpendicular to AC and equal to 1 unit and so on. Then $AB^2 = OA^2 + OB^2 = 1^2 + 1^2 = 2$ $\therefore$ AB $= \sqrt{2}$. Again $AC^2 = AB^2 + BC^2 = (\sqrt{2})^2 + 1^2 = 2 + 1 = 3$. $\therefore$ AC $= \sqrt{3}$.

$AD^2 = AC^2 + CD^2 = (\sqrt{3})^2 + 1^2 = 3 + 1 = 4$. $\therefore$ AD $= \sqrt{4} = 2$

$AE^2 = AD^2 + DE^2 = (\sqrt{4})^2 + 1^2 = 4 + 1 = 5$. $\therefore$ AE $= \sqrt{5}$.

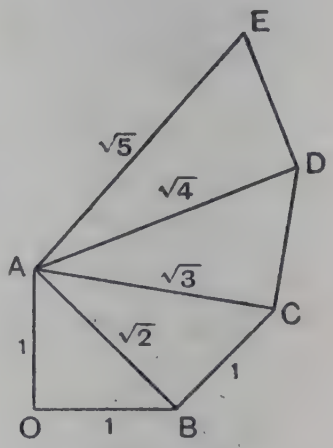


FIG. 140.

This method is very tedious as we have to construct many right-angled triangles, the number of triangles increasing as the number whose square root is to be got increases in value. Hence in cases of higher numbers such as 5, 7, 8, 10, etc., break up each into two numbers either both of which are perfect squares or at least one is a perfect square.

$5 = 4 + 1$ a right-angled triangle may be constructed as in Fig. 141.

$7 = 4 + 3$ a right-angled triangle may be constructed as in Fig. 142.

$8 = 4 + 4$ a right-angled triangle may be constructed as in Fig. 143

$10 = 9 + 1$ a right-angled triangle may be constructed as in Fig. 144.

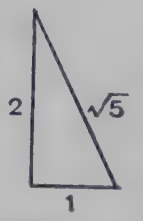


FIG. 141.

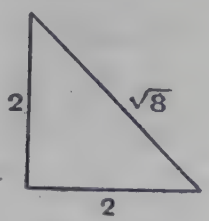


FIG. 143.

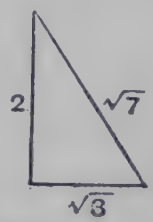


FIG. 142.

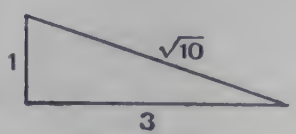


FIG. 144.

Method (2) The number whose root is to be found may be expressed as the difference of two square numbers also.

$12 = 16 - 4$. Hence a right-angled triangle may be constructed with the hypotenuse measuring 4 units and one of its sides measuring 2 units. Then third side must be $\sqrt{16-4}$ or $\sqrt{12}$. (*Vide* fig. 145).

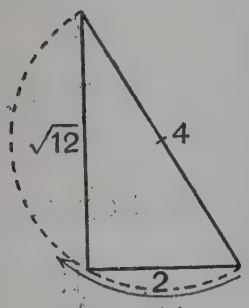


FIG. 145.

The general method is based on this relation:—

$$\left(\frac{n+1}{2}\right)^2 - \left(\frac{n-1}{2}\right)^2 = \frac{n^2+2n+1}{4}$$

$$-\frac{n^2-2n+1}{4} = \frac{4n}{4} = n$$

$$\therefore \left(\frac{n+1}{2}\right)^2 - \left(\frac{n-1}{2}\right)^2 = (\sqrt{n})^2$$

(*a*) Draw a line AB equal to $\frac{n+1}{2}$ units. On AB describe a semi-circle. With A as centre and a radius equal to $\frac{n-1}{2}$ units describe an arc cutting the semi-circle at P. Join PB. Measure the angle APB. It is 90° (for an angle in a semi-circle is a right angle which may be learned by varying the position of the point P on the circumference and measuring the ∠ APB in each case).

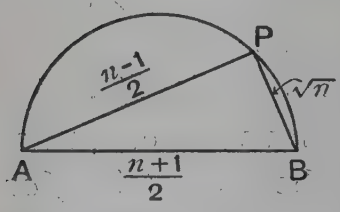


FIG. 146.

$$\therefore AP^2 + PB^2 = AB^2$$

$$\therefore PB^2 = AB^2 - AP^2$$

$$= \left(\frac{n+1}{2}\right)^2 - \left(\frac{n-1}{2}\right)^2 = \left(\frac{n-1}{2} + \frac{n+1}{2}\right)\left(\frac{n+1}{2} - \frac{n-1}{2}\right)$$

$$= \left(\frac{n+1+n-1}{2}\right) \times \left(\frac{n+1-n+1}{2}\right) = \frac{2n}{2} \times \frac{2}{2} = n \times 1 = n$$

$$\therefore PB = \sqrt{n}.$$

If we measure PB, its length will give the square root of the given number.

Model No. (13) Find $\sqrt{7}$ by the above method.

Here $n = 7$ $\therefore \frac{n+1}{2} = \frac{7+1}{2} = 4$ and $\frac{n-1}{2} = \frac{7-1}{2} = 3$.

Describe a circle on a finite straight line 4 cm. long as diameter. With one extremity as centre and a radius of 3 cm. describe an arc cutting the former circle. Join the other extremity to the point of intersection and measure it. It will be found to be 2·6.

$$\therefore \sqrt{7} = 2{\cdot}6 \text{ graphically.}$$

(*b*) Describe a circle with O as centre and $\frac{n+1}{2}$ units as radius.

Draw any diameter AB. From OB or OA cut off a length $ON = \frac{n-1}{2}$ units. From N draw NP perpendicular to AB meeting the circle at P. Then $PN = \sqrt{n}$.

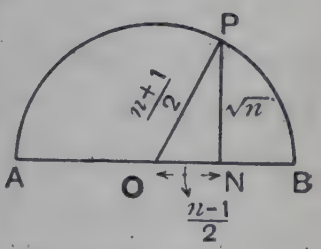


FIG. 147.

For $PN^2 + ON^2 = OP^2$.

$\therefore PN^2 = OP^2 - ON^2$.

$$= \left(\frac{n+1}{2}\right)^2 - \left(\frac{n-1}{2}\right)^2 = n.$$

$\therefore PN = \sqrt{n}$.

Model No. (14) Find $\sqrt{7}$ by the above method.

Describe with O as centre and a radius of $\frac{7+1}{2}$ cm. or 4 cm. a circle. Draw AB a diameter. Cut off along OB, $ON = \frac{7-1}{2}$ cm. or 3 cm. Draw NP perpendicular to AB meeting the circumference at P. PN when measured will give the root of 7.

The above method may be generalized. Then the above formula is a particular case of the still more general form.

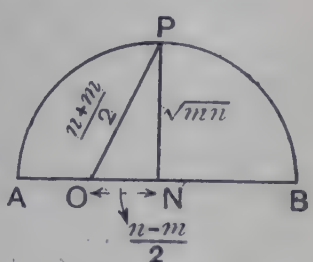


FIG. 148.

$$\left(\frac{n+m}{2}\right)^2 - \left(\frac{n-m}{2}\right)^2 = nm$$

for $\frac{n^2 + m^2 + 2nm}{4} - \frac{n^2 + m^2 - 2nm}{4} = \frac{4nm}{4} = nm$.

In the previous construction AO may be made equal to $\frac{n+m}{2}$, ON equal to $\frac{n-m}{2}$. Then PN will be equal to $\sqrt{mn}$.

$$\text{Now AN} = \text{AO} + \text{ON} = \frac{n+m}{2} + \frac{n-m}{2}$$

$$= \frac{2n}{2} = n \quad \ldots\ldots \quad (1)$$

$$\text{and NB} = \text{OB} - \text{ON} = \frac{n+m}{2} - \frac{n-m}{2}$$

$$= \frac{2m}{2} = m \quad \ldots\ldots \quad (2)$$

(Figure 148 is inaccurate. Imagine O is at the centre of the semicircle and PN is farther away as in Fig. 147).

Thus if AN represents n units, BN m units, PN represents $\sqrt{mn}$ units. This aspect enables us to find the square root of numbers geometrically thus :—

Model No. (15) Find $\sqrt{10}$ by this method.

Find the factors of 10 ; say 5 and 2. Draw a line ANB = 7 cm. such that AN = 5 cm. Then NB must be 2 cm. Describe a semicircle on AB or 7 cm. as diameter. From N draw NP at right angles to AB meeting the circle at P. Then $\text{NP} = \sqrt{10}$.

For $\text{PN}^2 = \text{OP}^2 - \text{ON}^2 = \text{OA}^2 - \text{ON}^2 = (\text{OA} + \text{ON})(\text{OA} - \text{ON})$.

$= \text{AN}(\text{OB} - \text{ON})$.

$= \text{AN}.\text{NB}$.

$= 5.2$.

$\therefore \text{PN} = \sqrt{10}$.

PN when measured will be found to be 3·2 in which is the square root of 10.

144. Approximations in square root.

When n digits in the square root of a number containing $2n - 1$ digits have been obtained by evolution, $n - 1$ more digits can be obtained by ordinary division.

Let N be the number and a the part of the square root containing n digits followed by $n - 1$ cyphers and p the part containing $n - 1$ digits yet to be obtained.

Then $\text{N} = (a + p)^2 = a^2 + 2ap + p^2$.

$\text{N} - a^2 = 2ap + p^2$.

$\frac{\text{N} - a^2}{2a} = p + \frac{p^2}{2a}$.

p contains $n-1$ digits. $\therefore$ p is not greater than 10^{n-1} $\therefore$ p^2 is not greater than 10^{2n-2} and a is not less than 10^{2n-2} $\therefore$ $2a$ is not less than 2.10^{2n-2} $\therefore$ $\frac{p^2}{2a}$ can never be greater than $\frac{10^{2n-2}}{2\cdot 10^{2n-2}}$. $\therefore$ it must be a proper fraction. Hence if a^2 is subtracted from N and the remainder divided by $2a$ the integral part of the quotient will be p.

Model No. (16) Extract the square root of 7656425001.

Ordinary method

```
7656425001 | 87501
64
167 | 1256
      1169
1745 | 8742
       8725
175001 | 175001
         175001
```

Division at the end.

```
7656425001 | 87501
64
167 | 1256
      1169
1745 | 8742   87501 nearly.
       8725
1750 | 17
       18
```

The five figures of the square root are $2n-1$ digits. Hence $n=3$. So when 3 figures have been got by the extraction of the square root $n-1$ i.e. two more figures can be got by contracted division.

Model No. (17) Extract the square root of 3 to 5 places of decimals.

The integral part of the root contains one figure. The answer should contain 6 figures and for approximation 1 figure more should be obtained. Hence obtain 4 figures by extraction of root and 3 more figures by contracted division.

```
3·000000 | 1·732051
1
27 | 2·00
     1 89
343 | 1100
       1029
3462 | 7100
       6924
        176
        173
          3
          3
```

$\therefore$ the required answer is 1·73205.

Model No. (18) Extract the square root of 1·005. $(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x$ when x is a very small fraction compared with 1.

$$\sqrt{1\cdot005} = (1\cdot005)^{\frac{1}{2}} = (1 + \cdot005)^{\frac{1}{2}} = 1 + \tfrac{1}{2} \text{ of } \cdot005 = 1\cdot0025.$$

Verification by ordinary Method.

$$\begin{array}{r|l} \dot{1}\cdot00\dot{5}0\dot{0}0\dot{0}0\dot{0}0 & 1\cdot00249 \\ 1 & \\ \hline 2002 \mid 5000 & \\ \quad 4004 & \\ \hline 20044 \mid 99600 & \\ \quad 80176 & \\ \hline 200489 \mid 1942400 & \\ \quad 1804401 & \\ \hline \end{array}$$

Answer = 1·0025 correct to four places.

Model No. (19) Extract the square root of 102.

$$102 = 100 \times \frac{102}{100} = 100\ (1\cdot02).$$

$$\therefore \sqrt{102} = \sqrt{100\ (1\cdot02)} = 10\sqrt{1\cdot02} = 10\ (1 + \cdot02)^{\frac{1}{2}}.$$
$$= 10\ (1 + \tfrac{1}{2} \times \cdot02).$$
$$= 10\ (1 + \cdot01) = 10\cdot1.$$

Thus $N = 100 + 2$, i.e. $N = p^2 + 2$ per cent. of p^2.

$\therefore \sqrt{N} = 10 + \cdot1$, i.e. $\sqrt{N} = p + 1$ per cent. of p.

In general if $N = p^2 + x$ per cent. of p^2

$$\therefore \sqrt{N} = p + \frac{x}{2} \text{ per cent. of } p.$$

Model No. (20) Extract the square root of 626.

$626 = 625 + 1 = 625 + \cdot2$ per cent. of 625 (roughly).

$\therefore \sqrt{626} = 25 + \cdot1$ per cent. of 25.

$= 25\cdot025$ approximately.

Model No. (21) Extract the square root of 96.

$96 = 100 - 4$ per cent. of 100.

$\therefore \sqrt{96} = 10 - 2$ per cent. of $10 = 9\cdot8$ roughly.

Model No. (22) The diagonal of a square field is 100 yds. Find its side to the nearest foot.

Let a side be x ft. Then $x^2 + x^2 = (300)^2$.

$\therefore 2x^2 = 90000.$

$\therefore x^2 = 45000.$

$\therefore x = \sqrt{45000}.$

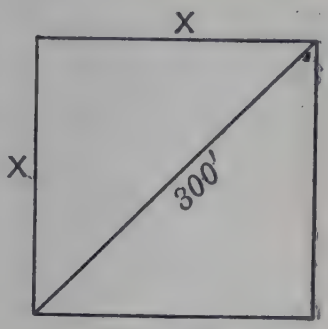


FIG. 149.

∵ the answer is required to the nearest foot, the root should be extracted to get the tenths' place.

$$\begin{array}{r|l} \dot{4}5\dot{0}0\dot{0}\cdot00 & 212\cdot1 \\ 4 & \\ \hline 41 \mid 50 & \\ \quad 41 & \\ \hline 422 \mid 900 & \\ \quad 844 & \\ \hline 4241 \mid 5600 & \\ \quad 4241 & \\ \hline \quad 1359 & \end{array}$$

= 212 correct to the nearest foot.

Model No. (23) Two ships start from a harbour at the same instant and sail south and east at the rate of 15 and 17 miles an hour. How far will they be apart in three hours?

Let O be the starting point. S_1 and S_2 the positions of the ships at the end of three hours after starting. Then $OS_1 = 3 \times 15 = 45$ miles and $OS_2 = 3 \times 17 = 51$ miles (the figure has not been drawn to scale)

$\therefore (S_1 S_2)^2 = O S_1^2 + O S_2^2$ (Pythagoras).

$= 45^2 + 51^2.$

$= 3^2 (15^2 + 17^2).$

$= 9 (225 + 289).$

$= 9.514 = 4626.$

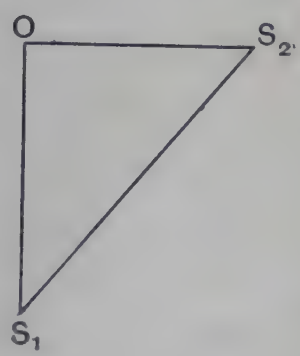


FIG. 150.

$\therefore S_1 S_2 = \sqrt{4626} = 68{\cdot}01$ miles $= 68$ miles nearly.

Note.—45^2 is got thus easily $4.5 = 20$, then add the digits 25 to the right; the result $= 2025$.

$51^2 = (50 + 1)^2 = 2500 + 100 + 1 = 2601.$

Also $4626 = 4900 - 274 = 4900 - 6$ per cent. of 4900 roughly.

$\therefore \sqrt{4626} = 70 - 3$ per cent. of $70 = 70 - 2{\cdot}1 = 68$ miles roughly.

Model No. (24) A zinc shed is in the form shown in the accompanying figure and supported on iron posts each 8 ft. high. The height of the highest point of the shed above the ground is 14 ft. and the breadth of the shed is 35 ft. Find the length of the slope of the shed.

In the rightangled triangle SLM,

$SL^2 = SM^2 + LM^2$.

$= 6^2 + (17\frac{1}{2})^2$

$= 36 + \frac{1225}{4}$.

$= \frac{144 + 1225}{4} = \frac{1369}{4}$.

$\therefore SL = \sqrt{\frac{1369}{4}} = \frac{37}{2} = 18\frac{1}{2}$ ft.

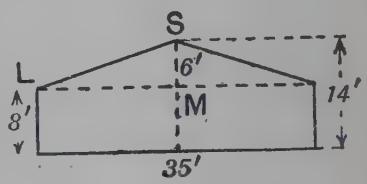


FIG. 151.

Note.—LM = ½ the breadth of the shed.
SM = the height of S − the height of M or L = 14−8 = 6 ft.

Exercise XXII (B).

Find the square root of :—

1. 441.
2. 676.
3. 1024.
4. 1521.
5. 1849.
6. 2209.
7. 3136.
8. 3969.
9. 4761.
10. 5329.
11. 6724.
12. 8464.
13. 9801.
14. 11025.
15. 56169.
16. 231361.
17. 393129.
18. 644809.
19. 923521.
20. 40972801.
21. 64032004.
22. 89321401.
23. 259371025.
24. 2127423376.
25. 9999400009.

Find the square root of the following decimals :—

26. 96·6289.
27. ·185761.
28. 25·5025.
29. 109627·21.
30. 36·58193289.
31. ·00465124.
32. 18·326961.
33. 4096·640025.
34. 4097·25449604.
35. ·000049014001.
36. 65035031·2249.

Find the square root of the following correct to three places of decimals :—

37. 3. 41. 10. 45. 221·36. 49. 144·08.
38. 5. 42. 3·4. 46. ·05. 50. 0·678.
39. 6. 43. 8·76. 47. ·0831. 51. 335·3.
40. 8. 44. 11·25. 48. ·16. 52. 1000.

Find the square root of :—

53. $12\frac{1}{4}$. 56. $\frac{14641}{22201}$. 59. $1\frac{3}{7}\frac{3}{1}\frac{9}{0}\frac{9}{6}\frac{7}{4}\frac{6}{9}$.
54. $1\frac{6}{8}\frac{3}{1}$. 57. $\frac{309976}{170569}$. 60. $2145\frac{13}{50}\frac{68}{43}$.
55. $12\frac{37}{49}$. 58. $37\frac{3771}{20449}$.

Find correct to three places of decimals, the square root of :—

61. $\frac{1}{7}$. 62. $\frac{8}{9}$. 63. $4\frac{13}{17}$. 64. $\frac{3}{11}$. 65. $\frac{1}{0{\cdot}3}$.
66. $\frac{4{\cdot}8}{0{\cdot}63}$.

Find the square root of the following using partly evolution and partly contracted division :—

67. ·0864 to six places of decimals.
68. 1433·8905 to four places of decimals.
69. $\frac{7}{17}$ to four places of decimals.
70. $\sqrt{38{\cdot}08765}$ to six places of decimals.

Evaluate the following correct to four places of decimals :—

71. $\frac{5}{\sqrt{3}}$. 76. $\frac{42}{5\sqrt{7}}$. 81. $\frac{2+\sqrt{3}}{2-\sqrt{3}}$.
72. $\frac{2}{4\sqrt{5}}$. 77. $\frac{1}{4\sqrt{3}}$. 82. $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$.
73. $\frac{\sqrt{5}}{3\sqrt{7}}$. 78. $\frac{1}{2+\sqrt{5}}$. 83. $\frac{24}{7-\sqrt{3}}$.
74. $\frac{4}{\sqrt{15}}$. 79. $\frac{1}{\sqrt{3}+\sqrt{2}}$. 84. (*a*) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$.
75. $\frac{9}{\sqrt{12}}$. 80. $\frac{5}{\sqrt{5}+2}$.

(*b*) $\sqrt{108} + \sqrt{192} - \sqrt{147} + \sqrt{300} - \sqrt{75}$.

(*c*) $\sqrt{5000} - \sqrt{2592} + \sqrt{578} - \sqrt{50}$.

Find the fourth root of :—

85. 194481. 86. 418161601.

Find the fourth root, to four significant digits of :—

87. 154·986. 88. 2241·968.

Write down approximations to the square roots of :—

89. 1525, the nearest square number being 1521.
90. 2000, ,, ,, ,, 1936.
91. 3080, ,, ,, ,, 3025.
92. 1020, ,, ,, ,, 1024.
93. 2300, ,, ,, ,, 2304.
94. 2501, ,, ,, ,, 2500.
95. 3596, ,, ,, ,, 3600.
96. 300, ,, ,, ,, 289.

97. Find the third side of the following right-angled triangles, the other two sides being given and each side of every triangle being a commensurable quantity : (*a*) 60, 48 ; (*b*) 27, 36 ; (*c*) 42, 70 ; (*d*) 96, 40 ; (*e*) 72, 78 ; (*f*) 60, 156 ; (*g*) 100, 28 ; (*h*) 625, 600 and (*i*) 112, 384.

98. $n^2 - 1$, $2n$, $n^2 + 1$ are the sides of a right-angled triangle. This formula can be used to find various sets of values for the sides of a right-angled triangle. With the help of the formula, find the lengths of the other two sides, the lengths of the hypotenuse being (*a*) 1765, (*b*) 226, (*c*) 730, (*d*) 2305, (*e*) 2501, (*f*) 3601 and (*g*) 2602.

99. If a and b are two consecutive numbers whose sum is equal to n^2 where n is an odd number, show by giving different values to n, that a, b and n units are the lengths of the sides of a right-angled triangle.

100. The length of a rectangular field is 78 yds. and breadth 24 yds. Find the length of its diagonal.

101. The adjacent sides of a rectangular field are 9999 yds. and 200 yds. Find the length of a diagonal.

102. The adjacent sides of a rectangular field are in the ratio of 3 : 4. The length of the diagonal is 2005 yds. Find the sides of the field.

103. The adjacent sides of a rectangular field are in the

ratio of 12 : 5. The length of the diagonal is 1352 km. Find the sides of the field.

104. The area of a square field is 2 acres. Find the length of its side to the nearest yard.

105. The area of a square field is 15 acres. Find the length of its side to the nearest yard.

106. In the formula $d = \sqrt{l^2 + b^2}$ when $d = 1370$, $l = 1368$, find b; if $d = 3364$, $b = 116$, find l.

107. The area of a rectangular field is 3 acres 484 sq yds. If the length is three times the breadth, find the dimensions of the field.

108. The area of a rectangular field is 5 hectares 14 ares. If the length is ten times the breadth, find the dimensions of the field to the nearest metre.

109. A vertical flag staff is 35 ft. long. If a guy rope is secured to the ground at a point 21 ft. from the pole and fastened to the pole at a point 7 ft. below the top, what length of rope is required?

110. The owner of a rectangular field 75 yds. $\times$ 35 yds. exchanges it for a square field and Rs. 40, an acre being worth Rs. 484. What is the length of the square field?

111. The sides of a rectangle are 5·43 km. and 3·28 km. Find to the nearest metre the side and diagonal of a square of equal area.

112. Solve the following equations :—

(*a*) $\frac{x}{68} = \frac{612}{x}$; (*b*) $\frac{x}{186} = \frac{22506}{4x}$; (*c*) $x : 100 :: 64 : x$; (*d*) $99 : x :: x : 396$.

113. (*a*) Find a mean proportional between 16 and 49.
(*b*) Do. do. 11 and 176.
(*c*) Work out (*a*) geometrically.
(*d*) Do. (*b*) do.

114. The areas of two square bunks are in the ratio of 389376 : 126025. Find the ratio of their sides.

115. The areas of two similar rectilineal figures are in the ratio of the squares of their sides. Find the ratio of

the corresponding sides of (*a*) two similar triangles of areas of 169 sq. ft. and 144 sq. ft. respectively; (*b*) two similar parallelograms of areas 5625 sq. ft. and 1225 sq. ft.; (*c*) two similar pentagons of areas 11025 sq. ft. and 7921 sq. ft. respectively; (*d*) two similar octagons of areas 93636 sq. ft. and 11664 sq. ft.

116. The side of a square is 13 inches. The middle points of the sides of the square are joined in order. The middle points of the sides of the figure so formed are joined again and so on. If this is done 4 times, find the side of the square formed last.

117. A body dropped from above descends s ft. in t seconds, s and t being connected by the relation $s = \frac{1}{2} gt^2$ where $g = 32{\cdot}2$. Find in what time it would descend (*a*) 1516·9 ft. and (*b*) 2084 ft. (Answer to the nearest second.)

118. The marks of three candidates in an examination are such that the ratio of the marks of the first candidate to those of the second is the same as the ratio of the marks of the second to those of the third. The marks of the first and third are 64 and 36. What marks were obtained by the second boy?

119. The time of oscillation of a simple pendulum varies as the square root of the length of the pendulum. If the lengths of two pendulums be 98 cms. and 87 cms., compare the times of their oscillations.

120. In the formula $T = 2\pi \sqrt{l/g}$, find T given $\pi = 3\frac{1}{7}$, $l = 85$ cms., $g = 981$ cm. per sec. per sec.

121. Two sources of light, whose candle powers are c_1 and c_2 placed at distances d_1 and d_2 from a screen, illuminate it to the same degree. Then the relation connecting these quantities is $\frac{c_1}{c_2} = \frac{{d_2}^2}{{d_1}^2}$. Find d_2 if $c : c_2 :: 3 : 4$ and $d_1 = 85$ cm.

122. The area of a triangle whose sides are a, b and c

is given by the formula, $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$. Find the area of a triangle to the nearest sq. unit when $a = 85$ units, $b = 35$ units and $c = 110$ units.

123. The area of a quadrilateral inscribed in a circle is $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where $2s = a + b + c + d$ and a, b, c and d are the lengths of the sides of the quadrilateral. Find the area of the quadrilateral when $a = 55$, $b = 75$, $c = 40$ and $d = 60$ units.

124. Given an altitude of an equilateral triangle of side a is $\frac{\sqrt{3}}{2}a$, find the side given that the area = 178 sq. in. (Answer to two decimal places.)

125. The following are the two formulæ giving the velocity of flow (V ft. per second) and quantity (Q ft. per second) of discharge of water in pipes of diameter (d) inches, whose hydraulic gradient is s:—

$$\left.\begin{aligned} V &= 39\sqrt{ds} \\ Q &= \pi \frac{d^2}{4} V \end{aligned}\right\} \quad \text{Find } Q \text{ if } d = 12'', \ s = \tfrac{1}{440} \text{ and } \pi = 3{\cdot}1416.$$

126. The discharge of an escape weir of a tank is given by the formula $Q = 3{\cdot}1\, hl\sqrt{h}$ where Q cubic feet is the quantity of discharge per second, l = length of weir in ft. and h height of water in ft. above the surface weir wall. Find the discharge per hour if $l = 62$ ft. and $h = 3$ ft.

127. Evaluate r from the relation Rs. $375\left(1 + \frac{r}{100}\right)^2$ = Rs. 421-5-7$\frac{1}{5}$.

128. A ladder is 5 ft. from the foot of a wall and rests on it at a point 12 ft. above the bottom of the floor. If the top of the ladder slips through 3 feet, find by what distance the bottom of the ladder has gone farther from the wall.

129. The projection of a triangular cardboard on the floor is an equilateral triangle of side 18″. If the perpendiculars from the corners of the cardboard on the floor

be 20″, 18″ and 16″, find the sides of the triangular cardboard.

130. If a rectangular block of wood is $a'' \times b'' \times c''$, show that the diagonal joining any two opposite corners of the cuboid $= \sqrt{a^2 + b^2 + c^2}$. Find the length of the diagonal if $a = 8''$, $b = 6''$ and $c = 4''$. Find the length of the diagonal of a cube of side 12″.

131. The total surface of a cube is 216 sq. inches. Find its edge and also the volume of the cube.

132. (*a*) Given $\sqrt{5} = 2{\cdot}236$, find the square root of 8000.
(*b*) ,, $\sqrt{3} = 1{\cdot}732$, ,, ,, 972.

133. (*a*) ,, $\sqrt[3]{3} = 1{\cdot}442$, find $\sqrt[6]{192}$.
(*b*) ,, $\sqrt[3]{9} = 2{\cdot}08$ find $\sqrt[6]{576}$.

134. Find graphically the square root of (1) $\sqrt{5}$, (2) $\sqrt{6}$, (3) $\sqrt{7}$ and (4) $\sqrt{12}$ (*a*) by constructing a series of right-angled triangles in each case (*b*) by constructing only one right-angled triangle and (*c*) by the method of drawing an ordinate in a semi-circle.

135. The height to which waves near a dam will rise above the normal surface of water in a lake is given by the formula $h = 2{\cdot}5 + 1{\cdot}5\sqrt{F} - \sqrt[4]{F}$ where h is the height of water in ft. and F the fetch in miles, i.e. the longest straight length from the dam face of the water surface exposed to the wind. Find h if $F = 3{\cdot}87$ miles.

136. The intensity of normal pressure P upon a circular or square surface of small area perpendicular to the direction of an artificial air current is given by the formula $P = {\cdot}0027\ V^2$ where V is the velocity of wind in miles per hour and P is the pressure. If $P = 2{\cdot}76481$, find V.

137. $P = \dfrac{f_c\, A}{1 + c\left(\dfrac{l}{d}\right)^2}$ gives the weight P in tons that a column of length l'' and least diameter d'' can safely bear, its cross-sectional area being A square inches, f_c being the safe stress allowed per square inch in tons and c a constant

differing with the material of the column. Evaluate l if A $= 144$, $d = 9$, $c = \frac{1}{100}$, $f_c = 6$ and P $= 173$.

138. The area of a tennis field is 3200 sq. ft. The length is twice the breadth. If a margin of 10 ft. is left at the sides and 20 ft. at the ends, find what length of fencing would be required?

139. A number of soldiers are drawn up into a solid square. From this square 484 men are taken away from the centre. If it is 5 deep find the number of soldiers in the back and front rows.

140. The span of a hipped roof is 14 ft. The rise of the roof is 4 ft. 8 in. Find the length of common rafters and also the hip rafters.

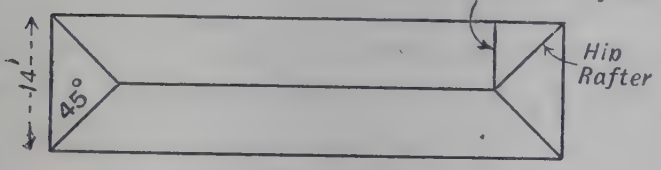


FIG. 152.

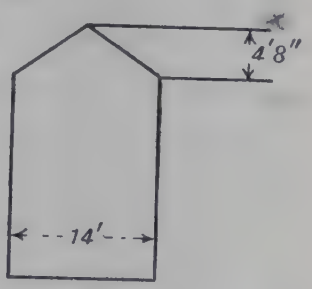


FIG. 153.

141. The semi-perimeter of an ellipse whose axes are $2a$ and $2b$ is $\pi \sqrt{\frac{a^2 + b^2}{2}}$ approximately. Supposing the earth is a prolate spheroid whose equatorial radius $a =$ 20923600 ft. and its polar radius b is 20853616 ft., find the length of a meridian in miles to the nearest ten, assuming $\pi = 3{\cdot}14159$.

142. Graham's law of diffusion states that the velocities of molecules of different gases at a given temperature are inversely proportional to the square roots of the densities of the respective gases. Compare the velocities of molecules of hydrogen and carbonic acid gas if their densities relative to that of air are $\frac{1}{14{\cdot}4}$ and $1{\cdot}529$.

143. Find π to three decimal places in the formula $g = \pi^2 l$ if $g = 980{\cdot}22$ and $l = 99{\cdot}319$ cm.

144. The safe load a beam can carry is directly proportional to the breadth of the beam, to the square of its depth and inversely proportional to the length of the beam and is given by the formula $W = \frac{bd^2}{L}$ where b and d are in inches, L is in feet and W in cwt. What depth of beam 20 ft. long and 12 in. broad is required so that it may support a safe load of 86·4 cwts.

145. A company of sepoys 990 strong is to be formed into the largest possible square. What is the number in the side of the square? How many men are left over?

146. Find the length of wire required to enclose a square lawn of area 2 acres. (Answer to the nearest yard.)

147. From a rectangular piece of brass sheet, the largest square piece is cut off. The areas of the square piece and the remaining rectangle are 1497·69 sq. cm. and 2310·3989 sq. cm. Find the dimensions of the rectangular metal sheet.

148. Three square silver sheets of sides 5 ft. 6 in., 4 ft. 8 in., and 8 ft. 9 in. respectively are melted and cast into one silver square sheet of the same thickness. What must be the side of the square correct to an inch?

149. A gentleman owns three square plots of land of sides 4000, 4500, and 5000 yds. in three different places. He sells away these plots and buys one square plot equal in area to these three. Find, graphically, the length of a side of the new square plot purchased. Calculate its length and verify your answer.

150. There is a square of side 440 yds. What should be the dimensions of a rectangle equal in area to the square? How many solutions are possible? Which of these has the least perimeter?

151. In a garden there are as many trees in a row as there are rows in the garden. The number of trees in the garden is 3481. Find the number of rows in it.

152. In the twentieth century there is a year which is a square number. Find it.

153. A party of pilgrims set out for Thirupathy and each pilgrim doles out 1 pie to each beggar. There are six times as many beggars as there are pilgrims in the party. The total money distributed to beggars is Rs. 78-2-0. Find the number of pilgrims.

154. On a railway incline the vertical rise is 1 ft. for every 500 ft. (horizontal). What is the length of the incline for each 500 ft. (horizontal)? Use the method of approximations to find your answer.

155. In an incline a vertical rise of 2 ft. occurs in every 1000 ft. (horizontal). What is the length of the incline if the rise is 4 ft.?

156. In the seventh century of the Christian era there was a leap year. The number represented by it was also a square number. Find the number.

157. The area of a circle of radius $r = \pi r^2$ where $\pi = 3{\cdot}14159$. Find the radius of a circle whose area is 8000 sq. cms. (to the nearest millimetre).

158. Find the radius of a circle whose area is 100 sq. ft. with the datum in Question 157. (Answer to the nearest inch.)

159. The co-ordinates of a point are 99, 20. Find its distance from the origin.

160. The length l of the straight line joining two points whose co-ordinates are (x_1, y_1), (x_2, y_2), is given by the formula $l = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Use this formula to find the length of the line joining (30, 42) and (54, 49).

CHAPTER XXIII.

COMPOUND INTEREST.

145. If a sum is lent at a certain rate of interest for a number of years without any condition regarding the

payment of interest, then the principal and the interest remain the same for every year. For example, if Rs. 100 is lent for three years at 5 per cent., the principal remains the same each of these three years and the interest charged every year is Rs. 5 so that the money, if repaid at the end of three years, will amount to Rs. 115.

But suppose Rs. 100 is invested in a bank at 5 per cent. The bankers, before they receive the sum, ascertain whether it is to be placed in the current account or fixed deposit account and if it is the latter how long it is to run whether for a year or for six months. If it is taken as a fixed deposit for a year the interest will be added to the principal at the end of a year from the date of investment and the amount of Rs. 105 will be ready for payment to the deposit holder. If the deposit is however renewed and the interest is not withdrawn, the amount of the deposit for the second year is Rs. 105. At the end of the second year, the deposit holder is entitled to get Rs. 105 + the interest on Rs. 105 for one year at 5 per cent., i.e. $105 + \frac{105 \times 5 \times 1}{100}$ or Rs. 110-4-0. If the deposit holder again renews the deposit for a year and does not withdraw any portion of the interest or of the amount, he gets at the end of the third year Rs. $110\frac{1}{4}$ and the interest thereon for one year, i.e., $110\frac{1}{4} + \frac{110\frac{1}{4} \times 1 \times 5}{100}$ or $110\frac{1}{4} + \frac{441}{4} \times \frac{5}{100}$ or $\frac{441}{4} + \frac{441}{80}$ or $\frac{8820 + 441}{80}$ or $\frac{9261}{80}$ i.e., Rs. 115-12-2.

This method of calculating interest in which the principal for each year is the amount at the end of the previous year is called the compound interest system. It is clear

that **the compound interest should be greater than the simple interest for the same principal, time and rate.**

Model No. (1) Find the compound interest on Rs. 450 at $6\frac{1}{4}$ per cent. per annum for three years.

Method (I) *Decimal system.*

450·0	× 0 6·25	= Principal for the first year.
2700·0	0	
90·0	0	Interest for one year $= \frac{P \times R}{100}$.
22·5	0	
2812·5	0	
∴ 28·125		= Interest for the first year.
450		
478·1	× 25 6·25	= Principal for the second year.
2868·7	5	
95·6	2	
23·9	1	
2988·2	8	
∴ 29·883		= Interest for the second year.
478·125		
508·0	× 08 6·25	= Principal for the third year.
3048·0	5	
101·6	0	
25·4	0	
3175·0	5	
∴ 31·751		= Interest for the third year.
508·008		
Rs. 539·759		= Amount at the end of the third year.
450		= Original principal.
Rs. 89·759		= Compound interest.
16		
As. 12·144		
12		
Ps. 1 728		Rs. 89-12-2.

N.B.—It is enough if each sum of money is worked correct to

three places of decimals. By a rough estimate it is seen that the integral portion of the interest contains two figures. Hence $\frac{450 \times 6{\cdot}25}{100}$ should be evaluated to the first five significant figures. Therefore the numerator should be worked correct to one place of decimals as the denominator is 100 or the following method may be adopted :—

$\because \frac{6{\cdot}25}{100} = {\cdot}0625$, the multiplier at each stage may be ·0625 and the result evaluated correct to three decimal places.

	×	
450·000	0	= Principal for the first year.
	0·0625	
27·000	0	
·900	0	
·225	0	
28·125	0	= Interest for the first year.
450		
478·125	0	= Principal for the second year.
	0·0625	
28·687	5	
·956	2	
·239	1	
29·882	8	= Interest for the second year.
478·125		
508·007	8	= Principal for the third year.
508·007	8	
	0·0625	
30·480	4	
1·016	0	
254	0	
31·750	4	= Interest for the third year.
508·008		
539·758	4	= Amount at the end of the third year.
450		= Original principal.
Rs. 89·758		= Compound interest for three years.
16		
As. 12·128		
12		
Ps. 1·536		∴ compound interest = Rs. 89-12-2.

Method II—

(*a*) By formula $A = P\left(1 + \frac{R}{100}\right)^n$ (Art. 146, page 74)

$$\text{Amount} = 450\left(1 + \frac{6\frac{1}{4}}{100}\right)^3 = 450\left(1 + \frac{25}{400}\right)^3 = 450\left(\frac{425}{400}\right)^3 = 450\left(\frac{4{\cdot}25}{4}\right)^3 = 450\,(1{\cdot}0625)^3.$$

It is clear, by forming a rough estimate of the answer, that there are three figures in the integral portion of the result [for $450 \times (1{\cdot}1)^3 = 450 \times 1{\cdot}3 = 585$]. To give the answer correct to the nearest pie, we must find the answer correct to three decimal places. We must evaluate the answer to six significant figures.

i.e., $450 \times (1{\cdot}0625)^3$ must be worked out correct to three places of decimals.

$\therefore$ $(1{\cdot}0625)^3$ must be worked out correct to 6 places of decimals.

For $(1{\cdot}0625)^3$ is to be multiplied by 450 which lies between 100 and 1000.

	×
1·062500	0
	1·0625
1·062500	0
63750	0
2125	0
531	3
1·128906	3
	1·0625
1·128906	3
67734	4
2257	8
564	5
1·199463	0

	×
119·946	30
	4·5
479·785	2
59·973	2
539·758	4
450	

Rs. 89·758
16
As. 12·128
12
Ps. 1·536

Rs. 89-12-2.

$\therefore$ $450 \times 1{\cdot}1994630 = 4{\cdot}50 \times 119{\cdot}94630$.

Method II (*b*) By the method of vulgar fractions

$$A = 450\left(1 + \frac{25}{400}\right)^3 = 450\left(\frac{17}{16}\right)^3 = 450 \times \frac{4913}{4096} = \frac{2210850}{4096}$$

$$\begin{array}{r|l} 4096\overline{)2210850}(539\text{-}12\text{-}1 & 49696 \\ \underline{20480} & 4096 \\ 16285 & 8736 \\ \underline{12288} & \underline{8192} \\ 39970 & 544 \\ \underline{36864} & 12 \\ 3106 & \underline{6528} \\ \underline{16} & 4096 \\ & \underline{2432} \end{array}$$

= Rs. 539-12-2 to the nearest pie.

∴ compound Interest required.

= Rs. 89-12-2.

Method III (The simplest of all)

∵ $6\frac{1}{4}$ per cent. $= \frac{1}{16}$, divide the principal at the beginning of each year by 16. The quotient is the interest. This is left as an exercise to the student.

146. How the formula $P\left(1 + \frac{r}{100}\right)^n = A$ is derived.

On 100 for 1 year the interest is r.

∴ ... 1 for 1 do. $\frac{r}{100}$.

∴ Re. 1 amounts to Rs. $\left(1 + \frac{r}{100}\right)$ at the end of one year.

∴ P do. $P\left(1 + \frac{r}{100}\right)$ do.

∴ $P\left(1 + \frac{r}{100}\right)$ is the principal for the second year.

As before 1 amounts to $\left(1 + \frac{r}{100}\right)$ at the end of the year.

∴ ,, $P\left(1 + \frac{r}{100}\right)$ amounts to $P\left(1 + \frac{r}{100}\right)\left(1 + \frac{r}{100}\right)$ or to $P\left(1 + \frac{r}{100}\right)^2$ at the end of the year.

This is the amount at the end of the second year and consequently the principal for the third year.

As before 1 amounts to $\left(1 + \frac{r}{100}\right)$ at the end of the third year.

$\therefore$ P $\left(1 + \frac{r}{100}\right)^2$ amounts to P $\left(1 + \frac{r}{100}\right)^2 \left(1 + \frac{r}{100}\right)$

or to P $\left(1 + \frac{r}{100}\right)^3$ at the end of the third year.

In general P amounts to P $\left(1 + \frac{r}{100}\right)^n$ at the end of the n^{th} year.

Model No. (2). What principal will amount to Rs. 521-8-0 at 12 per cent. per annum compound interest in two years ?

$$P\left(1 + \frac{r}{100}\right)^2 = 521{\cdot}5. \qquad \therefore P\,(1{\cdot}12)^2 = 521{\cdot}5.$$

$$\therefore P\left(1 + \frac{12}{100}\right)^2 = 521{\cdot}5. \qquad \therefore P = \frac{521{\cdot}5}{1{\cdot}2544} = \text{Rs. } 415\text{-}11\text{-}10.$$

The answer is to be obtained correct to three decimal figures and there are three figures in the integral part. Hence the result is to be evaluated to six significant figures, i.e., seven figures should be got in the quotient.

```
                415·7367
12544 ) 52150
        50176
        -----
         19740
         12544
         -----
          71960
          62720
          -----
           9240
           8781
           ----
            459
            376
            ---
             83
             75
             --
              8
              8
              -
```

Rs. 415·737 is the answer.

```
·737
 16
-----
11·792
 12
-----
9·504
```

$\therefore$ the answer is Rs. 415–11–10.

Model No. (3) Find what Rs. 600 will amount to at 6 per cent. per annum in a year interest payable every six months.

Rate for six months = 3 per cent. and there are two periods of six months in one year.

Hence according to the formula, $A = P\left(1 + \frac{3}{100}\right)^2$.

$= 600\ (1{\cdot}03)^2$.
$= 600 \times 1{\cdot}0609$.
$=$ Rs. $636{\cdot}54$.
$=$ Rs. 636-8-8 nearly.

Model No. (4) Find the difference between the simple and compound interest on Rs. 2,500 for two years at $10\frac{1}{2}$ per cent. per annum.

Method (1) Compound interest $= P\left(1 + \frac{r}{100}\right)^2 - P$.

$= 2500\left(1 + \frac{10{\cdot}5}{100}\right)^2 - 2500$.

$= 2500\ (1{\cdot}105)^2 - 2500$.
$= 2500 \times 1{\cdot}221025 - 2500$.
$= 25 \times 122{\cdot}1025 - 2500$.
$= 3052{\cdot}5625 - 2500$.
$= 552{\cdot}5625$.

·5625	·563
16	16
As. 9·0000	9·008
	12
	Ps. ·096

Simple interest $= \frac{2500 \times 2 \times 21}{2 \times 100} = 525$.

∴ the difference = Rs. $27{\cdot}5625$ = Rs. 27-9-0 nearly.

Method (2) Difference of interests = the difference of the amounts.

$= P\left(1 + \frac{r}{100}\right)^2 - P\left(1 + \frac{2r}{100}\right)$.

$= P\left(1 + \frac{2r}{100} + \frac{r^2}{100^2}\right) - P\left(1 + \frac{2r}{100}\right)$.

$= P\ \frac{r^2}{10000}$.

$= 2500 \times \frac{10{\cdot}5 \times 10{\cdot}5}{10000}$.

$= 25 \times 1{\cdot}05 \times 1{\cdot}05$.
$= 25\ (1 + {\cdot}0025 + {\cdot}1)$.
$= 25 \times 1{\cdot}1025$.
$= 27{\cdot}5625$ = Rs. 27-9-0 Answer.

Model No. (5) I borrow Rs. 1,000 at 9 per cent. at compound interest on condition that I should pay back Rs. 300 at the end of each year. What does the last instalment amount to ?

Method (1)

	1000	= Principal for the first year.
	9	
	90·00	= Interest for the first year.
	1090	= Amount due at end of the first year.
Deduct	300	= Paid at the end of the first year.
	790	= Amount due at the beginning of the second year.
	9	
	71·10	= Interest for the second year.
	790	
	861·1	= Amount due at the end of the second year.
Deduct	300	= Paid at the end of the second year.
	561·1	= Amount due at the beginning of the third year.
	9	
	50·499	= Interest for the third year.
	561·1	
	611·599	= Amount at the end of the third year.
Deduct	300	
	311·599	= Amount due at the beginning of the fourth year.
	9	
	28·04391	
	or 28·044	= Interest for the fourth year.
	311·599	
	339·643	= Amount due at the end of the fourth year.
Deduct	300	
	39·643	= Amount due at the beginning of the fifth year
	9	
	3·56787	= Intrest for the fifth year.
	39·643	
	43·21087	= Amount due at the end of the fifth year. This is the last instalment.

Rs. 43·211
16
As. 3·376
12
Ps. 4·512

∴ the required answer is Rs. 43-3-5.

Method (2).

Let Rs. x be the last instalment paid.

It is obvious that a debt of Rs. 1000 will be cleared in not less than 3 years, at the rate of Rs. 300 a year even if the interest is not taken

into consideration. If simple interest is reckoned, 1000 in 4 years amounts to Rs. 1,360 ; at the rate of Rs. 300 per year it will take not less than 5 years.

1000 in 5 years at 9 per cent. per annum will amount to

$$\text{Rs. } 1{,}000\left(1+\frac{9}{100}\right)^5.$$

Also 300 paid at the end of the first year will amount to

$300\left(1+\frac{9}{100}\right)^4$ at the end of the fifth year.

Similarly 300 paid at the end of the second year will amount to

$300\left(1+\frac{9}{100}\right)^3$ at the end of the fifth year.

,, 300 paid at the end of the third year will amount to

$300\left(1+\frac{9}{100}\right)^2$ at the end of the fifth year.

,, 300 paid at the end of the fourth year will amount to

$300\left(1+\frac{9}{100}\right)$ at the end of the fifth year.

,, x paid at the end of the fifth year will amount to x only.

∵ both these arrangements are equally profitable,

$$1000\left(1+\frac{9}{100}\right)^5 = 300\left(1+\frac{9}{100}\right)^4 + 300\left(1+\frac{9}{100}\right)^3 + 300\left(1+\frac{9}{100}\right)^2 + 300\left(1+\frac{9}{100}\right) + x.$$

$$= 300\left(1+\frac{9}{100}\right)^4 + 300\left(1+\frac{9}{100}\right)^3 + 300\left(1+\frac{9}{100}\right)^2 + 300\left(1+\frac{9}{100}\right) + 300 - 300 + x.$$

$$= 300\left\{\frac{(1+\frac{9}{100})^5 - 1}{(1+\frac{9}{100}) - 1}\right\} + x - 300.$$

$$\therefore 1000\left(1+\frac{9}{100}\right)^5 \cdot \frac{9}{100} = 300\left(1+\frac{9}{100}\right)^5 - 300 + (x-300) \times \frac{9}{100}.$$

$$\therefore 90\ (1{\cdot}09)^5 = 300\ (1{\cdot}09)^5 - 300 + \frac{9x}{100} - 27.$$

$$-\frac{9x}{100} = 210\ (1{\cdot}09)^5 - 327.$$

$$-\frac{9x}{100} = 323{\cdot}111019 - 327 = -3{\cdot}888981.$$

$$\therefore x = \frac{3{\cdot}888981 \times 100}{9} = \frac{388{\cdot}8981}{9} = 43{\cdot}2109.$$

Same answer as before.

Note 1.—This method is based on the principle that if the borrower's payments made on different dates be allowed to accrue with compound interest till the date of the clearing of the final debt they must amount to the same sum as the lender would be entitled to get had the loan been paid in a lump sum with compound interest.

Note 2.—$1 + r + r^2 + r^3 + r^4 = \frac{1 - r^5}{1 - r}$, which may be ascertained by ordinary division.

Model No. (6) If a person is to be paid Rs. 100 at the end of each year for three years from this date, what sum invested now may be regarded as equivalent to all these payments compound interest being calculated at the rate of 6 per cent. per annum ?

Method (1) Let the sum that is to be invested be Rs. x. It amounts in three years at 6 per cent. to $x\left(1 + \frac{6}{100}\right)^3$ or $x\,(1{\cdot}06)^3$ rupees.

The other arrangement is that Rs. 100 should be paid at the end of every year for three years.

The first payment of 100 (paid at the end of the first year) amounts to $100\,(1{\cdot}06)^2$ at the end of the third year ; the second payment of Rs. 100 (paid at the end of the second year) amounts to $100\,(1{\cdot}06)$ at the end of the third year. The third payment is made at the end of the third year.

These instalments amount to $100\,(1{\cdot}06)^2 + 100\,(1{\cdot}06) + 100$ at the end of the third year.

According to note (1) the lump sum arrangement should be equivalent to the instalments system (with interest in each case).

$$\therefore x\,(1{\cdot}06)^3 = 100\,(1{\cdot}06)^2 + 100\,(1{\cdot}06) + 100.$$
$$x \times 1{\cdot}191016 = 100\,(1{\cdot}1236) + 100\,(1{\cdot}06) + 100.$$
$$= 100 \times 3{\cdot}1836 = 318{\cdot}36.$$
$$\therefore x = \frac{318{\cdot}36}{1{\cdot}191016}.$$

It is clear that there are three figures in the integral part of the quotient. To give the answer to the nearest pie the first six significant figures in the quotient should be

obtained. To evaluate the answer correct to the sixth significant figure, we must get seven figures in the quotient.

$$
\begin{array}{r|l}
1{\cdot}\not{X}\not{\emptyset}\not{X}\not{\emptyset}\not{X}\not{\emptyset} & 267{\cdot}3012 \\
\hline
3183600 & \cdot 301 \\
2382032 & 16 \\
\hline
801568 & \text{As. } 4{\cdot}816 \\
714610 & 12 \\
\hline
86958 & \text{Ps. } 9{\cdot}792 \\
83371 & \\
\hline
3587 & \\
3573 & \\
\hline
14 & = \text{Rs. } 267{\cdot}301. \\
12 & \\
\hline
2 & = \text{Rs. } 267\text{-}4\text{-}10. \\
2 & \\
\hline
\end{array}
$$

Method (2) Find the present worth or value of each future payment and add all these present values together. Let the present values of the three payments to be made at the end of the first, the second and the third years be Rs. x, Rs. y and Rs. z. Then the present lump sum to be paid is $x + y + z$

$$
\begin{array}{l|l}
\text{where } x\left(1+\frac{6}{100}\right) = 100, & \therefore x = \frac{100}{1{\cdot}06} = 94{\cdot}3397. \\
y\left(1+\frac{6}{100}\right)^2 = 100, & \therefore y = \frac{100}{1{\cdot}1236} = 88{\cdot}9997. \\
\text{and } z\left(1+\frac{6}{100}\right)^3 = 100. & \therefore z = \frac{100}{1{\cdot}191016} = 83{\cdot}9619. \\
 & \therefore x + y + z = 267{\cdot}3013. \\
 & = \text{Rs. } 267\text{-}4\text{-}10.
\end{array}
$$

$$
\begin{array}{r|l}
1{\cdot}\not{\emptyset}\not{\emptyset} & 94{\cdot}3397 \\
\hline
1000 & \\
954 & \\
\hline
460 & \\
424 & \\
\hline
360 & \\
318 & \\
\hline
420 & \\
318 & \\
\hline
102 & \\
95 & \\
\hline
7 & \\
7 & \\
\hline
0 &
\end{array}
\qquad
\begin{array}{r|l}
1{\cdot}\not{X}\not{2}\not{3}\not{\emptyset} & 88{\cdot}9997 \\
\hline
100000 & \\
89888 & \\
\hline
101120 & \\
89888 & \\
\hline
11232 & \\
10112 & \\
\hline
1120 & \\
1011 & \\
\hline
109 & \\
101 & \\
\hline
8 & \\
8 & \\
\hline
0 &
\end{array}
\qquad
\begin{array}{r|l}
1\,1\not{\emptyset}\not{X}\not{\emptyset}\not{X}\not{\emptyset} & 83{\cdot}9619 \\
\hline
10000000 & \\
9528128 & \\
\hline
471872 & \\
357305 & \\
\hline
114567 & \\
107191 & \\
\hline
7376 & \\
7146 & \\
\hline
230 & \\
119 & \\
\hline
111 & \\
107 & \\
\hline
4 &
\end{array}
$$

Exercise XXIII.

1. Find, to the nearest pie, the amount at compound interest on :—

(*a*) Rs. 250 for 2 yrs. at 6 per cent. per annum interest being payable annually.

(*b*) Rs. 4,000 for $2\frac{1}{2}$ yrs. at 5 per cent. per annum interest being payable annually.

(*c*) Rs. 375 for 3 yrs. at $5\frac{1}{2}$ per cent. per annum interest being payable annually.

(*d*) £750 for 2 yrs. at 6 per cent. per annum interest being payable annually.

(*e*) Rs. 875 for $2\frac{3}{4}$ yrs. at $4\frac{3}{4}$ per cent. per annum interest being payable annually.

(*f*) Rs. 2,500 for $2\frac{5}{8}$ yrs. at 8 per cent. per annum interest being payable annually.

(*g*) Rs. 365-8-6 for 2 yrs. at $7\frac{1}{2}$ per cent. per annum interest being payable half-yearly.

(*h*) Rs. 1,734-13-8 for 9 months at $4\frac{1}{2}$ per cent. per annum interest being payable quarterly.

(*i*) Rs. 3,565-8-0 for 2 yrs. at $3\frac{1}{2}$ per cent. per annum interest being payable half-yearly.

2. Find, to the nearest pie, the compound interest on :—

(*a*) Rs. 615 for $1\frac{1}{2}$ yr. at 8 per cent. per annum interest being payable annually.

(*b*) £800 for 3 yrs. at 5 per cent. per annum interest being payable annually.

(*c*) Rs. 340-8-0 for $1\frac{1}{2}$ yr. at $6\frac{1}{2}$ per cent. per annum interest being payable half-yearly.

(*d*) Rs. 2,000 for $2\frac{1}{2}$ yrs. at 10 per cent. per annum interest being payable annually.

(*e*) Rs. 845-12-6 for 1 yr. at $5\frac{1}{2}$ per cent. per annum interest being payable quarterly.

(*f*) Rs. 1,728 for 4 yrs. at $8\frac{1}{3}$ per cent. per annum interest being payable annually.

(*g*) Rs. 1,834-5-4 for 2 yrs. at 9 per cent. per annum interest being payable annually.

3. A certain sum of money amounts to Rs. 405-0-8·4 in 2 yrs. at 6 per cent. compound interest payable annually. Find the sum.

4. A certain sum of money amounts to Rs. 1,390-1-6 at the end of $1\frac{1}{2}$ yr. at $8\frac{1}{2}$ per cent. per annum compound interest payable half-yearly. Find the sum.

5. A certain sum of money amounts to Rs. 539-10-2·36 in $2\frac{1}{2}$ yrs. at $7\frac{1}{2}$ per cent. per annum compound interest payable annually. Find the sum.

6. By how much will the compound interest on Rs. 1,350 at $5\frac{1}{2}$ per cent. for 3 years exceed the simple interest on the same sum for the same period at the same rate?

7. Find the difference between the simple and the compound interest on Rs. 850 for 2 yrs. at 9 per cent. per annum (compound interest payable annually).

8. A gentleman invests Rs. 1,000 at the beginning of every year for three years in a bank at 6 per cent. per annum compound interest, with the object of sending his son to England to be educated there. If the sum accumulated at the end of three years is handed to his son for journey to England, find what sum he received from the bank to the nearest pie.

9. A house is sold for Rs. 12,000 payable in 4 equal instalments at intervals of six months, the first instalment being paid on the date of the sale of the house. Another similar house is sold for Rs. 12,000 on the same day. Find what the difference in money values of the two transactions will be $1\frac{1}{2}$ year hence, reckoning compound interest payable yearly at 12 per cent. per annum.

10. A garden was sold in 1916 for Rs. 45,500. If the owner had sold it on the same date in 1919 he could have got for it Rs. 72,500. Find how much he estimated the loss to be in 1919 by his hasty sale, reckoning compound interest at 9 per cent. per annum.

11. A person lends a sum of Rs. 5,000, on mortgage of some machinery and plant worth Rs. 6,000, at $10\frac{1}{2}$ per cent.

compound interest. The machinery depreciates each year at the rate of 5 per cent. of the value at the beginning of the year. After 3 years the machinery is sold to clear off the debt. Has the lender gained or lost? Give your answer to the nearest pie.

12. A borrows Rs. 300 from B at 8 per cent. simple interest and immediately lends it at 8 per cent. compound interest payable half-yearly. Find how much A gains on the transaction at the end of two years.

13. What sum would amount to Rs. 908-6-6·34 at compound interest in 3 years if the rate of interest for the first year be $5\frac{1}{2}$ per cent., for the second year $6\frac{1}{2}$ per cent., and for the third year $7\frac{1}{2}$ per cent.?

14. The difference between the simple and compound interest on a certain sum for $2\frac{1}{2}$ years at $6\frac{1}{2}$ per cent. per annum is Re. 1. Find the sum.

15. Supposing the population in Bombay States to have had a uniform increase of $12\frac{1}{2}$ per cent. per annum, calculate the population in 1909 if it was 7,411, 675 in 1911.

16. The value of the tea exported in 1916 from India to England was £10,095,978. If it increased at a uniform rate of 8 per cent. per annum from 1912, find, correct to the nearest £, the value of the tea exported in 1912.

17. A person invests Rs. 850 at the beginning of each year in a certain bank which pays compound interest at $6\frac{1}{2}$ per cent. per annum. Find what his investments will have amounted to, at the end of three years.

18. A person buys a shop worth Rs. 13,450 and pays the owner Rs. 10,000 at once, agreeing to pay the rest at the end of $2\frac{1}{2}$ years. Find how much he will have to pay if compound interest is calculated at 9 per cent. per annum.

19. A person borrows a sum of Rs. 750 at 8 per cent. compound interest and pays Rs. 125 every half-year. Find what he will have to pay at the end of two years (interest is payable every half-year).

20. A bungalow and a garden around it cost Rs. 25,000. If the value of the bungalow depreciates at 12 per cent. per annum and that of the land steadily increases at 8 per cent. per annum, what will they cost at the end of 3 years, the costs of the bungalow and the garden being in the proportion of 3 : 2 ?

21. Find at what rate of interest Rs. 345 will amount to Rs. 380·3625 in two years at compound interest.

22. Find at what rate of interest Rs. 800 will amount to Rs. 972·405 in two years (interest payable half yearly).

23. In a town-planning scheme provision is made for an increase of population at a uniform rate of 8 per cent. per annum. If the number of houses built be proportional to the population, what will be the number of houses at the end of four years if there were 1086 houses in a suburb of the city prior to its extension ?

24. Light loses $\frac{1}{5}$ of its intensity at every reflection. What fraction of the original intensity remains after the seventh reflection ?

25. A tennis ball let fall from a height rises to $\frac{3}{4}$ of its original height after impinging on the ground. If the ball be dropped from a terrace 20 ft. high, find the height of the ball above the ground just after the fifth impact.

26. Using the formula $A = P\left(1 + \frac{r}{100}\right)^n$, find the fourth quantity, given the other three in the following :—

(*a*) P = Rs. 750, $r = 4\frac{1}{2}$, $n = 2$, find A.

(*b*) P = Rs. 8,192, $r = 12\frac{1}{2}$, $n = 3\frac{1}{2}$, find A.

(*c*) A = Rs. 1,596-8-4, $r = 5\frac{3}{4}$, $n = 3$, find P.

(*d*) A = Rs. 14,641, $r = 10$, $n = 4$, find P.

(*e*) A = Rs. 2,101·346625, P = 1785, $r = 8\frac{1}{2}$, find n.

(*f*) A = Rs. 1,900, P = 1458, $r = 11\frac{1}{9}$, find n.

(*g*) A = Rs. 9,553-4-0, P = 8750, $r = 6$, find n.

(*h*) P = Rs. 2,000, A = Rs. 2,205, $n = 2$, find r.

27. A sum is lent at 9 per cent. compound interest for two years, interest being payable yearly. If the interest is paid half yearly, find what the rate of interest should be so that it may amount to the same sum in the same period.

28. A person expects a profit of Rs. 6,000 at the end of three years in a certain business. As he is in urgent need of money he borrows money at the rate of 7 per cent. per annum compound interest. What is the sum borrowed if his expectation is fulfilled and his capital improved by 30 per cent. every year (to the nearest rupee) ?

29. A person borrows a certain sum of money at $5\frac{1}{2}$ per cent. per annum compound interest and pays it off in three equal half-yearly instalments of Rs. 150 each. Find the sum borrowed.

30. A merchant and customer buys a motor car from a company for Rs. 20,000 and wishes to pay in four equal annual instalments. If compound interest is charged at $10\frac{1}{2}$ per cent. per annum, find what amount is paid every time.

31. What should be the rate of compound interest in order that a sum of money may double itself in 10 years.

Assume $\sqrt[10]{2} = 1{\cdot}07$.

32. What should be the rate of compound interest in order that a sum of money may treble itself in eight years.

Assume $\sqrt[8]{3} = 1{\cdot}0596$.

33. The simple interest on a certain sum for 3 years at 5 per cent. per annum exceeds the compound interest on the same sum for the same period at $4\frac{1}{2}$ per cent. per annum by Rs. 3-3-3. Find the sum.

34. What sum of money lent out at simple interest at 7·2 per cent. per annum for 5 years will yield the same interest as Rs. 750 lent out at compound interest at $4\frac{1}{2}$ per cent. per annum for $2\frac{1}{2}$ years ?

35. A sum of money amounts to Rs. 938-3-8 and Rs. 985-2-3 at the end of two successive years. What is the

rate of interest (compound)? If the years in question be second and third, find the principal.

36. Divide Rs. 1,210·662 between A and B now so that they might get equal sums at the end of the second and third years respectively, compound interest being allowed at 6 per cent. per annum.

37. A fat person who weighs 210 lbs. loses $\frac{1}{2\frac{1}{5}}$ of his weight every year owing to a particular diet he is put under and some exercise he has been made to take. Find what he will weigh at the end of four years.

38. A person borrows Rs. 190 and agrees to pay off the debt in two equal annual instalments of Rs. 100 each or four equal half-yearly instalments of Rs. 50 each. Which is more profitable to the lender and by how much?

39. A merchant buys goods for Rs. 6,000, which do not sell for three years. He finds that, at the end of three years, a portion of the goods worth Rs. 1,750 gets damaged, and sells it only at a loss of 50 per cent. and the remainder for Rs. 5,550. Find his loss, money being worth 4 per cent. per annum compound interest.

40. A student buys books worth Rs. 200 (if cash payment is made) from a bookseller on condition that he pays Rs. 120 at the end of one year and also Rs. 120 at the end of two years. If the bookseller charges compound interest at r per cent., show that

$$\frac{120}{1+\frac{r}{100}}+\frac{120}{\left(1+\frac{r}{100}\right)^2}=200.$$

41. A man borrows Rs. 1,000 to be repaid by instalments, for a marriage and agrees to pay 12 per cent. interest on his dues at any time. He pays off Rs. 400 including interest at the end of each year until the last instalment, less than Rs. 400, has to be paid. What is the last instalment and the total of the repayments.

42. The administrators of a property belonging to the two minor sons aged 18 and 15 years, of a deceased

gentleman gave to the elder son a sum of Rs. 7,200 at the beginning of his ninteenth year and the younger, three equal sums at the end of his 16th, 17th and 18th year. If 9 per cent. per annum compound interest was reckoned and if the amounts of their shares were equal when their ages were 21 and 18 respectively, show that each of the equal sums is given by the relation

$7200 \times (1{\cdot}09)^3 = x \{(1{\cdot}09)^2 + (1{\cdot}09) + 1\}$ and find x.

43. An estate encumbered with a debt of Rs. 18,650, interest on which was payable yearly at 12 per cent. per annum, was placed in charge of the Administrator-General. Instead of paying the interest alone at the end of every year, the court paid Rs. 4,975 at the end of every year. In how many years would the loan have been repaid ?

44. A bank receives deposits running for 3 years and offers to pay compound interest at 9 per cent. per annum. A constituent of the bank deposits Rs. 10,000 on these terms but is put to the necessity of borrowing Rs. 5,000 twice at the end of the first and second years from the date of the investment of his money in the bank, at a higher rate of interest, viz., 12 per cent. per annum compound interest. When the deposit matures, what is his financial position in the bank ?

45. Suppose a labourer invests a sum of Re. 1 now in the post office savings bank at 3 per cent. per annum. What will he be entitled to get at the end of 50 years supposing the bank allows compound interest payable yearly ? Assume $(1{\cdot}03)^{50} = 4{\cdot}383906$. From this, deduce the compound interest (as a compound quantity) on (1) Rs. 100 and (2) Rs. 65.

46. Assume, in the previous question, the rate of interest the post office charges after 30 years to be 4 per cent., and after 40 years to be 5 per cent. Then what will the investment of Re. 1 amount to in 50 years, given

$(1{\cdot}03)^{30} = 2{\cdot}427262$, $(1{\cdot}04)^{10} = 1{\cdot}480244$ and $(1{\cdot}05)^{10} = 1{\cdot}628895$?

47. A middle school borrows Rs. 12,000 to raise it to a high school but agrees to repay it with compound interest in two equal payments at the end of 1 and 2 years. If the rate of interest be 6 per cent. payable annually, find the amount of each payment to the nearest anna.

48. A man proposes to invest £200 at 3 per cent. and to reinvest the interest at the same rate each year. If he continues this process to the end of the fortieth year, by how much will his original sum of Rs. 200 have multiplied itself ? Assume $(1{\cdot}03)^{30} = 2{\cdot}427262$ and $(1{\cdot}03)^{10} = 1{\cdot}343914$.

49. An under-graduate borrows Rs. 120 from a money-lender and agrees to pay Rs. 144 in four quarterly instalments of Rs. 36 each the first instalment to be paid 3 months after the date of the loan. The money-lender invests each repayment at 12 per cent. compound interest as soon as he receives it. Calculate how much he has received by the end of the year in return for his loan. At what rate of simple interest has the money of loan improved ?

50. A sum of Rs. 7,500 was invested at 5 per cent. per annum compound interest by the Trustees of an orphan's estate when he was 12 years old. When the orphan became a major (21 years old) what did the sum amount to, given $(1{\cdot}05)^9 = 1{\cdot}551328$?

CHAPTER XXIV.

Harder Equations of One Unknown Quantity and Problems thereon

147. There are some equations which involve a knowledge of fractions.

The following examples will illustrate how fractions are dealt with in equations :—

Model No. (1) Solve : $\frac{x}{2} - \frac{x}{3} + \frac{x}{4} = \frac{x}{12} + 8.$

The L.C.M. of the denominators is 12. Multiply every term of the equation by 12 to get rid of fractions.

$\therefore \quad \frac{x}{2} \times 12 - \frac{x}{3} \times 12 + \frac{x}{4} \times 12 = \frac{x}{12} \times 12 + 8 \times 12.$

$\therefore \quad 6x - 4x + 3x = x + 96.$

$\therefore \quad 4x = 96.$

$\therefore \quad x = 24.$

Verification.—Left hand side $= \frac{24}{2} - \frac{24}{3} + \frac{24}{4} = 12 - 8 + 6 = 10.$

Right hand side $= \frac{24}{12} + 8 = 2 + 8 = 10.$

$\therefore$ both sides are equal. Hence the answer is correct.

Model No. (2) Solve : $\frac{3(1 - 4x)}{5} + \frac{1}{8}\left(2 - \frac{x - 3}{3}\right) = -21.$

The L. C. M. of the denominators 5, 8, 3 is 120.
Multiply every term by 120.

$\therefore \quad 3\frac{(1 - 4x)}{5} \times 120 + \frac{1}{8} \times 120 \left(2 - \frac{x - 3}{3}\right) = -21 \times 120$

$\therefore \quad 72(1 - 4x) + 15\left(2 - \frac{x - 3}{3}\right) = -2520$

$\therefore \quad 72 - 288x + 30 - 5x + 15 = -2520.$

$\therefore \quad -288x - 5x = -2520 - 72 - 30 - 15.$

$\therefore \quad -293x = -2637.$

$\therefore \quad x = \frac{-2637}{-293} = 9.$

The answer may be verified.

Model No. (3) Solve : $\frac{x}{p} + \frac{x}{q} = \frac{x}{r} + \frac{1}{s}.$

The L.C.M. of the denominators is $pqrs$.
Multiply every term by $pqrs$.

$\therefore \quad \frac{x}{p} \times pqrs + \frac{x}{q} \times pqrs = \frac{x}{r} \times pqrs + \frac{1}{s} \times pqrs.$

$\therefore \quad xqrs + xprs = xpqs + pqr.$

$xqrs + xprs - xpqs = pqr.$

$xs(qr + pr - pq) = pqr.$

$\therefore \quad x = \frac{pqr}{s(qr + pr - pq)}.$

Model No. (4) Solve: $\frac{x+3}{x+4} = \frac{3x+1}{3x+3}$.

Divide each numerator by the corresponding denominator.

$$1 - \frac{1}{x+4} = 1 - \frac{2}{3x+3}.$$

$$\therefore -\frac{1}{x+4} = -\frac{2}{3x+3}.$$

$$\therefore \frac{1}{x+4} = \frac{2}{3x+3}.$$

The L.C.M. of the denominators is $(x+4)(3x+3)$.

Multiply every term by $(x+4)(3x+3)$

$\therefore 3x+3 = 2(x+4)$.

$\therefore 3x+3 = 2x+8$.

$\therefore x = 5$.

Verification.—Left hand side $= \frac{5+3}{5+4} = \frac{8}{9}$; right hand side $= \frac{3.5+1}{3.5+3} = \frac{16}{18} = \frac{8}{9}$.

Model No. (5) Solve: $(2x+3)(x+8) + (x-3)(x+6) = (3x+5)(x-1)$.

Removing the brackets by multiplying the factors in each term, we get

$$2x^2 + 19x + 24 + x^2 + 3x - 18 = 3x^2 + 2x - 5.$$

$$\therefore 3x^2 + 22x + 6 = 3x^2 + 2x - 5.$$

$$\therefore 22x - 2x = -5 - 6.$$

$$\therefore 20x = -11.$$

$$\therefore x = -\tfrac{11}{20}.$$

Model No. (6) Solve: $\frac{x+1}{x-1} + \frac{x+2}{x-3} = \frac{2x-15}{x-11}$.

Dividing each numerator by the corresponding denominator we get

$$1 + \frac{2}{x-1} + 1 + \frac{5}{x-3} = 2 + \frac{7}{x-11}.$$

$$\therefore \frac{2}{x-1} + \frac{5}{x-3} = \frac{7}{x-11}.$$ (For $+1+1$ cancels 2 on the other side.)

$$\therefore \frac{2x-6+5x-5}{(x-1)(x-3)} = \frac{7}{x-11}.$$

$$\therefore \frac{7x-11}{(x-1)(x-3)} = \frac{7}{x-11}.$$

When two fractions are equal, the cross products are equal.*

$$\therefore (7x - 11)(x - 11) = 7(x - 1)(x - 3).$$
$$\therefore 7x^2 - 88x + 121 = 7x^2 - 28x + 21.$$
$$\therefore -88x + 28x = 21 - 121.$$
$$\therefore -60x = -100.$$
$$\therefore x = \frac{100}{60} = \frac{5}{3} = 1\frac{2}{3}.$$

Model No. (7)

$$\text{Solve: } \frac{2x}{2x-3} + \frac{6x+26}{3x+11} = \frac{3x+7}{3x+5} + \frac{8x+9}{4x+3}.$$

Dividing each numerator by the corresponding denominator we get

$$1 + \frac{3}{2x-3} + 2 + \frac{4}{3x+11} = 1 + \frac{2}{3x+5} + 2 + \frac{3}{4x+3}.$$
$$\therefore \frac{3}{2x-3} + \frac{4}{3x+11} = \frac{2}{3x+5} + \frac{3}{4x+3}.$$

For $1 + 2$ cancels $1 + 2$ on the other side.

$$\frac{9x + 33 + 8x - 12}{(2x-3)(3x+11)} = \frac{8x + 6 + 9x + 15}{(3x+5)(4x+3)}.$$
$$\therefore \frac{17x + 21}{(2x-3)(3x+11)} = \frac{17x + 21}{(3x+5)(4x+3)}.$$
$$\therefore \frac{17x + 21}{(2x-3)(3x+11)} - \frac{17x + 21}{(3x+5)(4x+3)} = 0.$$
$$\therefore (17x + 21)\left\{\frac{1}{(2x-3)(3x+11)} - \frac{1}{(3x+5)(4x+3)}\right\} = 0.$$
$$\therefore 17x + 21 = 0.\dagger$$
$$\therefore x = -\frac{21}{17}.$$

---

* To prove this multiply every term in the equation by the L.C.M. of the denominators.

† If the product of two factors each of which contains x is zero, any one of them may be zero.

Model No. (8) Solve : $\frac{1}{x+3} - \frac{4}{x+8} + \frac{3}{x+4} = 0.$

$\therefore \frac{(x+8)(x+4) - 4(x+3)(x+4) + 3(x+3)(x+8)}{(x+3)(x+8)(x+4)} = 0$

$\therefore (x+8)(x+4) - 4(x+3)(x+4) + 3(x+3)(x+8) = 0$

$\therefore x^2 + 12x + 32 - 4(x^2 + 7x + 12) + 3(x^2 + 11x + 24) = 0.$

$\therefore 12x - 28x + 33x = -32 + 48 - 72.$

$\therefore 17x = -56.$

$x = -\frac{56}{17}.$

Exercise XXIV (A).

Solve :—

1. $\frac{x-1}{2} + \frac{2x-3}{4} = \frac{3x-1}{6}.$
2. $\frac{4x-5}{3} + \frac{6x-2}{8} + \frac{3x-4}{5} = \frac{1-13x}{4}.$
3. $\frac{x+1}{3} + \frac{2x-4}{9} + \frac{4x-8}{8} = x.$
4. $\frac{x-2}{11} + \frac{x-4}{20} + \frac{x-5}{19} = \frac{x-12}{3}.$
5. $\frac{3x+1}{4} - \frac{5x-1}{6} = \frac{7x+3}{2} - \frac{15x+1}{4}.$
6. $\frac{x}{8} - \frac{2x+1}{7} = \frac{x}{6} - \frac{1}{3}x.$
7. $\frac{3x}{8} - \frac{5x-6}{12} = \frac{2x}{3} + \frac{9x-8}{20}.$
8. $\frac{3x-1}{5} + \frac{4x-7}{6} = \frac{8x-3}{2} + \frac{12x-1}{10}.$
9. $\frac{1}{3}(x-4) + \frac{1}{4}(4x-1) = \frac{1}{5}(x-6) + \frac{2}{5}(3x-4).$
10. $\frac{x-3}{4} + \frac{3x-1}{11} = \frac{1}{3} + \frac{2x+10}{6}.$
11. $\frac{3x+5}{5} - \frac{4x+8}{8} = \frac{4x+5}{24} - \frac{7}{8}.$
12. $\frac{x}{2} + \frac{x}{3} - \frac{x}{5} - \frac{x}{6} = \frac{x}{15} + \frac{x}{20} + 84.$

13. $2x + 1 - \frac{4x - 5}{6} - \frac{38 - 4x}{7} = \frac{3x - 4}{8} + \frac{4x - 7}{9}$.

14. $3x - \frac{1}{2} + \frac{5x - 3}{3} + \frac{8x - 9}{4} = \frac{6x - 1}{5} + \frac{7x - 2}{8} - \frac{1}{3}$.

15. $5 - \frac{7(x - 8)}{6} = \frac{6x + 2}{7}$.

16. $9 - \frac{8(x - 3)}{5} = \frac{3(x + 5)}{10}$.

17. $6 - \frac{3x - 5}{5} + \frac{2x}{3} = \frac{x}{4} + \frac{5x}{6\frac{2}{3}} - 1$.

18. $\frac{5x}{8} - \frac{3x - 7}{17} - (2x - 6) = \frac{1}{12}(x + 24) - 4\frac{1}{5}$.

19. $\frac{1}{2}(x - 2) + \frac{1}{3}(3x - 1) = \frac{1}{4}(5x - 8) + \frac{1}{6}(3x - 7)$.

20. $\frac{x - 3}{9} - \frac{x + 4}{17} + \frac{x - 5}{5} + \frac{x - 12}{18} = 7$.

21. $\frac{1}{5}(x - 6) + \frac{1}{6}(x - 7) + 1\frac{1}{7}(x - 8) = \frac{1}{9}(x - 1)$.

22. $\cdot 2(x + 3) + \cdot 5(x - 4) = \cdot 8(x - 5)$.

23. $\frac{12 - x}{\cdot 75} + \frac{14 - 2x}{\cdot 25} = \frac{16 - 5x}{\cdot 125}$.

24. $\frac{x}{\cdot 8} + \frac{1}{\cdot 05} = \frac{x}{\cdot 9} - \frac{1}{\cdot 025}$.

25. $\frac{\cdot 02x - \cdot 34}{\cdot 8} = \frac{\cdot 54}{\cdot 9}x - 1\cdot 5$.

26. $\cdot 3(x - 5) + \cdot 4(8x - 3) + 6 = \cdot 4(2x - 1) + \cdot 25x$.

27. $x(x + 4) + x(x - 3) = (2x - 1)(x - 5)$.

28. $(x + 3)x + 2(x - 4)x = (3x + 5)(x - 1) + 4$.

29. $(x + 2)^2 + 3(x - 4)^2 = (4x - 1)(x + 3) - 7$.

30. $(x - 4)^2 + (x - 3)(x - 7) = (x - 5)^2 + (x - 6)(x - 7)$.

31. $(x-3)(x-8)+(x-2)(3x-1)=(2x-3)(2x-7)+17$.

32. $(x-2)(2x-1)+(x-5)(3x-2)=(x+6)(5x-1)+1$.

33. $\frac{1}{2}(x-2)(x-5)+\frac{1}{3}(x-3)(x-4)=\frac{1}{6}(5x+2)(x-2)+\frac{2}{3}$.

34. $(2x-1)(3x-4)+(4x-1)(6x-5)=(5x-3)(6x+1)+4(x-8)-12$.

35. $(x+8)^2+(2x-1)^2+(8x^2-3)=6(x-2)^2+7(x-3)^2+1$.

36. $(x+1)^3+(x-1)^3=2(x+2)^3-12x^2$.

37. $(x+6)(x+7)(x-4)=(x+3)^3-10$.

38. $\dfrac{x^2+2x-3}{x+2}=\dfrac{x^2+6x-9}{x+6}$.

39. $\dfrac{3x^2+5x-6}{3x+5}=\dfrac{7x^2-8x+3}{7x-8}$.

40. $\dfrac{(x-1)(x-2)}{x-3}=\dfrac{(x+8)(x+7)}{x+15}$.

148. Several questions in Arithmetic can be solved by introducing x. The following harder examples will further illustrate the application of equations to problems.

Model No. (9) Divide Rs. a into two parts so that they are in the ratio $1:b$. Find the parts. Evaluate your answer if $a=1000$ and $b=4$.

Let one part be x rupees. Then the other part is $a-x$ rupees.

$\therefore$ the ratio of the two parts $= x : a-x$.

By the question this ratio $= 1 : b$.

$\therefore \dfrac{x}{a-x}=\dfrac{1}{b}$

$\therefore bx = a-x$ (by cross multiplication).

$\therefore bx+x=a$.

$x(b+1)=a$.

$\therefore x=\dfrac{a}{b+1}$ i.e. one part $=\dfrac{a}{b+1}$.

$\therefore$ the other part is $a-x=a-\dfrac{a}{b+1}=\dfrac{ab+a-a}{b+1}=\dfrac{ab}{b+1}$.

$\therefore$ the parts $=\dfrac{1000}{4+1}$ and $\dfrac{1000\times 4}{4+1}$, i.e. 200 and 800.

Model No. (10) If I pay away $\frac{1}{3}$ of my money, then $\frac{1}{9}$ of what is left and again $\frac{1}{3\frac{1}{2}}$ of the then remainder, I have Rs. 155. What sum had I at first?

Let x rupees be the sum I had at first. I pay away $\frac{1}{3}x$.

$\therefore$ the remainder is $x - \frac{1}{3}x$ or $\frac{2}{3}x$.

Then I pay away $\frac{1}{9} \times \frac{2}{3}x$ $\therefore$ the remainder is $\frac{2}{3}x - \frac{2}{27}x$

$= \frac{18x - 2x}{27} = \frac{16x}{27}$.

Again I pay away $\frac{1}{32} \times \frac{16x}{27} = \frac{x}{54}$.

$\therefore$ the remainder is $\frac{16x}{27} - \frac{x}{54} = \frac{32x - x}{54} = \frac{31x}{54}$.

By the question, the final remainder is Rs. 155.

$\therefore \frac{31x}{54} = 155.$

$\therefore x = \frac{\overset{5}{\cancel{155}} \times 54}{\cancel{31}} = 270.$ $\therefore$ I had first Rs. 270.

The answer may be verified.

Model No. (11) There are a certain number of roses in a basket. One-third of the number and 1 more are wanted to make a garland. Two-thirds of the remainder and 3 more are required for making boquets and the remaining flowers are reserved for puja (worship), which are 20 in number. Find the number of roses in the basket.

Let x be the number of roses in the basket.

The number intended for making a garland $= \frac{1}{3}x + 1$.

$\therefore$ the remainder is $x - \left(\frac{1}{3}x + 1\right) = x - \frac{1}{3}x - 1$ or $\frac{2}{3}x - 1$.

The number intended for making boquets is $\frac{2}{3}\left(\frac{2}{3}x - 1\right) + 3$

$$\text{or } \frac{4}{9}x - \frac{2}{3} + 3 = \frac{4}{9}x + 2\frac{1}{3} = \frac{4}{9}x + \frac{7}{3}$$

$\therefore$ the remainder is $\frac{2}{3}x - 1 - \left(\frac{4}{9}x + \frac{7}{3}\right)$

$$\text{or } \frac{2}{3}x - 1 - \frac{4}{9}x - \frac{7}{3} \text{ or } \frac{2}{9}x - \frac{10}{3}.$$

But by the question this remainder is 20.

$\therefore \frac{2}{9}x - \frac{10}{3} = 20.$

$\therefore 2x - 30 = 180.$

$\therefore 2x = 210.$

$\therefore x = 105.$ $\therefore$ the number of roses in the basket is 105.

The answer may be verified.

Model No. (12) The cost of carpeting a room 20 ft. by 12 ft. leaving a border to be covered by linoleum is Rs. 30-12-0 including the cost of linoleum. The cost of carpeting is 4 as. per sq. ft. and that of covering the border with linoleum is 1 a. per sq. ft. Find the area of the carpeted portion.

Let x sq. ft. be the area of the carpet. Then the area of the border to be covered with linoleum is $12 \times 20 - x$ or $(240 - x)$ sq. ft.

$\therefore$ the cost of the carpet is Rs. $x \times \frac{1}{4}$;

and that of covering the border with linoleum is $(240 - x)$ as. or Rs. $\frac{1}{16}(240 - x)$.

$\therefore$ the total cost $= \left\{\frac{1}{4}x + \frac{1}{16}(240 - x)\right\}$ rupees.

By the question this is Rs. $30\frac{3}{4}$ or $\frac{123}{4}$.

$\therefore \frac{1}{4}x + \frac{1}{16}(240 - x) = \frac{123}{4}.$

Multiplying every term by 16 we get

$\therefore 4x + 240 - x = 492.$

$\therefore 3x + 240 = 492.$

$\therefore 3x = 252.$

$\therefore x = 84.$ $\therefore$ the carpeted portion is 84 sq. ft.

Model No. (13) On a certain railway line, the telegraph poles are placed 55 yds. apart and a passenger counts 35 in 2 minutes as he passes them. At what rate is he travelling ?

Let the rate be x miles per hour.

$\therefore$ in 1 minute he travels $\frac{x}{60}$ miles.

$\therefore$,, 2 minutes ,, $\frac{x}{60} \times 2 \times 1760$ yds.

By the question he travels 55×34 yds. for there are only 34 intervals of space when 35 posts are passed.

$$\therefore \frac{x}{60} \times 2 \times 1760 = 34 \times 55.$$

$$\therefore x = \frac{34 \times \overset{5}{\cancel{55}} \times \overset{3}{\cancel{60}}}{\underset{16}{\cancel{2}} \times \cancel{1760}} = \frac{510}{16} = 31\frac{14}{16} \text{ or } 31\frac{7}{8}.$$

$\therefore$ the rate is $31\frac{7}{8}$ miles per hour.

Model No. (14) An up train usually travels at the rate of 30 miles an hour and meets a down train 48 miles from the terminus. On one occasion owing to an accident it went only at the rate of 25 miles an hour and after travelling for 8 hours met the down train 70 miles from the terminus. Find the rate at which the down train travels.

Let A and B be the two terminii; M the place of the usual meeting of the trains and N that on the day of accident. Then BM = 48 miles and BN = 70 miles. Let the rate of the down train be x miles per hour.

FIG. 154.

The time taken by the down train to travel BM $= \frac{48}{x}$ hrs.

and that ,, ,, ,, BN $= \frac{70}{x}$ hrs.

$\therefore$ the time taken by it to travel the distance MN $= \frac{70}{x} - \frac{48}{x}$

$$= \frac{22}{x} \text{ hrs.}$$

It met the up train $\frac{22}{x}$ hours later than usual. . . . . (1)

The up train travels AN in 8 hours at 25 miles per hour.

$\therefore$ AN $= 25 \times 8 = 200$.

$\therefore$ AM $=$ AN $+$ NM $= 200 + 22 = 222$ miles.

The usual time taken by the up train to reach M $= \frac{222}{30} = 7\frac{2}{5}$ hours after its starting. But it has now met the down train at N 8 hours after starting.

$\therefore$ the delay in meeting $= 8 - 7\frac{2}{5}$ or $\frac{3}{5}$ hour. . . . . (2)

$\therefore$ the equation is $\frac{22}{x} = \frac{3}{5}$.

$\therefore 3x = 110$.

$\therefore x = \frac{110}{3} = 36\frac{2}{3}$. $\therefore$ the down train travels at $36\frac{2}{3}$ miles an hour.

Model No. (15) A train 88 yds. in length travels at 24 miles an hour. In what time will it pass a person who travels at 6 miles an hour (1) in the opposite direction and (2) in the same direction?

Part I.—At the instant when the train meets the man the positions of the train and the man are indicated in figure (1). G and M are separated by 88 yds. At the instant when the train passes the man their positions are indicated in fig. (2). G and M are side by side. Hence the question is 'In what time will G and M, 88 yds. apart at first, meet, their rates of motion being 24 miles and 6 miles per hour?'

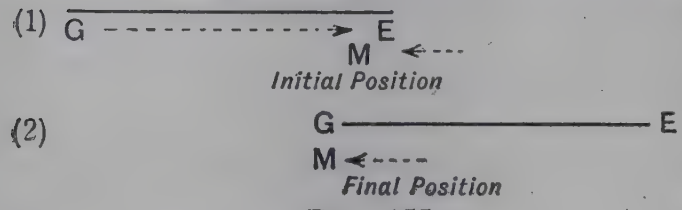


FIG. 155.

Let them meet in x seconds.

In x'' the train travels $\frac{24 \times 1760}{60 \times 60} \times x$ yds. (the distance travelled by G).

In x'' the man travels $\frac{6 \times 1760}{60 \times 60} \times x$ yds. (the distance travelled by M).

The sum of these two distances is 88 yds.

$$\therefore \frac{24 \times 1760}{60 \times 60} x + \frac{6 \times 1760}{60 \times 60} x = 88.$$

$$\therefore 24 \times 1760x + 6 \times 1760 \times x = 88 \times 60 \times 60.$$

$$\therefore 1760x\,(24 + 6) = 88 \times 60 \times 60.$$

$$\therefore x = \frac{88 \times 60 \times 60}{1760 \times 30} = 6.$$ $\therefore$ in 6 seconds they pass each other.

Arithmetical Solution

If G and M are $24 + 6$ or 30 miles apart, they meet in 1 hr.

i.e. if ,, ,, $\frac{22}{15} \times 30$ or 44 ft. ,, ,, 1 sec.

$\therefore$,, ,, 88 yds. or 88×3 ft. $\frac{1}{\cancel{44}} \times \overset{2}{\cancel{88}} \times 3$ or 6 sec.

Part II.—The positions of the train and the man are exactly as indicated in fig. (3) and fig. (4) at the instants of overtaking and passing respectively. Hence the question is this. 'In what time will G overtake M 88 yds. ahead, their rates of motion being 24 miles and 6 miles per hour respectively?'

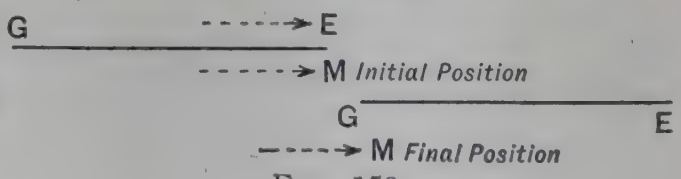


FIG. 156.

Let G overtake M in x seconds.

In x seconds G travels $\frac{24 \times 1760}{60 \times 60} \times x$ yds.

$\therefore$ In x ,, M travels $\frac{6 \times 1760}{60 \times 60} \times x$ yds.

The difference of these two distances is 88 yds. as G gains over M 88 yds.

$$\therefore \frac{24 \times 1760}{60 \times 60} \times x - \frac{6 \times 1760}{60 \times 60} x = 88.$$

$$\therefore \frac{1760}{60 \times 60} \times x\,(24 - 6) = 88.$$

$$\therefore x = \frac{88 \times 60 \times 60}{1760 \times 18} = 10. \quad \therefore \text{ they pass each other in 10 sec.}$$

Arithmetical Solution

If G and M are 24 — 6 or 18 miles apart, G overtakes M in 1 hr.

i.e. if , ,, $\frac{22}{15} \times 18$ ft. ,, ,, 1 sec.

$\therefore$,, ,, 88×3 ft. ,, ,, $\frac{\overset{5}{\cancel{11}} \times \overset{4}{\cancel{8}}\overset{2}{\cancel{8}}}{\underset{1}{\cancel{22}} \times \underset{1}{\cancel{18}}} \times \overset{1}{\cancel{3}}$

$\not{0}$ or=10 sec.

Model No. (16) Two trains 60 yds. and 72 yds. long travel at 36 and 24 miles respectively on parallel rails. In how many seconds will they pass each other if they are moving (1) in opposite directions and (2) in the same direction?

Part I.—Figure (1) indicates the positions of the guards' vans of the trains at the instant when they meet, i.e. G_1 and G_2 are separated by a distance of 60 + 72 or 132 yds. Figure (2) indicates the positions of the guards' vans of the trains at the instant of passing each other, i.e. G_1 and G_2 are side by side.

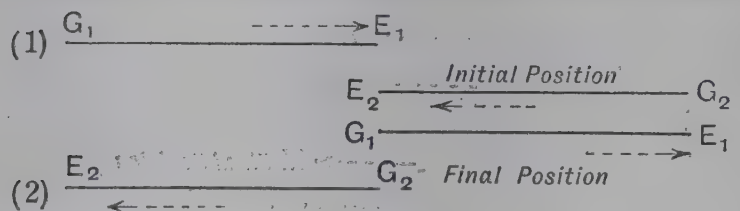


FIG. 157.

The question is this: 'When will G_1 and G_2 132 yds. apart at first meet if they travel at the rate of 36 and 24 miles an hour respectively?'

Let them pass each other in x seconds.

In x sec. G_1 travels $\frac{36 \times 1760}{60 \times 60} \times x$ yds.

,, x ,, G_2 ,, $\frac{24 \times 1760}{60 \times 60} \times x$ yds.

The sum of the two distances is 132 yds.

$\therefore \frac{36 \times 1760}{60 \times 60} x + \frac{24 \times 1760}{60 \times 60} x = 132.$

$\therefore \frac{1760}{60 \times 60} x\,(36 + 24) = 132.$

$\therefore x = \frac{132 \times 60 \times 60}{1760 \times 60} = \frac{18}{4} = 4\frac{1}{2}.$

$\therefore$ they pass each other in $4\frac{1}{2}$ seconds.

Arithmetical Solution

If G_1 and G_2 are 36 + 24 or 60 miles apart, they meet in 1 hr.

i.e. ,, ,, ,, $\frac{22}{15} \times 60$ ft. ,, ,, 1 sec.

$\therefore$,, ,, 132 yds. or 132×3 ft. $\frac{\cancel{15}}{\cancel{2}\cancel{2} \times \cancel{6}\cancel{0}_{\cancel{4}}} \times \frac{\overset{3}{\cancel{3}\cancel{3}}}{\cancel{3}\cancel{2} \times 3} = \frac{9}{2}$ or $4\frac{1}{2}$ sec.

Part II.—The positions of the trains at the instant of overtaking, are indicated in Figure (3) i.e. G_1 and E_2 are 132 yds. apart. Their positions at the instant of passing, are indicated in Figure (4) i.e. G_1 and E_2 are abreast. The question then is this: 'In what time will G_1 overtake E_2 132 yds. ahead, the rates of their motions being 36 miles and 24 miles an hour respectively?'

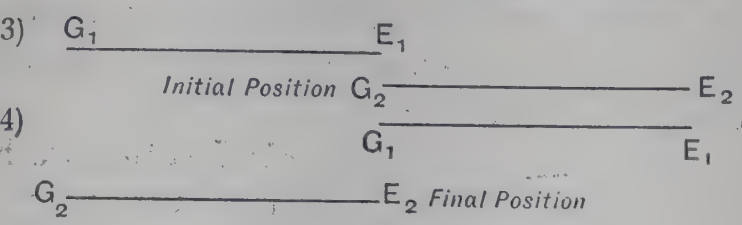


FIG. 158.

Let them pass each other in x seconds.

In x'' G_1 travels $\frac{36 \times 1760}{60 \times 60} x$ yds.

,, E_2 ,, $\frac{24 \times 1760}{60 \times 60} x$ yds.

The difference of these two distances = 132 yds. as G_1 gains over E_2 132 yds.

$$\therefore \frac{36 \times 1760}{60 \times 60} x - \frac{24 \times 1760}{60 \times 60} x = 132.$$

$$\therefore \frac{1760\, x}{60 \times 60} (36 - 24) = 132.$$

$$\therefore x = \frac{132 \times 60 \times 60}{1760 \times 12} = \frac{360}{16} \text{ or } 22\tfrac{1}{2} \text{ seconds.}$$

They pass each other in $22\frac{1}{2}$ seconds.

Arithmetical Solution

If G_1 and E_2 are 36–24 or 12 miles apart, G_1 overtakes E_2 in 1 hr.

i.e. ,, ,, $\frac{22}{15} \times 12$ ft. ,, ,, 1 sec,

$$\therefore \text{ ,, } 132 \text{ yds. or } 132 \times 3 \text{ ft. ,, } \frac{\overset{3}{\cancel{15}} \times \cancel{11}\cancel{2} \times \cancel{3}}{\cancel{22} \times \cancel{12}_{\cancel{4}\ 2}} = \frac{45}{2}$$

or $22\frac{1}{2}$ sec.

Model No. (17) At what time between 4 and 5 o'clock are the hour and minute hands of a watch (1) coincident (2) at right angles to each other and (3) exactly opposite to each other?

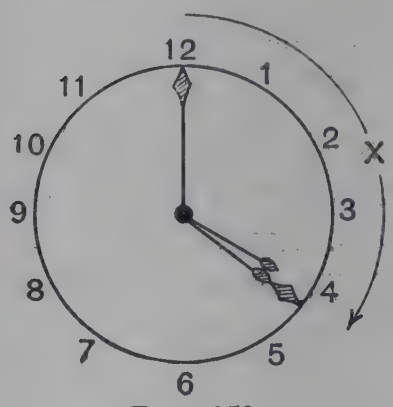


FIG. 159.

(1) Let the two hands be together at x minutes past 4.

The minute hand has travelled x minute divisions. ∴ the hour hand has moved $x - 20$ minute divisions.

$$\therefore \frac{x}{x-20} = \frac{60}{5} = \frac{12}{1}. \quad (\because \text{distances traversed by the hands are proportional to their speeds which are 60 minute divisions and 5 minute divisions per hour}).$$

$$\therefore 12x - 240 = x.$$

$$\therefore 11x = 240.$$

$\therefore x = 21\frac{9}{11}$. At $21\frac{9}{11}$ minutes past 4 they are coincident.

Arithmetical Solution

The minute hand should gain over the hour hand 20 divisions to come together.

It gains 55 minute divisions in 60 minutes,

$$\therefore \quad ,, \quad 20 \quad ,, \quad \frac{60}{\cancel{55}_{11}} \times \overset{4}{\cancel{20}} = \frac{204}{11} = 21\frac{9}{11} \text{ minutes.}$$

(2) (*a*) Let them be at right angles at x minutes past 4.

FIG. 160.

The minute hand has moved x minute divisions from 12. The distance from 12 of the hour hand at the required instant is $x + 15$ minutes. But at 4 it was at a distance of 20 minute divisions from 12. ∴ it has moved $x + 15 - 20$ minute divisions.

$$\therefore \frac{x}{x+15-20} = \frac{60}{5} \quad (\because \text{distances traversed are in the ratio of their speeds.})$$

$$\therefore \frac{x}{x-5} = \frac{12}{1}.$$

Therefore $12x - 60 = x$.

$\therefore 11x = 60$.

$\therefore x = \frac{60}{11} = 5\frac{5}{11}$ minutes past 4.

Arithmetical Solution

The two hands are 20 minute divisions apart at 4. To be 15 divisions apart the m. hand, should gain over the h. hand 5 minute divisions.

It gains 55 minute divisions in 60 minutes.

$\therefore$ ...5 ,, ,, $\frac{60}{\cancel{55}_{11}} \times \cancel{5} = 5\frac{5}{11}$ minutes.

(*b*) The two hands will be at right angles in another position between 4 and 5.

Let them be at right angles at x minutes past 4. The minute hand has moved x minute divisions from 12. The hour hand is now at a distance of $x - 15$ minute divisions from 12. But at 4 it was at a distance of 20 minute divisions from 4. $\therefore$ it has moved $x - 15 - 20$ minute divisions.

FIG. 161.

$\therefore \frac{x}{x - 15 - 20} = \frac{60}{5}$ ($\because$ distances traversed are in the ratio of their rates.)

$\therefore \frac{x}{x - 35} = \frac{12}{1}$.

Therefore $12x - 420 = x$.

$\therefore 11x = 420$.

$\therefore x = 38\frac{2}{11}$. They are at right angles at $38\frac{2}{11}$ minutes past 4.

Arithmetical Solution

The two hands are 20 divisions apart at 4. To be 15 divisions apart a second time the m. hand should gain over the h. hand $20 + 15$ or 35 divisions. It gains 55 minute divisions in 60 minutes,

$$\therefore \quad ,, \quad 35 \ldots \frac{\overset{12}{\cancel{60}}}{\cancel{55}_{11}} \times 35 = \frac{420}{11} = 38\frac{2}{11} \text{ minutes.}$$

(3) Let the two hands be opposite to each other at x minutes past 4.

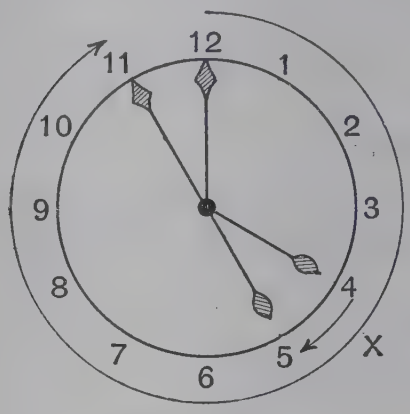


FIG. 162.

The minute hand has moved x minute divisions from 12.

The distance from 12 to where the hour hand is at the required instant is $x - 30$ (because they are opposite to one another). But at 4 it was 20 minute divisions from 12.

$\therefore$ it has moved only $x - 30 - 20$ or $x - 50$ minute divisions.

$$\therefore \frac{x}{x-50} = \frac{60}{5}$$

$$\therefore \frac{x}{x-50} = \frac{12}{1}.$$

$$\therefore x = 12x - 600.$$

$$\therefore 600 = 11x.$$

$$\therefore \frac{600}{11} = x.$$ They are opposite at $54\frac{6}{11}$ minutes past 4.

Arithmetical Solution

At 4 the two hands are 20 minute divisions apart. To be opposite to one another they must be 30 minutes apart; so the m. hand should gain over the h. hand 20 minute divisions + 30 minute divisions or 50 minute divisions.

55 minute divisions are gained in 1 hr. or 60 minutes.

$$\therefore 50 \quad ,, \quad ,, \quad ,, \quad \frac{\overset{12}{\cancel{60}}}{\underset{11}{\cancel{55}}} \times 50 = \frac{600}{11} = 54\frac{6}{11} \text{ minutes.}$$

Model No. (18) A man rows down stream a distance of 35 miles and up stream a distance of 30 miles. He takes twice as much time for the up stream journey as for the down stream journey. At what rate does he row in still water if the rate of the current is 2 miles per hour?

Let x miles per hour be his rate in still water. Then the rate

up stream is $x - 2$ miles per hour and that down stream is $x + 2$ miles per hour.

$\therefore$ the time taken for the up journey $= \frac{30}{x-2}$ hrs.

$\therefore$,, ,, ,, down ,, $= \frac{35}{x+2}$ hrs.

$\therefore$ by the question $\frac{30}{x-2} = 2\frac{35}{x+2}$.

$\therefore 70(x-2) = 30(x+2)$.

$70x - 140 = 30x + 60$.

$\therefore 40x = 200$.

$\therefore x = 5$.

The rate in still water is 5 miles per hour.

Model No. (19) There are three numbers. The difference between the first and the second is 1 and that between the second and third is 1, the third being the greatest. The sum of the quotients got by dividing the first by the second and the second by the third is 2. Find the numbers. Work out the question by supposing (*a*) $x + 1$ to be the smallest number (*b*) x to be the smallest number.

(*a*) Let $x + 1$ be the smallest number.

$\frac{x+1}{x+2} + \frac{x+2}{x+3} = 2$.

$\therefore 1 - \frac{1}{x+2} + 1 - \frac{1}{x+3} = 2$.

$\therefore -\frac{1}{x+2} = \frac{1}{x+3}$.

$\therefore x + 2 = -x - 3$.

$\therefore 2x = -3 - 2$.

$\therefore x = -2\frac{1}{2}$.

$\therefore$ the three numbers are $x + 1$, $x + 2$, $x + 3$ *i.e.* $-2\frac{1}{2} + 1$, $-2\frac{1}{2} + 2$, $-2\frac{1}{2} + 3$ *i.e.* $-1\frac{1}{2}$, $-\frac{1}{2}$, $\frac{1}{2}$.

(*b*) Let x be the smallest number

$\frac{x}{x+1} + \frac{x+1}{x+2} = 2$.

$\therefore 1 - \frac{1}{x+1} + 1 - \frac{1}{x+2} = 2$

$\therefore -\frac{1}{x+1} = \frac{1}{x+2}$.

$\therefore -(x+2) = x+1$.

$\therefore -3 = 2x$.

$\therefore -\frac{3}{2} = x$.

$\therefore$ the three numbers are $-1\frac{1}{2}$, $-\frac{1}{2}$, $\frac{1}{2}$.

Verification.—

Left hand side $\frac{-2\frac{1}{2}+1}{-2\frac{1}{2}+2} + \frac{-2\frac{1}{2}+2}{-2\frac{1}{2}+3} = \frac{-1\frac{1}{2}}{-\frac{1}{2}} + \frac{-\frac{1}{2}}{\frac{1}{2}} = 3 - 1 = 2$.

The right hand side $= 2$.

$\therefore$ both sides are equal.

Model No. (20) A person buys a watch for a certain sum and sells it so as to gain 25 per cent. Had he bought it for Rs. 5 more and sold it for the same price he would have gained $11\frac{1}{9}$ per cent. What was the price of the watch?

Let Rs. x be the price of the watch.

The S. P. $= \frac{125}{100}$ C. P. $=$ Rs. $\frac{5}{4}x$.

The supposed C. P. $= x + 5$, the supposed S. P. $= \frac{5}{4}x$.

$\therefore$ the supposed gain $= \frac{5}{4}x - x - 5$. This sum he is supposed to gain on $x + 5$.

$\therefore$ rate of gain $= \frac{\frac{5}{4}x - x - 5}{x + 5}$. This by the question $= \frac{11\frac{1}{9}}{100}$.

$\therefore \frac{\frac{5}{4}x - x - 5}{x + 5} = \frac{100}{9 \times 100} = \frac{1}{9}$.

$\therefore \frac{\frac{1}{4}x - 5}{x + 5} = \frac{1}{9}$.

$\therefore 9(\frac{1}{4}x - 5) = x + 5$.

$\therefore \frac{9}{4}x - 45 = x + 5$.

$\therefore \frac{5}{4}x = 50$.

$\therefore x = \frac{50 \times 4}{5} = 40$.

$\therefore$ the C. P. of the watch is Rs. 40.

Arithmetical Solution

Let 100 be the C. P. Then S P. is 125.

By selling the article for 125, he would gain $11\frac{1}{9}$ per cent.

$\therefore$ If $\frac{100}{9}$ had been the S. P., the C. P. would have been 100.

...1 ,, ,, $\frac{100 \times 9}{1000}$

$\therefore$ ...125 ,, ,, $\frac{100 \times 9}{1000} \times 125 = \frac{900}{8} = 112\frac{1}{2}$

so the C. P. should have been increased by $12\frac{1}{2}$.

If the article had been bought for $12\frac{1}{2}$ more, the original C. P. must be 100.

,, ,, 5 ,, ,, $\frac{100}{12\frac{1}{2}} \times 5 =$ Rs. 40

Model No. (21) A reduction of 25 per cent. in the price of eggs will enable a man to get 32 more for 6 rupees than before. What is the present price of an egg ?

Let x annas be the original price of an egg.

Then for Rs. 6, could be purchased $\frac{1}{x} \times 6 \times 16$ or $\frac{96}{x}$ eggs.

The reduced price is $\frac{75}{100}x$ or $\frac{3}{4}x$ anna per egg.

For Rs. 6, could be purchased $\frac{4}{3x} \times 6 \times 16$ or $\frac{128}{x}$ eggs.

$\therefore$ the excess in the number of eggs $= \frac{128}{x} - \frac{96}{x}$.

By the question this excess $= 32$.

$\therefore \frac{128}{x} - \frac{96}{x} = 32.$

$\therefore \frac{32}{x} = 32.$

$\therefore x = 1.$ $\therefore$ the original price is 1 anna. Present price $= \frac{3}{4}x = \frac{3}{4}$ anna.

Arithmetical Solution

The altered price is $\frac{3}{4}$ of the original price (75 per cent. of the original price).

$\therefore$ the number bought must be $\frac{4}{3}$ times the original number.

$\therefore$ the increase $= \frac{1}{3}$ the original number.

by the question, the increase is 32 (for Rs. 6).

$\therefore \frac{1}{3}$ the original number $= 32$.

$\therefore$ the number $= 96$.

The cost of 96 is Rs. 6 or as. 96.

$\therefore$,, ,, egg is 1 an. $\therefore$ the reduced price is $\frac{3}{4}$ an.

Model No. (22) Divide Rs. 12,540 among A, B and C so that A shall receive $\frac{3}{7}$ as much as B and C together and B shall receive $\frac{20}{33}$ of what A receives.

Let Rs. x be A's share.

B's share $= \frac{20}{33}x$ rupees.

$\therefore$ C's share $= 12{,}540 - x - \frac{20}{33}x.$

$= 12{,}540 - \frac{53}{33}x.$

B's and C's shares amount to $\frac{20x}{33} + 12{,}540 - \frac{53}{33}x$

$12{,}540 - \frac{33}{33}x$ or $12{,}540 - x$ (Which could have been got directly from the question for A's share is x and the whole is Rs. 12,540.)

By the question $x = \frac{3}{7}(12{,}540 - x)$.

$\therefore 7x = 37{,}620 - 3x.$

$10x = 37{,}620.$

$\therefore x = 3{,}762.$

$\therefore$ A's share is Rs. 3,762.

B's share is $\frac{20}{33} \times 3762 =$ Rs. 2,280.

$\therefore$ C's share is $= 12{,}540 - 6{,}042 =$ Rs. 5198.

Exercise XXIV (B).

1. A bill of Rs. 165 is paid in rupees, half-rupees, quarter-rupees, two anna pieces and annas. The number of half-rupees is $\frac{9}{14}$ of the number of rupees; the number of quarter-rupees is $\frac{5}{9}$ of that of half-rupees. The number of two anna pieces is 23 more than that of rupees; and the number of annas 65 more than that of two anna pieces. How many of each kind are there?

2. A sum of £144-11-0 is made up of sovereigns, guineas, crowns and shillings. The total number of coins is 271. The number of guineas is $\frac{23}{100}$ of that of sovereigns while the number of crowns is $\frac{13}{20}$ of the number of sovereigns. Find the number of coins of each kind.

3. A purse contains 10 rupee notes, 5 rupee notes, rupee notes and half-rupees amounting to Rs. 550-8-0. The number of 5 rupee notes is $\frac{3}{10}$ of that of the ten rupee notes. The number of rupee notes is three more than double the number of 10 rupee notes. The number of half-rupees is $\frac{15}{83}$ of that of rupee notes. Find the number of each sort.

4. The road from A to B consists of (1) an uphill (2) a level and (3) a down hill portion. The total distance is 5,500 yds. The level is $\frac{29}{84}$ of the uphill portion of the road

which is $\frac{14}{27}$ of the downhill portion. Find the length of each portion of the road.

5. A house is 60 ft. longer than it is broad. If the length is increased by 5 ft. and breadth also increased by 10 ft., the area will be increased by 950 sq. ft. Find the length and breadth of the house.

6. The length of a field is 27 yds. more than its breadth. If the length were increased by 25 yds. and the breadth diminished by 3 yds., the area would have been increased by 1,450 sq. yds. Find the dimensions of the field.

7. A lawn is 100 yds. longer than it is broad. There is another lawn whose length is 65 yds. less and whose breadth is 15 yds. more than the corresponding dimensions of the former. The perimeters of the two are in the ratio of 23 : 18. Find the dimensions of the first lawn.

8. A hall is 50 ft. longer than it is broad. If it is converted into a square of equal area, its length is 30 ft. less than that of the hall. Find the dimensions of the original hall.

9. A number of two digits is such that the digit in the units' place is $\frac{3}{8}$ of that in the tens' place. If the digits were reversed, the number formed would be half of the difference of the original number and 7. Find the number.

10. A number of two digits is such that the digit in the tens' place is $\frac{1}{3}$ of that in the units' place. If 54 is added to the number, the sum is the number formed by reversing these digits. Find the number.

11. A number of two digits is such that the digit in the units' place is greater than that in the tens' place by 5. If the digits are reversed, the new number formed exceeds the original number by 45. Find the number.

12. A number of two digits is such that the difference of the digits is three. If the number is diminished by $\frac{27}{85}$ of itself, the digits are reversed. Find the number.

13. A number of two digits is such that the digit in the units' place is smaller than that in the tens' place by 4.

If the number be diminished by $\frac{18}{5}$ times the sum of the digits, the digits will be reversed. Find the number.

14. One-fifth of a number increased by 24 is equal to $\frac{1}{6}$ of the difference between twice that number and 4. What is that number?

15. One-fourth of a number exceeds $\frac{1}{5}$ of it by a number which exceeds the difference between the eighth and tenth parts of the number by 8. Find the number.

16. Three-eighths of a number exceeds $\frac{4}{19}$ of the number formed by the addition of 120 to itself by 80. Find the number.

17. A person bought a certain number of eggs at four for a penny and sold them at eight for 3*d*. gaining thereby 2 *sh*. 6*d*. How many eggs did he buy altogether?

18. A merchant buys 1,260 mds. of corn, $\frac{1}{5}$ of which he sells at a gain of 5 per cent., $\frac{1}{3}$ at a gain of 8 per cent. and the remainder at a gain of 12 per cent. If he had sold the whole at a gain of 10 per cent. he would have obtained £22-13-0 more. What was the cost price per maund?

19. A merchant sold 40 qrs. of barley at a profit of 5 per cent. and 80 qrs. at a profit of 10 per cent. Had he sold the whole at a uniform profit of $7\frac{1}{2}$ per cent., he would have lost Rs. 56. Find the cost price of a quarter.

20. A person sells 160 oxen at a gain of 10 per cent. and 260 oxen at a gain of 40 per cent. Had he sold all at a uniform gain of 20 per cent. he would have Rs. 4,320 less. Find the prime cost of each ox.

21. A person bought a French watch bearing a duty of 50 per cent. and sold it at a loss of 10 per cent. Had he sold it for £6 more he would have cleared 2 per cent. on the bargain. What had the French maker for it?

22. A boy buys a number of oranges at 3 for 2 as. and a third of the number at 2 for 1 anna. He sells them at $\frac{3}{4}$ anna per orange and gains Rs. 4. Find the number of oranges he purchased.

23. I bought 100 horses and sold 30 of them at a gain of 40 per cent., 50 of them at a gain of 32 per cent. and the rest at cost price. Had I sold all at a gain of 36 per cent. I would have gained Rs. 1,120 more than I actually did. Find the cost price of each horse.

24. A man sells an article at a profit of 8 per cent. If he had bought it at 8 per cent. less and sold it for Re. 1-4-0 more he would have gained 20 per cent. What was the cost price ?

25. Ice is manufactured at 12 pies per lb. and sold for 18 pies per lb. Two-third of the quantity made is kept at the factory and the remainder is sent to branch shops. If the average loss from melting be $12\frac{1}{2}$ per cent. of the former and 25 per cent. of the latter and if the profit made was Rs. 35, find the quantity manufactured.

26. A watch was sold so as to give a profit of 12 per cent. Had it been bought at 8 per cent. less and sold for Rs. 13 less there would have been a profit of 10 per cent. For how much was the watch bought originally ?

27. A person bought a number of mangoes at Rs. 7-8-0 per 100 and an equal number at Re. 1-4-0 a dozen and sold the whole lot at $1\frac{1}{2}$ anna per fruit, thereby making a profit of 13 as. 4 ps. How many mangoes did he buy altogether ?

28. A fruit seller buys a certain number of apples at Rs. 6 per 100, twice as many pine-apples at Rs. 10-8-0 per 100 and sells the whole lot at an average price of 2 as. per fruit gaining thereby Rs. 15-12-0. Find the number of each sort.

29. A retail dealer bought equal numbers of two books at Rs. 2 and Rs. $1\frac{1}{2}$ respectively. If he spent his money equally on the purchase of both the kinds of books, he could have got 25 more books than he did. How many did he buy ?

30. A farmer buys equal numbers of 2 kinds of bullocks for ploughing his lands at Rs. 150 and Rs. 200 each respectively.

If he had spent his money equally on the two kinds he could have got 1 bullock more than he did. Find how many he bought.

31. A person bought a house and a garden their prices being in the ratio of 5 : 4. If the garden had cost Rs. 2,000 more and the house Rs. 3,000 less, their prices would be in the ratio of 12 : 5. Find the cost of the house and the garden.

32. A person buys a motor and coach, their prices being in the ratio of 70 : 17. If the motor costs Rs. 500 less and the coach Rs. 250 more, their prices would be in the ratio of 20 : 7. Find the cost of the motor and that of the coach.

33. A gentleman buys a certain number of oranges and gives his eldest son a tenth of it and 5 more. He gives his second son a fifth of the remainder and 6 more. He gives to each of his two daughters a fourth of the original number he purchased and a score more. He finds he has only 60 left. Find the number of oranges he purchased.

34. A train starts with its full complement of passengers. At the first station it drops $\frac{1}{4}$ of these and takes in 75 more. At the next station it drops $\frac{1}{3}$ of the new total and takes in 60 more. On reaching the next station the number left was found to exceed half the original number by 110. With what number did it start?

35. A tramcar starts with a number of passengers. Between the starting place and the terminus there are three places where it stops. At the first stopping, it drops $\frac{1}{5}$ of the number and takes in 2 more. At the second stopping it drops a third of the new total and takes in 4 more. At the third stopping it drops $\frac{1}{2}$ the new total and takes in 1 more. If 37 arrive at the terminus, with how many did it start?

36. Think of a number, add $5\frac{2}{3}$ to it, multiply the sum by 54 and divide the product by 107. A certain result is got. Again multiply the number by $5\frac{1}{3}$, add 58 to the

product, multiply the sum by 64 and divide this product by 109. A certain result is got. The sum of the former and latter result is 146. Find the number thought of.

37. Think of a number, subtract 3 from twice the number and divide the difference by the excess of the number over 3. A certain result is got. Again multiply the number by 3 and subtract 2 from the product. Divide this difference by the number increased by 2. A certain result is got. The sum of the former and latter results is 5. Find the original number.

38. Think of some number. Two new numbers are formed by adding $\frac{1}{6}$ and $\frac{1}{2}$ to it respectively. The two new numbers are multiplied and a product is formed. Similarly two new numbers are formed by adding $\frac{1}{4}$ to double the number and $\frac{1}{3}$ to the number thought of, respectively. The two numbers now formed are multiplied and a product is formed. The former product is divided by the latter and the result is found to be unity. Find the number.

39. Think of some number. Two new numbers are formed by adding $8\frac{1}{2}$ and $7\frac{1}{2}$ to it respectively. The new numbers are squared and the difference of their squares is obtained. Similarly two new numbers are formed by adding $3\frac{1}{3}$ and $2\frac{2}{3}$ to twice the number thought of respectively. The new numbers are squared and the difference of the squares is obtained. The difference of the squares formerly got is eight times the latter difference of the squares. Find the number thought of.

40. The interest on a certain sum of money at 5 per cent. for 6 years is less than the interest on a larger sum at 9 per cent. for 5 years by Rs. 30. The two sums differ by Rs. 50. Find the first sum.

41. The interest on Rs. 500 at 9 per cent. per annum for a certain number of years is Rs. 18 less than the interest on Rs. 480 at 12 per cent., the time being 1 year less than in the former case. Find the time.

42. The interest on Rs. 600 for $5\frac{1}{2}$ years at a certain rate per cent. is Rs. $61\frac{1}{2}$ more than the interest on Rs. 500 for $6\frac{1}{3}$ years at a smaller rate of interest differing from the first by $1\frac{1}{2}$ per cent. Find the rate.

43. The incomes of two schools are in the ratio of 3 : 2. Their expenses amount to Rs. 2,200 and Rs. 1,850 respectively. Their savings are in the ratio of 16 : 3. Find the incomes of the two schools.

44. The incomes of two brothers are in the ratio of 6 : 2. They spend Rs. 450 and Rs. 125 respectively and it is found their savings are in the ratio of 2 : 1. Find their incomes.

45. A's age : B's age :: 5 : 3; 20 years hence their ages will be in the ratio of 15 : 11. Find their present ages.

46. The ages of a father and a son are in the ratio of 8 : 3. Five years ago their ages were in the ratio of 7 : 2. Find their present ages.

47. Six years ago, the ages of a husband and wife were in the ratio of 14 : 13. 12 years hence their ages will be in the ratio of 20 : 19. Find their present ages.

48. A gentleman owns a house, a garden and a farm. The garden costs Rs. 12,000 more than the house but Rs. 33,000 less than the farm. He has two sons A and B to whom he gives $\frac{2}{5}$ and $\frac{1}{3}$ portions of the house, $\frac{1}{5}$ and $\frac{2}{9}$ portions of the garden and $\frac{3}{8}$ and $\frac{4}{15}$ of the farm respectively. It is found that A's inheritance is Rs. 6,900 more than B's. Find the value of each kind of property.

49. A gentleman is a partner in a firm dealing in leather, indigo and groundnut. In a certain year its profits in indigo, exceeded those in leather by Rs. 7,000, but were less than those in groundnut by Rs. 6,000. Next year its profits in leather increased by a seventh, while those in indigo and groundnut showed an increase of Rs. 2,100 and Rs. 2,250 over the corresponding profits

in the previous year. The gentleman got $\frac{1}{5}$, $\frac{1}{7}$ and $\frac{1}{11}$ of the profits in leather, indigo, and groundnut in the first year, but obtained only $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{25}$ of the profits in them respectively in the second year. His total share of the profits in the second year was Rs. 350 less than that in the first year. Find the profits of the firm in the first of these two years in these lines of business.

50. The perimeter of a right-angled triangle is 30 units. One of the sides containing the right angle is 12 units. Find the lengths of the other sides.

51. The perimeter of a right-angled triangle is 36 units. One of the sides containing the right angle is 12 units. Find the lengths of the other sides.

52. The base of a right-angled triangle measures 6 units. The difference of the hypotenuse and the other side is 2 units. Find the sides of the right-angled triangle.

53. The sum of the sides containing the right angle in a right-angled triangle is 49 units. The difference between the hypotenuse and the larger side is 1. Find the lengths of the sides.

54. A railway train 66 yds. long is running at the rate of 20 miles an hour. In what time will it pass a telegraph post?

55. A railway train 66 yds. long travels at 15 miles an hour. In what time will it pass a pointsman?

56. A railway train 110 yds. long passes a passenger on the platform in x seconds. Find x if the rate of the train is 10 miles per hour.

57. A railway train 77 yds. long crosses a bridge 1 mile 880 yds. long. If it travels at the rate of 18 miles per hour, what time does it take to cross the bridge?

58. An express train 132 yds. long travelling at the rate of 30 miles per hour passes a station 110 yds. long where it does not stop. In what time will it clear the station?

59. A train 55 yds. in length travels at 28 miles an hour. In what time will it pass a man walking (*a*) in the same direction as the train, at 4 miles an hour and (*b*) in the opposite direction at 4 miles an hour?

60. A train 66 yds. in length travels at 27 miles an hour. In what time will it pass a man running (*a*) in the same direction as the train, at 8 miles an hour and (*b*) in the opposite direction at 8 miles an hour?

61. A train 99 yds. long travels at 30 miles an hour. In what time will it pass a man sitting in another train (*a*) moving in the same direction as the first and (*b*) in the opposite direction as the first? The rate of the latter train is 16 miles an hour.

62. A mail train 44 yds. long travels at 40 miles per hour. In what time will it pass a man sitting in a shuttle train moving in the same direction as the mail train? The rate of the shuttle is $17\frac{1}{2}$ miles per hour.

63. Two trains 60 yds. and 50 yds. long are going (*a*) in opposite directions and (*b*) in the same direction at 24 and 36 miles per hour respectively. In what time will they pass each other?

64. Two trains 84 yds. and 70 yds. long are going (*a*) in the same direction and (*b*) in opposite directions at 28 and 32 miles per hour respectively. In what time will they pass each other?

65. Find the respective times between 8 and 9 o'clock when the hour and minute hands of a watch (*a*) are exactly opposite to each other, (*b*) at right angles to each other and (*c*) are coincident.

66. Find the respective times between 7 and 8 o'clock when the hour and minute hands of a watch (*a*) are exactly opposite to each other, (*b*) at right angles and (*c*) are coincident.

67. It is observed that between 1 and 2 the number of minute divisions between the hour hand and minute hand at a certain instant is to the number of minute divisions

between them 10 minutes later as 7 : 15. What is the time ?

68. It is observed that between 8 and 9 o'clock that the number of minute divisions between the hour hand and minute hand at a certain instant : the number of minute divisions between them 20 minutes later as 37 : 15. What is the time ?

69. An astronomical clock has its dial divided into 24 divisions instead of 12. The small hand goes round once in 24 hours and the large hand once every hour. The 24th hour is noon. When will the two hands be (1) at right angles, (2) opposite to one another and (3) coincident (*a*) between 3 and 4 and (*b*) between 15 and 16 ?

70. In question 69 find the interval of time between two successive coincident positions of the hour and minute-hands. Hence find how often and when the two hands will be coincident between 12 and 6.

71. In question 69, find the interval of time between two successive exactly opposite directions of the hour and minute hands. Hence find how often and when the two hands will be exactly opposite to each other between 6 and 12.

72. In question 69, find the interval of time between two successive perpendicular positions of the hour and minute hands. Hence find how often and when the two hands will be at right angles between 3 and 9.

73. Find the interval of time between two successive coincident positions of the hour and minute hands in a watch.

74. Find the interval of time between two successive opposite directions of the hour hand and minute hand in a watch.

75. Find the interval of time between two successive perpendicular positions of the hour-hand and minute-hand in a watch.

CHAPTER XXV.

Solution of Simultaneous Equations of Two Unknown Quantities of the First Degree and Problems leading to them.

Model No. (1) A gentleman buys 15 second class tickets and 18 third class tickets for a marriage party proceeding from Madras to Thirupathy for Rs. $110\frac{1}{4}$. He spent Rs. $89\frac{1}{4}$ in buying 12 second class and 15 third class tickets when the party returned to Madras. Find the fare of each class of ticket.

Let the fare of a second class ticket be Rs. x.

,, ,, third ,, be Rs. y.

Then $15x + 18y = 110\frac{1}{4}$ . . . . . (1).

$12x + 15y = 89\frac{1}{4}$ . . . . . (2).

Equation (1) can be satisfied by many pairs* of values of x and y. Equation (2) can be satisfied by many pairs of values of x and y. We are now required to find a pair of particular values of x and y that will satisfy both the equations.

To find a pair of values of x and y that satisfy both the equations, we proceed thus :—

Method (1) The L.C.M.† of the co-efficients of x in both the equations is 60. To get $60x$ in equation (1) every term in it should be multiplied by 4.

$\therefore 60x + 72y = 441$ . . . . . (3)

---

* Such pairs can be easily found thus :—

$$15x = \frac{441}{4} - 18y.$$

$$\therefore x = \frac{441 - 72y}{60}.$$

If $y = 1$, then $x = \frac{441 - 72}{60} = \frac{369}{60} = 6\frac{3}{20}$.

If $y = 2$, then $x = \frac{441 - 144}{60} = \frac{297}{60} = 4\frac{57}{60}$.

Thus for any value of y, a corresponding value of x can be found. Hence several pairs of values of x and y can be obtained. Similarly several pairs of values of x and y can be found to satisfy the second equation.

† You may find the L.C.M. of the co-efficients of y and choose the proper multiplier in each case to get the same number of y as is indicated by the L.C.M. By subtraction y may be eliminated.

Similarly to get $60x$ in equation (2) every term in it should be multiplied by 5.

$\therefore 60x + 75y = \frac{357}{4} \times 5 = \frac{1785}{4}$ . . . . . (4).

Subtract (4) from (3).*

$60x + 72y - 60x - 75y = 441 - \frac{1785}{4} = \frac{1764 - 1785}{4} = \frac{-21}{4}$.

$\therefore -3y = -\frac{21}{4}$.

$\therefore y = \frac{21}{4 \times 3} = 1\frac{3}{4}$

To find the value of x, this value of y may be substituted in equation (1) or (2). Substituting $1\frac{3}{4}$ for y in equation (1) we get

$15x + 18 \times \frac{7}{4} = 110\frac{1}{4}$.

$\therefore 15x = 110\frac{1}{4} - 31\frac{1}{2} = 78\frac{3}{4} = \frac{315}{4}$.

$\therefore x = \frac{315}{4 \times 15} = 5\frac{1}{4}$.

$\therefore$ the fare of a third class ticket is Re. $1\frac{3}{4}$.

$\therefore$,, ,, second ,, Rs. $5\frac{1}{4}$.

Method (2) $15x + 18y = \frac{441}{4}$.

$\therefore 15x = \frac{441}{4} - 18y$.

$x = (\frac{441}{4} - 18y) / 15$ . . . . . (1).

Similarly $12x + 15y = \frac{357}{4}$.

$\therefore 12x = \frac{357}{4} - 15y$.

$\therefore x = (\frac{357}{4} - 15y) / 12$ . . . . . (2).

$\therefore \frac{\frac{441}{4} - 18y}{15} = \frac{\frac{357}{4} - 15y}{12}$ . . . . . from (1) and (2).

We have now eliminated x, thus deriving an equation of one unknown quantity. Multiply every term in the equation by the L.C.M. of the denominators 15 and 12, i.e., 60.

$\therefore 4(\frac{441}{4} - 18y) = 5(\frac{357}{4} - 15y)$.

$\therefore 441 - 72y = \frac{1785}{4} - 75y$.

$\therefore 75y - 72y = \frac{1785}{4} - 441 = \frac{1785 - 1764}{4} = \frac{21}{4}$.

$\therefore 3y = \frac{21}{4}$. $\therefore y = \frac{21}{4 \times 3} = 1\frac{3}{4}$.

---

* By subtraction we get rid of x. **Getting rid of a quantity is called elimination.**

The value of x may as before be obtained by substituting the value of y either in equation (1) or (2).

Method (3) $15x + 18y = \frac{441}{4}$ . . . . . (1).

$$12x + 15y = \frac{357}{4} \quad \text{. . . . . (2).}$$

To eliminate the absolute term, multiply (1) by 357 and (2) by 441 and subtract.

$$\therefore 15 \times 357x + 18 \times 357y = \frac{441 \times 357}{4} \quad \text{. . . . . (3)}$$

$$12 \times 441x + 15 \times 441y = \frac{357 \times 441}{4} \quad \text{. . . . . (4).}$$

$$\text{or } 5355x + 6426y = \frac{441 \times 357}{4} \quad \text{. . . . . (3)}$$

$$\text{and } 5292x + 6615y = \frac{441 \times 357}{4} \quad \text{. . . . . (4)}$$

$$\therefore 63x - 189y = 0$$

$$\therefore 63x = 189y$$

$$\therefore x = 3y.$$

Substitute this value of x in equation (1), i.e., $15x + 18y = 110\frac{1}{4}$.

We get $15 \times 3y + 18y = 110\frac{1}{4}$

$$\therefore 63y = \frac{441}{4}$$

$$\therefore y = \frac{\overset{47}{\cancel{441}}}{\cancel{4}} \times \frac{1}{\cancel{9}3} = \frac{7}{4}.$$

$$\therefore x = 3y = 3 \times \frac{7}{4} = \frac{21}{4} = 5\frac{1}{4}.$$

Thus either x or y or the absolute term may be eliminated. The student will have to determine in a particular problem which quantity will present the least trouble in its elimination.

Model No. (2) A can do a piece of work in 6 days. Assisted by B he can finish it in 4 days. Assisted by B and C he can finish it in 2 days. Find the number of days in which B and C can do the work separately.

Let B do the work in x days
and C ,, ,, y days

A in 6 days does 1 work. B in x days does 1 work.

$\therefore$,, 4 days ,, $\frac{4}{6}$ work. $\therefore$,, 4 ,, $\frac{4}{x}$ work.

$$\therefore \frac{4}{6} + \frac{4}{x} = 1 \quad \text{. . . . . (1).}$$

In 2 days A does $\frac{2}{6}$ work ; B does $\frac{2}{x}$ work ; and C does $\frac{2}{y}$ work.

$$\therefore \frac{2}{6} + \frac{2}{x} + \frac{2}{y} = 1 \quad \ldots\ldots\ldots \quad (2).$$

$$\therefore \frac{2}{6} + \frac{2}{x} = \tfrac{1}{2} \text{ from (1)} \quad \ldots\ldots\ldots \quad (3).$$

$\therefore$ subtracting (3) from (2), $\frac{2}{y} = 1 - \frac{1}{2} = \frac{1}{2}$.

$$\therefore y = 4.$$

Substitute this value of y in 2.

$$\therefore \frac{2}{6} + \frac{2}{x} + \frac{2}{4} = 1$$

$$\therefore \frac{2}{x} = 1 - \frac{2}{4} - \frac{2}{6} = 1 - \tfrac{1}{2} - \tfrac{1}{3} = \tfrac{1}{6}$$

$\therefore x = 12$ days $\}$ i.e. B does the work in 12 days.
$y = 4$,, $\}$ C ,, ,, 4 ,, $\}$

Model No. (3) There are a certain number of students in a college hostel. If two of them sleep in a room, 5 students have no accommodation. If however 3 sleep in a room, 31 rooms are empty and one of the remaining rooms is to be occupied by a single student. Find the number of rooms and students in the hostel.

Let x be the number of students in the college hostel
and y ,, ,, rooms in it.
In 1 room 2 students sleep.
$\therefore$ in y rooms $2y$ students sleep
$\therefore 2y + 5 = x \quad \ldots\ldots\ldots \quad (1)$
In 1 room 3 students sleep.
$\therefore$ in $(y - 32)$* rooms $3(y - 32)$ students sleep.
$\therefore 3(y - 32) = x - 1 \quad \ldots\ldots\ldots \quad (2).$
Substituting the value of x given by (1) in (2) we get
$3(y - 32) = 2y + 5 - 1.$
$\therefore 3y - 96 = 2y + 4.$
$\therefore y = 96 + 4 = 100.$
$\therefore x = 200 + 5 = 205.$ 205 students $\}$ and 100 rooms. $\}$

Model No. (4) A and B run a race of 440 yds. At first A gives B a start of 2 seconds and beats him by 11 yds.

---

* Out of the y rooms, 31 rooms are empty and one is occupied by 1 student. $\therefore$ $(y - 32)$ rooms are each occupied by 3 and there are $(x - 1)$ students in them.

Then A gives B a start of 33 yds. and is beaten by 1 second. How many yards per second does each run?

Let A run x yds. and B y yds. per second. A gives B a start of 2 sec., i.e., B has gone in advance of A, $2y$ yds. When A reaches the winning post, B is 11 yds. behind it.

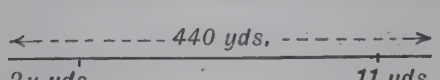


FIG. 163.

$\therefore$ B takes to run the remaining distance $(440 - 2y - 11)$ yds.

$\frac{440 - 2y - 11}{y}$ seconds. In that time A runs the whole distance of 440 yds., i.e., A takes $\frac{440}{x}$ seconds.

$$\therefore \frac{440 - 2y - 11}{y} = \frac{440}{x} \quad \ldots\ldots \quad (1).$$

When A gives B a start of 33 yds. A has to run 440 yds., and B has to run $(440 - 33)$ yds. and A takes 1 second more than B.

$$\therefore \frac{440}{x} = \frac{440 - 33}{y} + 1. \quad \ldots\ldots\ldots \quad (2).$$

From (1) $\frac{429 - 2y}{y} = \frac{440}{x}$. (3)

,, (2) $\frac{407}{y} + 1 = \frac{440}{x}$. (4)

Eliminating x from (1) and (2)

$$\frac{429 - 2y}{y} = \frac{407}{y} + 1.$$

$$\frac{429}{y} - \frac{407}{y} = 1 + 2\,\frac{y}{y} = 1 + 2$$

$$\therefore \frac{22}{y} = 3.$$

$\therefore 3y = 22$ $\therefore y = 7\frac{1}{3}$ yds.

i.e. B runs $7\frac{1}{3}$ yds. per second.

From (4) $\frac{407}{y} + 1 = \frac{440}{x}$.

$$\frac{407 \times 3}{22} + 1 = \frac{440}{x}.$$

$$\therefore \frac{111}{2} + 1 = \frac{440}{x}.$$

$$\therefore \frac{113}{2} = \frac{440}{x}.$$

$$\therefore 113x = 880.$$

$\therefore x = \frac{880}{113} = 7\frac{89}{113}$ yds. per second, i.e., A runs $7\frac{89}{113}$ yds. per second.

Exercise XXV.

Express x in terms of y in the first equation in questions 1 to 4 and substitute that value of x in the second equation and solve :—

1. (*a*) $6y + 4x = 8.$ 2. (*a*) $\frac{3x}{4} + 2y = 14.$

   (*b*) $3x - 8y = -1.$ (*b*) $x - \frac{y}{2} = \frac{13}{2}.$

3. (*a*) $5x - \frac{3y}{5} = 27.$ 4. (*a*) $\frac{x}{2} + \frac{y}{3} = 2.$

   (*b*) $\frac{4x}{7} + \frac{4y}{35} = 4.$ (*b*) $\frac{2x}{5} + \frac{3y}{7} = 2\frac{3}{35}.$

Express y in terms of x in the first equation and substitute that value of y in the second equation in questions 5 to 8 and solve :—

5. (*a*) $8x + 9y = 9.$ 6. (*a*) $\frac{2x}{3} + 4y = 8.$

   (*b*) $2x - 3y = 4.$ (*b*) $-\frac{4x}{5} + \frac{7y}{5} = 9.$

7. (*a*) $6x - \frac{7y}{8} = 106\frac{11}{16}.$ 8. (*a*) $\frac{x}{4} + \frac{y}{5} = 9.$

   (*b*) $\frac{7x}{9} + 4y = 20.$ (*b*) $\frac{3x}{5} - \frac{y}{2} = 2.$

Solve the equations in questions 9 to 12 by making the co-efficients of x the same in both equations :—

9. $6x + 5y = 20.$
   $7x + 4y = 49.$

10. $11x + 15y = 37.$
    $20x - 13y = 27.$

11. $3x - 9y = 30.$
    $7x + 4y = 20.$

12. $132x - 146y = 59.$
    $143x + 81y = 184.$

Solve the equations in questions 13 to 16 by making the co-efficients of y the same in both the equations :—

13. $2x - y = 1.$
    $4x + 3y = 9.$

14. $10x + 27y = 77.$
    $7x - 18y = 17.$

15. $\frac{1}{3}x + \frac{1}{4}y = 9\frac{1}{6}.$
    $\frac{1}{2}x - \frac{1}{5}y = 8.$

16. $24x + 441y = 465.$
    $500x - 483y = 17.$

Solve the equations in questions 17 to 20 by eliminating the independent terms and finding the ratio of x to y or y to x :—

17. $18x - 25y = 11.$
$19x + 12y = 50.$

18. $11\frac{1}{3}x + 14y = 150.$
$12\frac{1}{2}x - 12y = 138.$

19. $15x + 16\frac{1}{2}y = 264.$
$23x - 15\frac{1}{3}y = 162.$

20. $\frac{x}{5} + \frac{6y}{7} = 2.$
$\frac{x}{6} - \frac{8y}{3} = -22.$

Reduce the equations in questions 21 to 24 to the form $ax + by = c$ and then solve :—

21. $4(x + 1) - 5(y + 2) = 15.$
$6(x + \frac{1}{2}) - 8(y + \frac{1}{4}) = 20.$

22. $\frac{x+1}{2} - \frac{y+2}{3} = 8.$
$\frac{2x+6}{4} + \frac{y+3}{5} = 14.$

23. $\frac{3}{2}(2x + 1) + \frac{5}{4}(4y + 3) = 34\frac{3}{4}.$
$\frac{4}{3}(5x + 3) + \frac{6}{5}(2y - 5) = 20.$

24. $\frac{x}{4} + \frac{y}{5} - \frac{x}{3} - \frac{y}{2} = 1.$
$-\frac{9x}{4} + 4 = \frac{6y}{4} - 2.$

25. $4x + 6y = 0.$
$8x + 9y = 0.$

26. $ax + by = 0.$
$cx + dy = 0.$

What do you learn from Questions 25 and 26 ?

Solve :—

27. $3y = 4x + 5.$
$4x = 7y + 8.$

28. $5x + 6y - 7 = 0.$
$8x - 3y - 5 = 0.$

29. $11x - 9y + 4 = 0;$ $7x + 6y - 28 = 0.$

30. $23x - 11y + 29 = 0;$ $15x - 8y + 31 = 0.$

31. $12x + 15y - 22 = 0;$ $8x + 9y - 14 = 0.$

32. $ax + by + c = 0;$ $dx + ey + f = 0.$

33. $lx + my + n = 0;$ $px + qy + r = 0.$

34. $a_1x + b_1y + c_1 = 0;$ $a_2x + b_2y + c_2 = 0.$

35. $\frac{x}{4}+\frac{y}{3}=2\,;\ \frac{x}{5}-\frac{y}{6}=-1.$

36. $\frac{3x}{5}-\frac{4y}{7}=\frac{26}{7}\,;\ \frac{5x}{6}+\frac{11}{12}y=12.$

37. $\frac{x}{a}+\frac{y}{b}=c\,;\ \frac{x}{d}+\frac{y}{e}=f.$

38. $\frac{ax}{b}+\frac{by}{c}=d\,;\ \frac{bx}{a}+\frac{c}{b}y=e.$

39. $\frac{p-3}{4}+\frac{q-5}{6}=7\,;\ \frac{p}{10}+\frac{6(q-3)}{7}=3$ (p and q being the unknown quantities.)

40. $5p+3q=15\,;\ 4p-7q=21$ (p and q being the unknown quantities).

41. $5A+3B=10\,;\ 4A-9B=15.$

42. $5X+6Y=23\,;\ 9X-3Y=22.$

43. $9u+3v=44\,;\ 7u-4v=42.$

44. $3x+\frac{4}{y}=7\,;\ 5x-\frac{3}{y}=2.$

45. $\frac{8}{x}+3y=11\,;\ \frac{7}{x}-4y=3.$

46. $\frac{8}{x}-\frac{9}{y}=-1\,;\ \frac{4}{x}+\frac{11}{y}=15.$

47. $\frac{8}{u+v}-\frac{9}{u-v}=4\,;\ \frac{9}{u+v}+\frac{10}{u-v}=6.$

48. $(x+3)(y+4)=xy-10.$
$(x+8)(y-9)=xy+23.$

49. $\frac{1280}{x}-\frac{1320}{y}=20.$
$\frac{1600}{x}+\frac{2200}{y}=200.$

50. $\frac{5}{x+y}+\frac{6}{x-y}=12\tfrac{1}{2}\,;\ \frac{8}{x}=\frac{7}{y}.$

51. $8x+3y+14=6x-2y+18=3x+4y-7.$

52. $2\tfrac{1}{2}x-3\tfrac{1}{3}y+7=4\tfrac{1}{3}x-5\tfrac{1}{4}y+8=6\tfrac{1}{4}x-8\tfrac{1}{3}y+9.$

53. $8y+\frac{9x+3y}{6}=2\,;\ 7x+\frac{6x-3y}{2}=10.$

54. $(x + 9)(y - 3) = xy = (x - 20)(y + 12)$.

55. $3x + 4y = 28$; $(x - 10)(y - 8) = xy + 300$.

56. $7x + 6y = 13xy$; $8x + 9y = 17xy$.

57. $\frac{3}{x+1} - \frac{6}{y+2} = 2$; $\frac{4}{x+1} = \frac{3}{y+2}$.

58. $\frac{8}{x+1} - \frac{6}{y+1} = 4$; $\frac{3}{x+1} + \frac{4}{y+1} = 7$.

59. $x(a + b) + y(a - b) = 2a$; $3x + 5y = 8$.

60. $x(a^2 + ab + b^2) - y(a^2 - ab + b^2) = 2ab$; $7x - 3y = 4$.

61. A gentleman purchased a motor-car and a motor cycle for Rs. 4,000. The motor-car costs Rs. 2,400 more than the motor cycle. Find the cost of each.

62. A merchant sold his carriage and horse for Rs. 1,750. The horse was worth Rs. 250 more than the carriage. Find the price of each.

63. The sum of two numbers is 31. Their difference is 5. Find the numbers.

64. Twice a number plus thrice another number is equal to 17. 5 times the first number diminished by 4 times the second is equal to 8. Find the numbers.

65. A teacher had 5 sons, 2 of whom were reading in the III Form and 3 in the IV Form in a certain year. He then paid Rs. 23 for their school fees. One of the sons in the III Form was not promoted and all the sons in the IV Form were not promoted. The next year their total fees amounted to Rs. 29 at the same rates as before. Find the school fees for the III and IV Forms respectively.

66. A tailor supplied 5 khaki shirts and 6 khaki shorts for Rs. 22-12-0 to a scout troop. The same tailor supplied 12 khaki shirts and 13 khaki shorts to another troop for Rs. 52-8-0 at the same rates. What did he charge for a shirt and a pair of shorts ?

67. Three rickshaws and 4 jutkas were made for Rs. 660; another time 5 rickshaws and 6 jutkas were made for

Rs. 1,020 at the same rate. Find the price of a rickshaw and a jutka.

68. A bought a camp-cot and a camp-table for Rs. 19-12-0. B bought 4 camp-cots and 3 camp-tables for Rs. 74-4-0 from the same shop. Find the price of a cot and a table.

69. A and B played at cards and had together Rs. 9 when they commenced the play. At the end of the play A gained Re. 1 and B lost Rs. 2. A then had thrice as much money as B. What money had each at first?

70. Two gentlemen went to the Guindy races with an aggregate sum of Rs. 350. In betting for the horses, one gentleman gained Rs. 500 and the other lost Rs. 100. The fortunate gentleman had 14 times as much money as the other. With what sums did they start for the Guindy races?

71. A woman bought for Rs. 8-12-0 a number of plantain fruits at 12 as. per 100 and some mangoes at Rs. 10 per 100. 20 mangoes and 25 plantains were not sold and she gained 15 as. by selling the remaining mangoes at Re. 1-8-0 per dozen and the remaining plantains at 5 for 1 anna. Find the number of plantains and mangoes bought.

72. Three tennis bats and 5 presses cost Rs. 87; 5 tennis bats and 8 presses of the same kind cost Rs. 144. Find the price of a bat and of a press.

73. There are two tennis courts belonging to a school, one for the members of the staff and the other for students. In the month of April the total collections from the two courts were Rs. 36; but in the month of May $\frac{1}{6}$ of the original number of students and $\frac{7}{12}$ of the number of teachers left the place for the vacation and the total collections in that month amounted to Rs. 20. Find the number of members in each court assuming a teacher and a student pay Rs. 2 and Re. 1 respectively per month.

74. Two teachers of a school together earn Rs. 210. One pays 6 as. more for the subscription of the reading

room than the other. The formula for calculating the subscription to be paid by a teacher for the reading room is Re. $\left(1 + \frac{s - 30}{10} \times \frac{1}{8}\right)$ where s is the salary of a teacher. Find the salaries of the two teachers.

75. In a class $\frac{1}{3}$ of the number of second year students and $\frac{1}{4}$ of the number of the first year students are eligible for promotion and their total number is 100. The students that have not been promoted are 260. Find the number of first year students and second year students.

76. In a certain library $\frac{1}{3}$ of the number of books in the consulting branch and $\frac{1}{5}$ of the number of books in the other branches are bound and their number is 2,150. The rest are books that are not bound and number 5,900. Find the number of books in the consulting branch and other branches.

77. A doctor charges a certain fee for each visit he pays to a patient at his residence and another fee for each consultation in his own consulting room. The doctor treated 20 cases in his room, paid 15 outside visits and made an income of Rs. 75. The next day he treated 35 cases in his room, paid 30 visits and derived an income of Rs. 135. Find the amount charged for each consultation and each visit.

78. A number consisting of two digits is such that the difference of the digits is three and that the difference between thrice the smaller digit and the greater is equal to the greater digit diminished by one. Find the number. How many solutions are there ?

79. The difference between a number of two digits and that obtained by reversing them is 63. The greater digit increased by 1 is 5 times the smaller digit. Find the number.

80. A number consists of two digits. The number formed by reversing the digits is $\frac{2}{9}$ of the original number. The difference between the digits is 7. Find the number.

81. A number consists of two digits. The number formed by reversing the digits is greater than twice the original number by 7. The difference between the two numbers exceeds 4 times the sum of the digits by unity. Find the number.

82. A number consists of three digits. If the first and last digits are interchanged, the number so formed exceeds the original number by 396. The sum of the first and last digits is twice the middle digit and their difference is equal to the middle digit. Find the number.

83. Four times a son's age exceeds twice that of the father by 2 years. Ten years hence twice the father's age exceeds thrice the son's by 6 years. Find the ages of the father and the son.

84. The ages of a grandfather and his grandson are such that 4 times the grandson's age a year hence would be equal to that of the grandfather 3 years ago. Ten years hence the grandfather's age will exceed thrice that of his grandson by 1 year. Find the ages of the grandfather and the grandson.

85. Sixteen years ago, a father was 4 times as old as his son. Twelve years hence their ages will be in the ratio of 5 : 3. Find their present ages.

86. A straight line 25 cm. long is divided into two unequal parts such that 4 times the smaller exceeds thrice the greater by $1\frac{3}{4}$ times the smaller. Find the lengths of the parts.

87. A straight line AB 12 in. long is produced to C such that $2\frac{1}{2}$ times AC + $5\frac{1}{2}$ times BC is greater than 8 times AB by 2 in. Find the lengths of AC and BC.

88. C and D are two points on a given line AB 10 cm. long such that AB : CD = 9 : 5 and 3 AB + 5 CD = 10 BC. Find the lengths of the three parts of AB.

89. X is a point in a straight line AB 5·2 in. long such that AX : XB = 7 : 6. Find the lengths of AX and BX. (Use two unknown quantities to solve the problem.)

90. X is a point in a straight line AB (6 ft. 4 in. long) produced such that AX : XB = 7 : 3. Find the lengths of AX and BX. (Use two unknown quantities to solve the problem.)

91. AB is a straight line 9·6 cm. long. X and Y are points in AB and AB produced respectively such that $\frac{AX}{XB} = \frac{AY}{YB} = \frac{p}{q}$. Find the length of XY.

92. A daughter's age last year was $\frac{1}{3}$ of what the mother's age would be next year. 7 years hence the mother's age would be 2 years more than thrice the daughter's age. Find their ages.

93. If 1 is subtracted from the numerator and also from the denominator of a fraction, it becomes $\frac{1}{2}$. If 1 is added to the numerator and the denominator is doubled, it becomes $\frac{1}{3}$. Find the fraction.

94. A certain fraction is equal to $\frac{5}{12}$. If a new fraction is formed having for its denominator $\frac{2}{3}$ of the number formed by adding one to its original numerator and for its denominator $\frac{1}{3}$ of the number formed by adding 6 to the original denominator, the new fraction $= \frac{7}{9}$. Find the fraction.

95. The sum of the numerator and the denominator of a fraction is 24. If each is increased by 3 the fraction is equal to $\frac{1}{2}$. Find the fraction.

96. If you add to the numerator $\frac{1}{4}$ of itself and to the denominator $\frac{4}{5}$ of itself the fraction becomes $\frac{2}{5}$. If 2 is subtracted from the numerator and 5 from the denominator, then the fraction $= \frac{7}{12}$. Find the fraction.

97. The cost of 5 viss of ghee and 3 viss of oil is Rs. 17-12-0. The cost of 3 viss of ghee and 5 viss of oil is Rs. 16-4-0. Find the price of a viss of ghee and a viss of oil.

98. 7 men and 12 women earn Rs. 38-11-6 in $3\frac{1}{2}$ days. 4 men and 8 women earn Rs. 10-2-0 in $1\frac{1}{2}$ days. Find how much a man and a woman earn every day.

99. I have a certain number of rupees and a certain number of half-rupees amounting in value to Rs. 38½. If the numbers were interchanged I would have Rs. 10 more. Find how many coins of each sort I had.

100. I have a certain number of rupees and a certain number of nickel anna coins amounting in value to Rs. 45. If the numbers were interchanged I would have Rs. 356-4-0 more. Find how many of each sort I had?

101. Veerasawmy Achari, a carpenter and contractor, supplied to a school in the year 1921, 20 benches with backs and desks and 15 chairs for Rs. 655. The next year he supplied 12 benches and 10 chairs for Rs. 406, the prices having remained unchanged. Find what he charged for each bench and chair.

102.

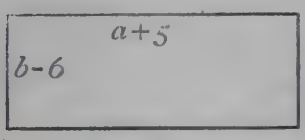


FIG. 164.

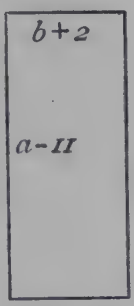


Fig. 165.

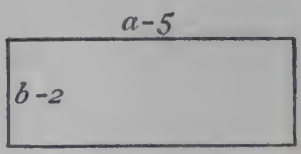


FIG. 166.

The above rectangles are equal in area. Find the length and breadth of each rectangle.

103.

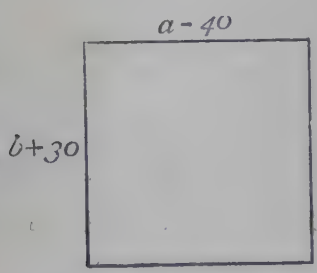


FIG. 167.

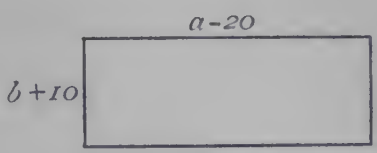


FIG. 168.

The above square and rectangle are equal in area. Find their dimensions.

104. Two fields of which one is rectangular and the

other square are equal in area. The length and breadth of the rectangle are 8 yds. greater and 7 yds. less than a side of the square respectively. Find the dimensions of the rectangle and the square. (Use two unknown quantities to frame the equations.)

105. The perimeters of a rectangle and a square are equal. The area of the square is 20736 sq. ft. The difference of the sides of the rectangle is $\frac{2}{9}$ of the side of the square. Find the sides of the rectangle.

106. If the length of a rectangle is diminished by 16 yds. and the breadth increased by 12 yds., it becomes a square equal in area to the rectangle. Find the dimensions of the rectangle and square.

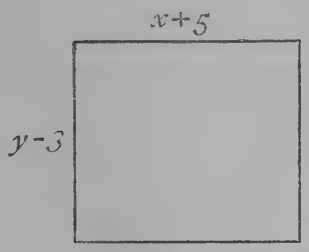


FIG. 169.

107. The side of a square may be represented by either $(x + 5)$ yds. or $(y - 3)$ yds. The perimeter of the square is 48 yds. Find the dimensions of the field.

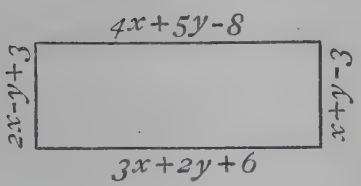


FIG. 170.

108. The sides of a rectangle are represented by the measurements shown in Fig. 170. Find the dimensions of the rectangle.

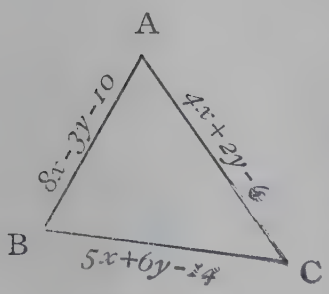


FIG. 171.

109. The sides of a scalene triangle are represented by the measurements in the figure. $8x-3y-10 : 5x+6y-14 : 4x+2y-6 :: 3 : 2 : 4$. Find the lengths of the sides of the triangle.

110. In the Mylapore Fund there are two kinds of loans, ordinary and special. A subscriber taking a special loan of Rs. 4,800 and an ordinary loan of Rs. 2,400 pays a total

sum of Rs. 40-8-0 for interest per month. Another subscriber taking a special loan of Rs. 3,200 and an ordinary loan of Rs. 800 pays a total sum of Rs. 22-13-4 for interest per month. Find the rate per cent. of interest charged on each kind of loan.

111. A school gets books for its library in two lots, each lot consisting of two sorts of books all being English publications. The exchange allowed for one sort of books is quite different from that for the other sort. The first lot included £50 worth of books of the first sort and £30 worth of books of the second sort and cost Rs. 1,275 in Indian money. The second lot included £48 worth of books of the first sort and £36 worth of books of the second sort and cost Rs. 1,350. Find the rate of exchange allowed for each sort in annas per shilling.

112. In the relation $y = ax + b$, $y = 3$ when $x = 0$; $y = 8$ when $x = 1$. Express the relation with values of a and b substituted in it.

113. In the relation $\frac{x}{a} + \frac{y}{b} = 1$, $y = 16$ when $x = 3$; and $y = 13\frac{1}{3}$ when $x = 5$. Express the relation with the values of a and b substituted in it.

114. $y = mx + c$ is satisfied by the values 0, 5 and 3, 2 of x and y respectively. Rewrite the equation with the values of m and c substituted in it.

115. $\frac{x}{h} + \frac{y}{k} = 1$ is satisfied by the values 10, 0 and 5, $-\frac{3}{2}$ of x and y respectively. Rewrite the equation with the values of h and k entered in it.

116. If $ax^2 + bx + c = 7$ when $x = 0$
$= 18$,, $x = 1$
$= 39$,, $x = 2$

} rewrite the expression putting in the values of a, b and c. Evaluate the expression when $x =$ (i) $\frac{1}{2}$ (ii) -3.

117. If $px^2 + qx + r = 8$ when $x = 0$
$= 8\frac{3}{4}$,, $x = \frac{1}{4}$
$= -1$,, $x = 3\frac{1}{2}$
} rewrite the expression putting in the values of p, q and r. Evaluate the expression when $x =$ (i) $\frac{2}{3}$ (ii) -8.

Given that the following relations are true for all values of x, in questions 118 and 119 find the values of A and B :—

118. $3x + 16 \equiv A(x + 4) + B(x + 6)$. You may give any two values you like to x. Which values will be most convenient ?

119. $7x - 2 = A(2x + 3) + B(x - 8)$.

120. Eliminate z from these two relations :—

$$8x + 3y - 9z = 16 \ldots\ldots (1).$$
$$4x + 5y - z = 20 \ldots\ldots (2).$$

121. Eliminate t from : $y = a + 4t$; $ty = x + 3$.

122. Are these equations consistent ?

$$8x + 6y - 1 = 0,\ 3x + 4y - 2 = 0,$$
$$7x + 6y - 5 = 0.$$

123. Examine the consistency of these equations :—

(*a*) $5x + 6y = 17$. $3x - 4y = -5$. $6x + y = 8$.

(*b*) $6x - 5y = 80$. $3x - 4y = 46$. $4x + 3y - 28 = 0$.

124. Solve the following equations :—

(*a*) $5x + 6y = 7$. $3(x - 2) + 2(x + 3y - \frac{1}{2}) = 0$. } What is the difficulty in solving them ?

(*b*) $8x - 9y = 15$. $\frac{8x}{3} - 3y = 5$. } Do.

125. One sample of milk contains 65 per cent. of milk and the rest water. Another sample contains 40 per cent. of milk and the rest water. How many ollocks of each sample should be taken and mixed in order to get another sample in which we may have $50\frac{5}{7}$ per cent. of milk.

126. A mass of gold and silver costs £318-13-6 ; if the proportion were interchanged, it would be worth £129-10-6.

Find the weight of the mixture given that 1 oz. of gold and 1 oz. of silver together cost £4-3-0. Is it possible to find the weight of gold and silver separately?

127. Gold costs £3-17-10½ per oz. and silver 5*s.* 6*d.* per oz. In what proportion must these metals be mixed so that a pound of the mixture may be worth £32-5-0. (Suppose x lbs. of gold and y lbs. of silver are mixed. Find the ratio $\frac{x}{y}$.)

128. A mass of gold and silver weighing 60 oz. is worth £158. Find the quantity of gold and silver in the mass, an oz. of gold being worth £3-16-0 and an oz. of silver 6*s.* (Use x and y to frame the equations.)

129. A vessel contains milk and water in the proportion of 3 : 7; 12 seers of milk are added and then the new mixture contains milk and water in the proportion of 13 : 21. How many seers of water does the mixture contain?

130. Two casks A and B contain mixtures of wine and water. In A the quantity of wine : the quantity of water :: 8 : 3. In B the like proportion is 15 : 8. In what ratio should A and B be combined so that the proportion of wine to water in the new mixture may be 100 : 47.

131. Gold weighing 19 units in air weighs only 18 units in water. Silver weighing 11 units in air weighs only 10 units in water. A mixture of gold and silver weighing 15 units in air weighs 14 units in water. If the mass of mixture of gold and silver weighs 64 gm. in air, find the weights of gold and silver separately in the mixture.

132. A nugget of gold mixed with quartz weighs 12·8 oz. in air, but loses $\frac{1}{6{\cdot}4}$ of its weight in water. Given that gold and quartz lose $\frac{1}{19{\cdot}35}$ and $\frac{1}{2{\cdot}15}$ of their weights when weighed in water, find the quantity of gold in the nugget.

133. A certain number of men can do a piece of work in a certain number of days. If the number of men is

diminished by 2, five more days will be taken to complete the work. If the number of men is increased by 2, the work will be finished 4 days sooner. Find the number of men and days.

134. A and B can do a work in 30 days; B and C can do the same work in 15 days. If A is $\frac{1}{3}$ as good a workman as B, in what time can C alone do the work?

135. A is half as good a workman as B; they both can do a piece of work in 20 days. B and C can do the same work in 10 days. In what time can C do the work?

136. Two taps A and B fill a cistern in x and y hrs. respectively. Both of them can fill $\frac{7}{12}$ cistern in 1 hour and it is found that in 1 hour A fills $\frac{1}{12}$ cistern more than B. Find x and y.

137. A journey is finished in a certain time at a certain speed. If the speed is diminished by 2 miles an hour, the journey takes $1\frac{2}{3}$ hour more. If the speed is increased by 4 miles an hour, the journey can be finished $2\frac{1}{2}$ hours sooner. Find the speed and also the time taken to finish the journey.

138. Two taps A and B fill and empty a cistern in x and y hours respectively. When the cistern is full, it is found that, if both taps are open, the cistern is only $\frac{19}{20}$ full at the end of 1 hour and that if B alone is open the cistern is $\frac{3}{4}$ full at the end of 1 hour. Find x and y.

139. I engage a carpenter for 60 days on condition that he should get 10 as. 8 ps. for every day he works and forfeit 7 as. 4 ps. for every day he is idle. In the end he is paid Rs. 23-2-0. How many days was he idle? (Use x and y.)

140. I have a sum of money to be distributed among a certain number of beggars. If I give 2 as. each, I shall have 12 as. too much; but if I give 2 as. 9 ps. each, I shall have 6 as. too short. Find the number of beggars and the amount that I have.

141. A dealer buys 100 tons of coal and after selling them at 1*s.* 5*d.* per sack, finds that he has gained £4. Had

he sold them at 1*s.* 4*d.* per sack, he would have lost £6. Find the weight of a sack of coal and the cost price of a ton.

142. A dealer buys 100 tons of coal and selling them at 1*s.* 7*d.* per sack gains £8-6-8 on the whole. Had he sold them at 1*s.* 5*d.* per sack he would have lost £8-6-8 on the whole. Find the cost price of a ton of coal and the weight of a sack.

143. Two persons A and B together earn Rs. 500 per month. A drawing less than Rs. 3,000 per year has to pay an income-tax at the rate of 5 ps. in the rupee and B drawing more than Rs. 3,000 per year has to pay an income-tax of 6 ps. in the rupee. A and B together draw net salaries amounting to Rs. 485-6-8 per month. Find their salaries.

144. Passengers are allowed a certain amount of luggage free on a train. Two passengers paid a total sum of 12 as. for excess luggage for the same journey. If one of them travels alone and takes the luggage of both he will have to pay Re. 1-4-0. If their combined luggage is $1\frac{3}{4}$ md., how much is each allowed free and what is the charge per md. of excess luggage ?

145. A party of scouts were given a number of buns and found that if each took four there were two buns over, but that if each took 5, the last two scouts had no buns. How many scouts and how many buns were there ?

146. A party of 8 railway passengers carrying an average load of $12\frac{1}{2}$ seers started for a certain place of pilgrimage and estimated they could have carried $1\frac{1}{2}$ mds. more without being charged for excess of luggage. Another party of 3 pilgrims proceeding to the same place was charged Rs. 7-8-0 for excess luggage. If the latter had joined the former party they could have just escaped the payment of excess luggage, their luggage being combined. Supposing Rs. 5 was the excess luggage charged per md., find how much was allowed free of charge and what average load was carried by a pilgrim of the second party. (1 md. = 40 seers).

147. A landholder owned a plot of land. He divided it between his two sons as follows :—(1) wet lands in the ratio of 5 : 4 and (2) dry lands in the ratio of 5 : 7. The sons were then worth Rs. 25,000 and Rs. 23,000 respectively. Find the value of the wet lands and dry lands.

148. There are two numbers whose sum is 145. Also $\frac{5}{6}$ of one number exceeds $\frac{4}{17}$ of the other by 30. Find them.

149. I have to perform a journey of 350 miles partly by train and partly by motor which takes 20 hours. Assuming the rates of the train and motor to be respectively 20 and 10 miles per hour, find how many miles I travelled by train.

150. A scout walks a few steps and then runs a few steps. By taking these alternate paces he travels 1 mile in 12 minutes. If his average rate of walking be 3 miles per hour and average rate of running 6 miles per hour, find how many yards he walked.

151. A person rows up stream from A to B a distance of 18 miles in a river and returns from B to A in 8 hrs. He again starts from A, goes down stream to C a distance of 12 miles in $1\frac{1}{3}$ hrs. Find the velocity of the stream and the speed of the boat.

152. A train starts 20 minutes after another and overtakes it in 1 hr. 10 minutes. If the former train had increased its speed by 2 miles an hour and the latter diminished its speed by 3 miles it would have overtaken it in double the time. Find the speeds of the trains.

153. Two trains 88 yds. and 132 yds. pass each other in the same direction in $8\frac{2}{11}$ seconds. If they had moved in opposite directions they would have passed each other in $5\frac{5}{11}$ seconds. Find the speeds of the trains.

154. Two trains 184 ft. and 168 ft. long are moving with uniform velocities on parallel rails. When they move in opposite directions they are observed to pass each other in 3 seconds, but when they move in the same

direction, the faster train is observed to pass the slower in 6 seconds. Find the rate at which each train moves.

155. Two trains 330 ft. and 264 ft. long run on parallel lines of rail. When travelling in opposite directions they are observed to pass each other in 9 seconds, but when they are running in the same direction at the same rates as before the faster train passes the other in $22\frac{1}{2}$ seconds. Find the speeds of the two trains in miles per hour.

156. I leave my house between 1 and 2 and return between 6 and 7 and find that the minute and hour hands of my watch have exactly interchanged positions. Find the time when I left the house.

157. A gentleman fell asleep between 4 and 5 p.m. and woke up between 7 and 8 p.m. He found that during the interval the hands of his watch had exactly interchanged positions. How long did he sleep?

158. A and B run a mile. First A gives B a start of 3 minutes and is beaten by 176 yds. At the second heat A gives B a start of 160 yds. and beats him by 1 minute. Find the rates of A and B per minute.

159. In a mile race A gives B 44 yds. start and wins by 6 seconds. In a three-quarter mile race he gives him 20 seconds start but is beaten by 29 yds. 1 ft. Find the rates per minute to the nearest yard.

160. A person whose rates of walking uphill, on level ground, and downhill are 3, 4 and 5 miles an hour goes from A to B in 6 hrs. 6 min. and returns from B to A in 6 hrs. 22 min. Find the distances walked uphill and downhill supposing the level is 20 miles long.

161. On a building under construction were employed x men, y women and $\frac{1}{2}y$ boys. The men were paid at 12 as. per head per day; the women and boys were paid at 6 as. per head per day. Then the total daily wages amounted to Rs. 33-12-0. Some time after, the wages were increased by 1 a. for each man and by 6 pies for each woman and for each boy and the total daily wages

amounted to Rs. 42-1-0. Is it possible to find x and y? If not, correct the second amount.

162. A and B are two adjacent stations on a railway. A train starts from A, goes on a level for some distance and gets up an incline till it reaches B. The train takes 54 min. to travel the distance AB. The same train travels from B to A on its return journey and does the distance in $44\frac{2}{5}$ minutes. Assuming that the rates of the train on the level ground, up the incline and down it are 20 miles, 15 miles and 25 miles per hour, find the length of the level road. (Use x and y.)

163. Two trains A and B start simultaneously from P and Q respectively and travel towards each other meeting at a station M. They reach Q and P respectively and again travel towards each other meeting again at a station N. The distance MN = 60 miles and the rates of the trains A and B are 40 miles and 30 miles respectively. Find the distance from P to Q.

164. If, in the previous question, the two trains A and B start at 7 a.m. and another train C leaves A at 8 a.m. and runs at 15 miles an hour, find when the three trains will be equidistant.

165. Three mounted sepoys P, Q and R leave Madras for Patalam at 6 a.m., 8 a.m. and 9 a.m. Q and R travel 1 mile and 3 miles an hour faster than P. If they all reach Patalam at the same time, find their speeds and the distance from Madras to Patalam along the road they rode on.

166. The manager of a hostel had 200 and 230 students respectively in two consecutive years. This addition of 30 students increased his yearly expenditure by Rs. 1,260 but diminished the average cost per head by Rs. 18. What was the total expenditure in each of the two years?

167. The storage of water in a reservoir would be just sufficient to supply water to the inhabitants of a village for a few days. If the families in the village received 500

more additions, it would have lasted for 6 days less. If however 400 of the men left the village it would have lasted for $7\frac{1}{2}$ days more. Find the number of men in the village and the number of days for which the storage was meant to last.

168. The total collections in a cinema which exhibited in 5 parts a certain film consisting of 30 reels amounted to Rs. 2,340. One part was shown every day in a week and the five parts were shown in 5 weeks. The tickets issued were of 5 classes (i) Box tickets, Rs. 3 each; (ii) 1st Class tickets, Re. 1 each; (iii) 2nd Class tickets, 8 as. each; (iv) 3rd Class tickets, 4 as. each; (v) 4th Class tickets, 2 as. each. 6 tickets were sold in Class I; the number in Class V being double the number in Class IV while the number in Class III being 180 more than in Class II. Supposing the total number that attended the film was 1646 in each week and that each saw the whole film, find the number in Classes II, III, IV and V separately.

169. From a Rs. b as. c ps. I deduct c Rs. b as. a ps. The remainder is 2 ps. less than 2 rupees. If I add 1 a. 6 ps. to the remainder it is equal to $20\ (c + a)$ pies. Find the values of c and a.

170. If in the previous question I add a Rs. b as. c ps. to c Rs. b as. a ps. the sum is 4 ps. less than Rs. 22. Find the value of b.

N.B.—For some more problems on simultaneous equations refer to the next chapter on graphs.

CHAPTER XXVI

The Straight Line Graph

149. Variables and their dependence. When the diameter of a circle changes, its circumference also changes. If the diameter is 1 inch, the circumference $= 3\frac{1}{7} \times 1$ or $3\frac{1}{7}$ in. If the diameter is 2 in., the circumference $= 3\frac{1}{7} \times 2$ or $6\frac{2}{7}$ in. If the diameter is x in. and the circumference y in.,

then $y = 3\frac{1}{7}x$ or $x = \frac{1}{3\frac{1}{7}}y$. In these relations x and y are said to be **variables** because they change and $3\frac{1}{7}$ is said to be a **constant** because it does not change. Here the change is governed by a simple law.

Take the case of a hostel student. His expenses are of several kinds:—room rent, boarding charges, dhoby charges, conveyance charges, etc. The first does not vary and the remaining charges may vary. If y represents his total charges $y = r + x$ a constant and a changing part. This is more complicated than the relation between the circumference and its diameter.

150. This dependence in mathematics is termed a **function.** Thus the fee collections in a school depend upon the number of pupils. In mathematics we say the fee collections are a function of the number of pupils; similarly the distance travelled is a function of the velocity if time remains constant or is a function of the time if velocity is constant. Generally **if a quantity y depends for its value upon another quantity x, then y is said to be a function of x.** By complicated laws the relation between y and x may be as follows:—

$$y = 3x,\ y = \frac{4x-5}{6},\ y = x^2,\ y = x^2 - 4x + 2.$$

Each of the expressions $3x$, $\frac{4x-5}{6}$, x^2, $x^2 - 4x + 2$ involving x is called a function of x because for a particular value of x each of the above expressions has a corresponding value or in other words varies with x.

151. $y = 3x + 4$. In this relation y or $3x + 4$ is a function of x. Hence we can prepare a tabular form of values of y corresponding to the values that we assign to x.

$x =$	7	5	3	1	0	—	— 5	— 7
$3x+4$ or y =	$3.7+4$ or 25	$3.5+4$ or 19	$3.3+4$ or 13	$3.1+4$ 7	$3.0+4$ 4	$-9+4$ or -5	$-15+4$ -11	$-21+4$ -17

It is clear that the pairs of values (7, 25), (5, 19), and (3, 13), (— 5,—11) etc. are inter-related.

To show this connection graphically we plot the points (7, 25), (5, 19) and (3, 13), etc. as indicated in Chapter IX (vol. I), i.e. join the points freely by a straight line. This line AB (Fig. 172) is called the graph of $y = 3x + 4$ because

(1) corresponding to any value of x, the value of $3x + 4$ or y may be read from that line. Note the point on the line whose abscissa is 2. Its ordinate is 10, i.e. when $x = 2$, y or $3x + 4$ is 10 corresponding to the process of evaluation of $3x + 4$ when $x = 2$;

(2) corresponding to any value of y or $3x + 4$, the value of x may be read from that line. Note the point on the line whose ordinate is 16. Its abscissa is 4 corresponding to the process of solving the equation $3x + 4 = 16$;

(3) any pair of values of x and y satisfying the relation $y = 3x + 4$ are the co-ordinates of some point on the line and not of any point outside the line; and

(4) the co-ordinates of every point on the line satisfy the relation $y = 3x + 4$ and those of any point outside the line do not satisfy the relation.

The graph is the geometrical locus of all points whose perpendicular distance from a straight line = 3 times the perpendicular distance from another straight line at right angles to it + 4 units.

152. It will be noted that of all the points on the line, two seem to deserve our particular attention. They are the points where the line crosses the axes, (1) where it cuts the axis of x, there is no ordinate, i.e. $y = 0$, then the corresponding abscissa or value of x is $-\frac{4}{3}$; (2) where it cuts the axis of y, there is no abscissa, i.e. $x = 0$; then the corresponding ordinate is 4. If it is remembered that a straight line is determined by two points, it is clear that these two points are enough to draw the graph completely

(though a third point is generally taken to draw an accurate figure).

In the figure $OA = -\frac{4}{3}$, $OB = 4$. OA and OB are called the intercepts made by the graph on the axes. It may be now considered whether it is not possible to get the lengths of these intercepts from the relation $y = 3x + 4$ itself without any reference to the graph.

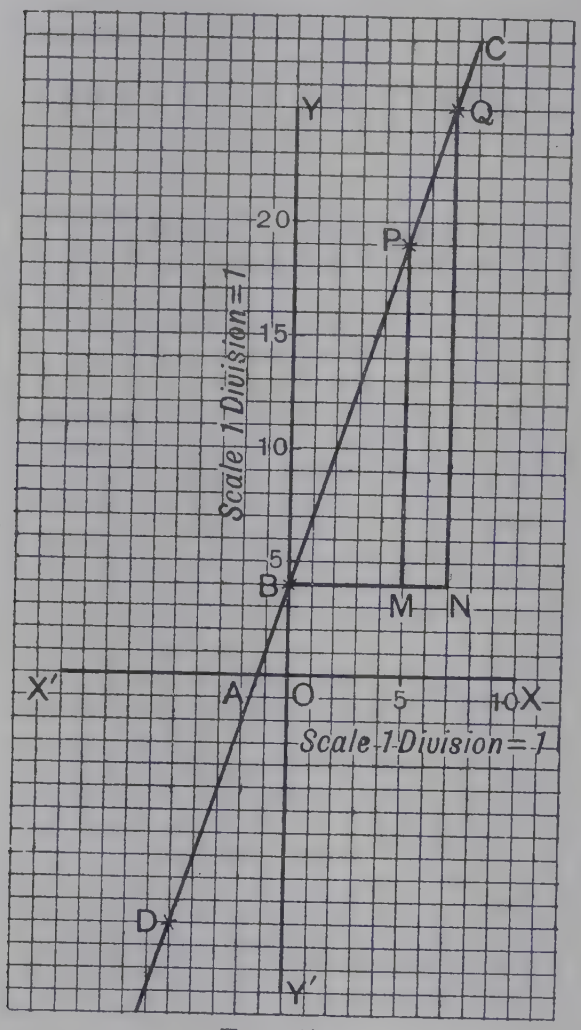


FIG. 172.

$$-3x + y = 4.$$

Divide both sides by 4.

$$-\tfrac{3}{4}x + \tfrac{1}{4}y = 1$$

which may be written

$$\frac{x}{-\frac{4}{3}} + \frac{y}{4} = 1.$$

$-\frac{4}{3}$ and 4 which were noted to be the intercepts on the x and y axes respectively appear as denominators under x and y respectively with 1 on the other side. If every such relation or equation can be reduced to this form, i.e. $\frac{x}{a} + \frac{y}{b} = 1$, it may be inferred that a and b are the intercepts on the x and y axes respectively. The student may take one or two more relations or equations of the sort, find the intercepts graphically and examine by algebraical transformation whether he gets the same values under x or y when they are cast in the form $\frac{x}{a} + \frac{y}{b} = 1$. Transformation of an equation to the standard form $\frac{x}{a} + \frac{y}{b} = 1$ will give the intercepts a and b.

Model No. (1) Reduce $5x + 6y + 3 = 0$ to the form $\frac{x}{a} + \frac{y}{b} = 1$.

$$5x + 6y = -3.$$

Divide by -3. $\therefore \frac{5x}{-3} + \frac{6y}{-3} = 1.$ $\therefore \frac{x}{-\frac{3}{5}} + \frac{y}{-\frac{3}{6}} = 1.$ $\therefore a = -\frac{3}{5}.$ $b = -\frac{3}{6}$ or $-\frac{1}{2}.$

153. Transformation of an equation to the standard form $y = mx + c$. In the graph of $y = 3x + 4$, it was noticed that 4 is the intercept on the y axis. It will be seen by drawing the graph of $y = 5x + 6$, 6 is the intercept on the y axis. Hence it may be inferred that c is the intercept on the y axis made by the straight line graph represented by $y = mx + c$. Then we have to find what 3 is in the relation $y = 3x + 4$, what 5 is in $5x + 6$ and what m is in $y = mx + c$. In Fig. (172) through B draw a line parallel to the axis of x and if PM and QN are the perpendiculars drawn, from P and Q on the parallel through B, then $\frac{BM}{PM} = \frac{5}{15}$, $\frac{BN}{QN} = \frac{7}{21}$, or $\frac{BM}{PM} = \frac{1}{3}$ and $\frac{BN}{QN} = \frac{1}{3}$ which shows that for an advance of 1 unit on the x axis, there is a rise of 3 units. The rise of the slope or the gradient is 3 to 1.

154. There are three methods of drawing a straight line graph. The first consists in preparing a tabular statement of corresponding values of x and y, plotting points and joining them. The second consists in transforming the linear equation into the form $\frac{x}{a} + \frac{y}{b} = 1$, marking off the intercepts on the x and y axes, and joining the extremities

of the intercepts. The third consists in transforming the equation to the form $y = mx + c$. On the y axis, measure from the origin a length OK equal to c units (*vide* fig. 175). Through the extremity K draw a line parallel to the x axis. Measure KR 1 unit on the parallel and then step off RS m units in a direction at right angles to the parallel line, i.e. parallel to the y axis. Join S.K. Then S.K. produced both ways represents the graph.

Model No. (2) Draw the graph of $8x + 5y + 9 = 0$.

Method (1) $5y = -8x - 9$.

$$\therefore y = -\frac{8}{5}x - \frac{9}{5}.$$

$x =$	0	2	4	7	-2	-3	
$y =$	$-\frac{9}{5}$	-5	$-8\frac{1}{5}$	-13	$\frac{7}{5}$	3	

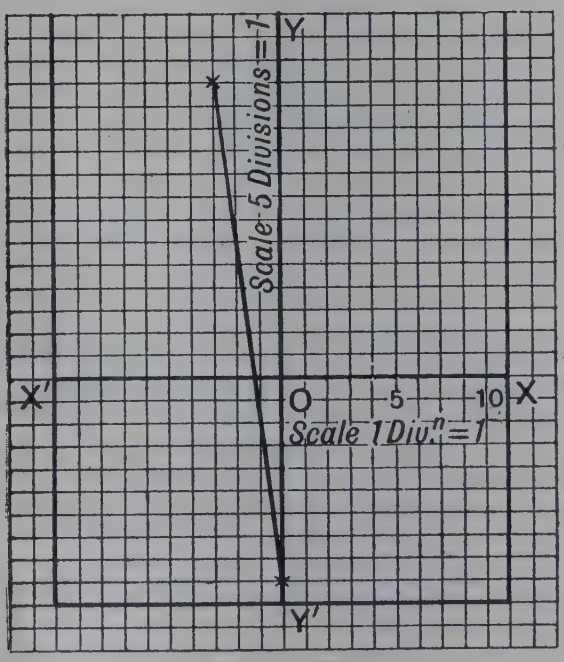


FIG. 173.

The scale on the x axis might be $1 = \frac{1}{10}$ of an inch.

The scale on the y axis might be $1 = \frac{5}{10}$ of an inch.

The points $(0, -\frac{9}{5})$; $(-3, 3)$ may be plotted and joined.

N.B.—In the figure -3, $\frac{13}{5}$ has been plotted instead of -3, 3. The pupil may draw a correct figure.

Method (2) $8x + 5y = -9$.

$\frac{8}{-9}x + \frac{5}{-9}y = 1.$

$\therefore \frac{x}{-\frac{9}{8}} + \frac{y}{-\frac{9}{5}} = 1.$

Hence on the x and the y axis mark intercepts equal to $-\frac{9}{8}$ and $-\frac{9}{5}$ units; join the extremities of the intercepts and produce it both ways.

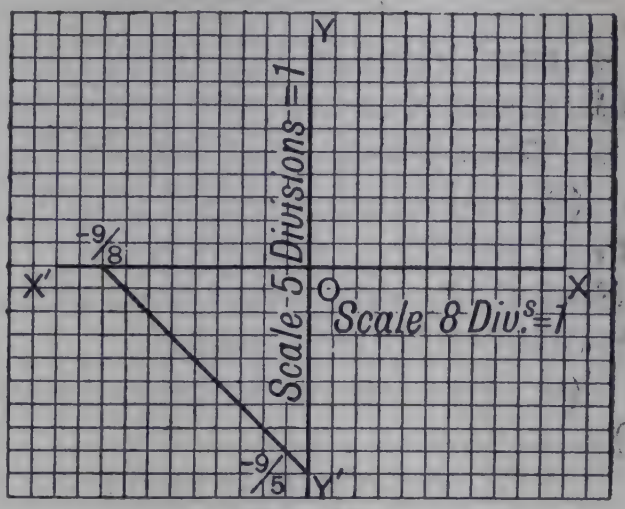


FIG. 174.

Note.—The scale on the x axis is $1 = \frac{8}{10}$ of an inch, on the y axis $1 = \frac{5}{10}$ of an inch.

Method (3) $8x + 5y + 9 = 0$.

$5y = -8x - 9.$

$y = -\frac{8}{5}x - \frac{9}{5}.$

Take on the y axis a length OK equal to $-\frac{9}{5}$. Through K draw a parallel to the x axis; KR = 1 unit, and draw RS parallel to the y axis in a negative direction equal to $\frac{8}{5}$ units. Then KS produced gives the straight line graph of the function $8x + 5y + 9 = 0$.

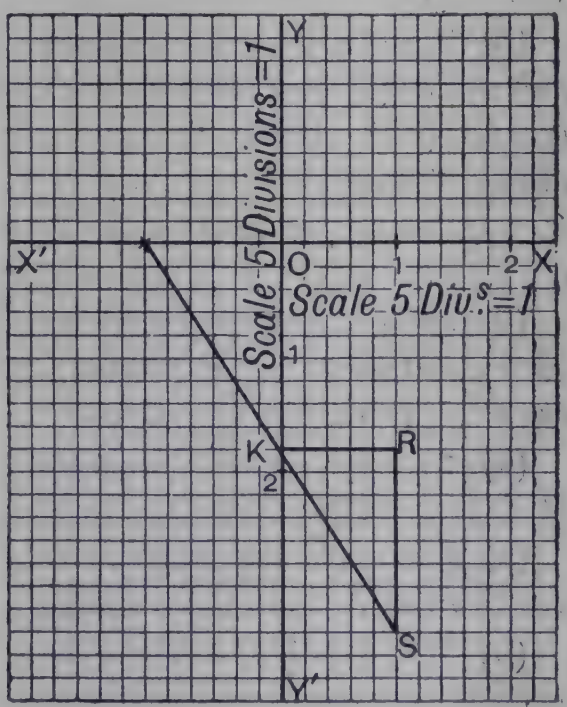


FIG. 175.

Note.—The scale on the y axis $1 = \frac{5}{10}$ of an inch.
x axis $1 = \frac{5}{10}$ of an inch.

Exercise XXVI (A).

In each question the several graphs should be drawn with reference to the same origin and axes.

1. Draw the graph of, by plotting points :—

(i) $y = x$, (ii) $y = x + 1$, (iii) $y = x - 1$, (iv) $y = 2x + 1$ and (v) $y = 2x + 1$.

Find from each graph the value of y when $x = 3$ and the value of x when $y = 8$.

(a) What do you notice about straight lines (i), (ii) and (iii) ?

What is the m in each case ?

(b) What do you notice about straight lines (iv) and (v) ?

What is the m in each case ?

(c) Hence what do you learn from (a) and (b) ?

Note that straight lines the m in whose equations is the same are parallel.

(d) What difference is there between the straight line denoted by (i) and the straight lines denoted by (ii), (iii), (iv) and (v) ? How can you infer the same by merely looking at the relation ?

2. Draw the graphs of the functions (by plotting points) $2x$, $3x$, $4x$, $5x$, $2x + 3$, $3x - 4$, $4x - 5$ and $5x - 6$.

(a) Find the value of each of the functions when $x = -3{\cdot}5$ and the value of x when each of the functions $= 10$, from the graph.

(b) In what quadrants does the graph of each of the first four functions lie ?

(c) Which of the above functions are parallel judging from their m ? Are they so in your graphs ?

(d) Which of the functions must pass through the origin ? (Judge from the forms of the equations.)

(e) From the form of the equation judge c in each case and verify from the graph whether the values of c are correct.

3. Draw the graphs of the following functions by plotting points :—

$y = -x$, $y = -2x$, $y = -3x$, $y = -4x$, $y = -x + 1$, $y = -2x + 1$, $y = -3x - 5$ and $y = -4x - 6$.

(*a*) Find the value of each of the functions when $x = -2{\cdot}5$ and the value of x when each of the functions $= 12$, from the graph.

(*b*) In which quadrants does the graph of each of the first four functions lie ?

(*c*) What is the m in each of the first four functions ?

(*d*) What difference do you notice between the 'm' of the first four functions in Question 2 and the 'm' of the first four functions in Question (3). What difference do you notice in the quadrants through which these pass ?

Hence learn that **in the relation $y = mx$ if m is positive the graph passes through the first and third quadrants and if m is negative the graph passes through the second and fourth quadrants.**

4. (*a*) What is the equation of the graphs passing through the origin parallel to $y = x - 2$, $y = 2x + 1$ and $y = 4x + 3$?

(*b*) What is the equation of the graphs passing through the origin parallel to $y = -x + 5$, $y = -2x - 3$, $y = -3x - 4$ and $y = -4x + 8$?

(*c*) OX and OY are two straight lines intersecting at O at right angles. A point P moves so that its distance from OX = its distance from OY. Find the locus of P (OX and OY may be regarded as axes of x and y and the perpendiculars on OX and OY from any point P as co-ordinates of P. Then $y = x$. The graph of $y = x$ is the required locus).

(*d*) OX and OY are two straight lines intersecting at O at right angles. A point P moves so that its distance from OX = 2 times the distance from OY. Find the locus of P. (The locus is the graph of $y = 2x$ as in 4*c*.)

(*e*) OX and OY are two straight lines intersecting at

O at right angles. A point P moves so that its distance from OX $= n$ times its distance from OY. Find the locus of P. (The locus is the graph of $y = nx$ as in 4*c* or 4*d*.)

5. Draw the graphs of the following equations :—

(*a*) $x - y = 0$ (same as $y = x$) ; (*b*) $x + y = 0$ (same as $y = -x$); (*c*) $y + 2x = 0$; (*d*) $y - 2x = 0$; (*e*) $y - \frac{1}{2}x = 0$; (*f*) $y + \frac{1}{2}x = 0$; (*g*) $3x + y = 0$; (*h*) $3x - y = 0$; (*i*) $2x + 4y = 0$; (*j*) $2x - 3y = 0$; (*k*) $3x + 5y = 0$ and (*l*) $3x - 5y = 0$.

What can you say about the graphs of these equations ? Which of them pass through the first and third quadrants and which of them through the second and fourth ?

6. Draw the graphs of the following equations without plotting the points but by reducing them to the form $\frac{x}{a} + \frac{y}{b} = 1$:—

(*a*) $3x + 2y + 1 = 0$ (choose the proper scale on each axis).

(*b*) $3x - 2y + 1 = 0$.

(*c*) $4x + 5y + 6 = 0$.

(*d*) $3x - 8y + 9 = 0$.

(*e*) $12x + 4y - 8 = 0$.

(*f*) $6x + 9y + 18 = 0$.

7. Draw the graphs of the following equations without plotting the points by reducing them to the form $y = mx + c$:—

(*a*) $3x + 2y + 4 = 0$ (choose the proper scale).

(*b*) $4x - 5y + 6 = 0$.

(*c*) $2x + y - 3 = 0$.

(*d*) $4x + 7y - 6 = 0$.

8. On squared paper mark 6 points which are at a constant distance of 1·2″ from the axis of x. What is the condition satisfied by these points ? Take some more points on the locus and see if they satisfy the condition of the locus ? What is the equation of the straight line parallel to the X axis at a distance of 1·2″ from it ?

9. Draw the graphs of $y = 5$, $y = 10$, $y = -15$, $-y = -20$ and see how they are situated with respect to the axes.

10. Draw the graphs of $x = 6$, $x = 15$, $x = -18$, $x = -24$ and see how they are situated with respect to the axes.

Hence learn that $y = b$ **represents a line parallel to the axis of** x **at a distance of** b **units from it and** $x = a$ **represents a line parallel to the axis of** y **at a distance of** a **units from it.**

155. Uses of Graphs. When equations are given their roots can be found graphically also.

Model No. (3) Solve the equations

$$\left.\begin{aligned} 4x + 3y &= 20 \\ 5x - 2y &= 2 \end{aligned}\right\} \text{ graphically.}$$

We know that the equation $4x + 3y = 20$ is represented by a straight line graph on which lie a number of points

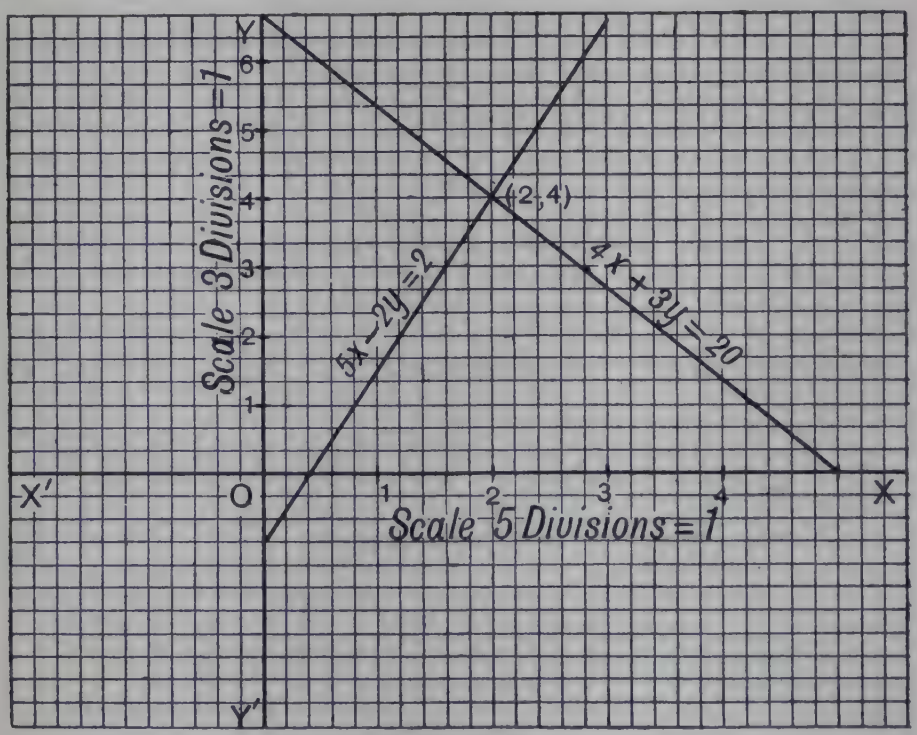


FIG. 176.

whose co-ordinates satisfy the equation; similarly we know that the equation $5x - 2y = 2$ is represented by

another straight line graph on which lie a number of points whose co-ordinates satisfy the equation. But solving graphically these two equations means finding a pair of values of x and y satisfying both the equations. Hence the pair of values of x and y satisfying both the equations must be the co-ordinates of a point on both the straight line graphs, i.e., must be the co-ordinates of a point common to the graphs, i.e., of the point of intersection of the two graphs.

Thus $x = +2$, $y = +4$ gives the solution required.

As two straight lines intersect only in one point, we can get only one set of values of x and y that satisfy the two linear equations.

Note (1).—

$\left.\begin{aligned} 4x + 3y &= 20 \\ 4x + 3y &= 16 \end{aligned}\right\}$ have no solution in common. For they may be reduced to the forms

$$\left.\begin{aligned} y &= -\frac{4x}{3} + \frac{20}{3} \\ \text{and } y &= -\frac{4x}{3} + \frac{16}{3} \end{aligned}\right\}$$ The m in both these cases is the same. Hence if the graphs are drawn they will be found to be parallel.

Note (2).—

$x + y = 4$, $2x + 2y = 8$. These two graphs apparently different are really identical. Hence no solution is possible.

Note (3).—

$3x - 5 = 4x - 7$. This equation may be solved graphically thus.

The function $3x - 5$ may be supposed to be equal to y.

Then $y = 3x - 5$ (1) is one equation.

$y = 4x - 7$ (2) is another equation.

The graphs may be drawn and the co-ordinates of the point of intersection may be found to be 2, 1.

Note (4).—

$4x + 3y = 20$ . . (1).

$\therefore y = \dfrac{20 - 4x}{3}$ (2).

The graph of the equation $4x + 3y = 20$ would give the different values assumed by the function $\frac{20 - 4x}{3}$ corresponding to the various values of x. Hence the graphs of (1) and (2) are not different.

156. Supposing there is a sort of relation between the two variables x and y, find the values of m and c from the following sets of values :—

x	= 4	8
y	= 3	11

Plot the points (4, 3), (8, 11) and join them. Produce the straight line to cut the axis of y. The intercept on the axis of $y = -5$. $\therefore c = -5$. Through the points (4, 3), (8, 11) draw straight lines parallel to the axis of x and that of y. The slope of the straight line $= \frac{8}{4} = 2$. $\therefore m = 2$.

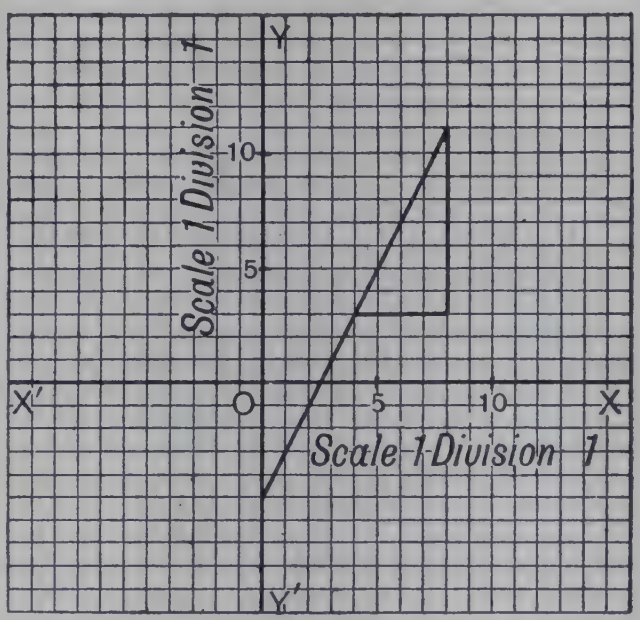


FIG. 177.

$\therefore y = 2x - 5$ is the relation connecting the variables.

157. A second use of the graphs is that they serve as ready reckoners and are used to convert one system of units of one kind into an equivalent system of some other units of the same kind. For example, a price expressed in annas per viss may be converted into one of rupees per maund; a velocity of ft. per second may be converted into one of miles per hour or a length in metres may be converted into one in inches and so on. A graph may be drawn to find out what distance a train will travel in a given time if its velocity is known.

Model No. (4) A train travels at 20 miles per hour. If it travels uniformly draw a graph to show the relation between the distance travelled and the time taken.

Let the distance be d miles and the time taken t hours. Then $d = 20t$ is the relation connecting the distance and time or $y = 20x$ if y miles is travelled in x hours. By giving values to t the corresponding values of d may be found and the following table prepared.

$t =$	0	2	5	6	7
$d =$	0	40	100	120	140

The times in hours may be denoted as abscissa along the x axis scale $\frac{3}{10}'' = 1$ hr.; the distance in miles may be denoted as ordinates along the y axis. Scale $\frac{1}{10}'' = 5$ miles. The points may be plotted and the graph drawn. Since the function is linear the graph must be a straight line.

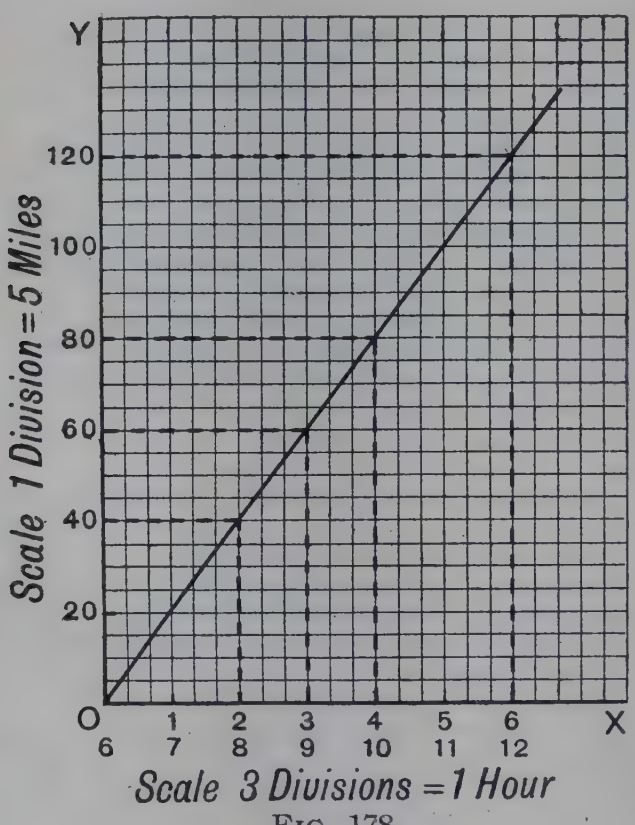


FIG. 178.

Note (1).—The figure at once shows what distance is travelled in a given time and in what time a given distance is travelled. It is clear that in 4 hours 80 miles is travelled for the ordinate of the point on the graph whose abscissa is 4 hours is 80 miles. It is also clear that 60 miles is travelled in 3 hours for the abscissa of the point on the graph whose ordinate is 60 miles is 3 hrs. So the graph is a ready-reckoner.

Note (2).—If the train starts at 6 a.m., to find what distance

it will have traversed by 8 a.m., we mark on the x axis 6 a.m., 7 a.m., 8 a.m. . . . . instead of 0, 1, 2, 3 . . . . and note the reading on the y axis corresponding to 8 a.m., on the x axis. . . . Thus we get 40 miles.

Note (3).—The graph gives us the products of 1 and 20, 2 and 20, 3 and 20, and so on as in the multiplication by 20 and also the products of $3\frac{1}{3} \times 20$, $3\frac{2}{3} \times 20$ and $4\frac{2}{3} \times 20$, etc. It also gives the quotients $\frac{100}{20}$, $\frac{90}{20}$, $\frac{60}{20}$, etc.

Note (4).—The graph can also be used as a *conversion graph* to find out the price per candy or barum when the price of a viss is given or *vice versa* the price per maund when the price of a candy is given. For example, if the price per maund is Rs. 5-5-4, the price per candy is a little more than Rs. 105. If the price per candy is Rs. 115, the price per maund is a little more than $5\frac{2}{3}$ rupees. £ can be converted into shillings and shillings into £ with the help of the above graph.

Note (5).—The gradient in this case is $\frac{4}{3}$. The graph enables us to find with the help of a scale the distance travelled by a train on an incline whose gradients $\frac{4}{3}$, i.e. which rises vertically 4 units to every 3 units advanced horizontally.

158. A third use of the graphs is that many problems can be easily solved graphically without stating the conditions in mathematical language, framing equations and solving them. Serious thinking is thereby sometimes saved.

Model No. (5) Two trains start from A and B at 6 and 8 p.m., and travel in opposite directions to B and A at the rate of 30 and 45 miles per hour respectively. Find when and where they meet. The distance AB is 360 miles.

Solution. Hours are represented on the X axis, the scale is $\frac{2}{10}''$ = 1 hour ; and miles on the y axis, the scale is $\frac{1}{10}''$ = 10 miles. The distance is a function of the time. If d be the distance, t the time, then $d = 30t$ in one case and $d = 45t$ in the other case.

(1) To draw the graph of $d = 30t$, plot a point O 6 p.m. to show that at 6 p.m. the train is at A. The gradient should indicate a rise of 30 miles in every hour. Proceed from A 1 unit (in hours) then go 30 units (miles) upwards,

plot a point C on the graph. Similarly plot other points and draw the graph.

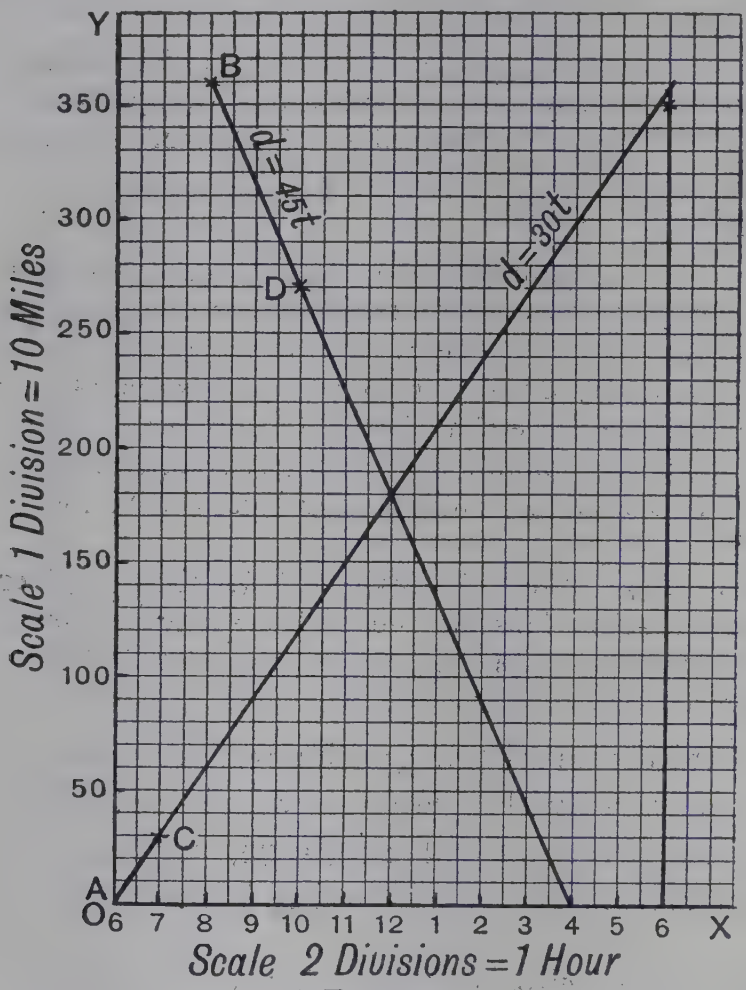


FIG. 179.

(2) To draw the graph of $d = 45t$, plot a point 8 p.m., 360 and call it B. since this train runs opposite to the first the gradient of the graph should indicate a fall of 90 miles in every two hours. Proceed from B 2 units (2 hours) horizontally, then go 90 units (90 miles) downwards and plot a point D. Similarly plot other points and draw the graph.

From the graphs the following observations can at once be made at a mere glance:—

(1) The trains meet at 12 midnight at a place 180 miles from A.

(2) The train starting from A reaches B at 6 a.m.

(3) ,, ,, B ,, A ,, 4 a.m.

(4) The trains are separated by a distance of 150 miles at 10 p.m. and 2 a.m.

Exercise XXVI (B).

Solve graphically and check your results otherwise :—

11. $y = 8x + 7$; $y = 7x + 5$.
12. $y = -3x - 4$; $y = 4x - 6$.
13. $y = 6x - 14$; $y = -5x + 17$.
14. $y = 7x + 15$; $y = \frac{3}{4}x - \frac{1}{5}$.
15. $y = \frac{7}{8}x - \frac{1}{9}$; $y = \cdot 85x + \cdot 75$.
16. $1 \cdot 8x - 4 \cdot 5 = y = -9 \cdot 3x + 4 \cdot 3$.

Put each function of x equal to y in examples 17 to 25 :—

17. $3x + \frac{4}{5} = 5x + \frac{6}{7}$.
18. $3x + 18 = 5x + 16$.
19. $\frac{7}{8}x - 9 = \frac{7}{5}x + 1 \cdot 4$.
20. $6 \cdot 4x - 7 \cdot 8 = 5 \cdot 6x + 8 \cdot 9$.
21. $6 \cdot 4\,(x - 8) - (7 - 8x) \times \cdot 5 = \dfrac{8 \cdot 6 - x}{3} + \frac{1}{5}$.
22. $\dfrac{x - 15}{4} + \dfrac{6 - x}{3} = \dfrac{3 - 4x}{5} + \dfrac{6 - 8x}{8}$.
23. $1 + \dfrac{4}{5}x + 3\,(6 - x) = 6$. $\dfrac{7x + 5}{4} + 13$.
24. $\dfrac{6 \cdot 5x + 5 \cdot 4}{8} - \dfrac{8 \cdot 6x - 3}{2} = \dfrac{3 \cdot 4 + 6x}{5} + \dfrac{9x - 7}{3}$.
25. $\frac{1}{3}\,(x - 6) + \frac{3}{5}\,(x - 8) = \frac{5}{8}\,(x - 10) + \frac{8}{9}\,(x - 12)$.
26. $3x + 4y = 7$.
    $6x - 8y = 9$.
27. $5x - 6y = 8$.
    $3x + 7y = 17$.
28. $5x + 8y = 47$.
    $-7x + 3y = -9$.
29. $3y - 4x + 13 = 0$.
    $21x - 4y + 14 = 0$.
30. $10x + 7y - 47\frac{1}{2} = 0$.
    $4y - 3x + 29\frac{1}{2} = 0$.
31. $3\frac{1}{2}x + 7\frac{3}{4}y = 121$.
    $-1\frac{7}{8}x + 6\frac{5}{6}y = 67$.
32. $1 \cdot 5x + 3 \cdot 7y = 16 \cdot 26$.
    $-6 \cdot 4x + 1 \cdot 8y = -37 \cdot 72$.
33. $6x + 8y = 31$.
    $7 \cdot 5x - 1 \cdot 6y = 15 \cdot 65$.
34. $1\frac{1}{3}x + 7 \cdot 5y = 34$.
    $8 \cdot 6x - 7\frac{3}{4}y = -5 \cdot 2$.

Solve the following problems graphically :—

35. 5 chairs and 3 tables cost Rs. 107-8-0. Four tables and 7 chairs cost Rs. 145-8-0. Find the cost of a table and a chair.

36. 5 pairs of bulls and 8 pairs of buffaloes cost Rs. 1,360. 3 pairs of buffaloes and 4 pairs of bulls cost at the same rate Rs. 680. Find the cost of a pair of bulls and a pair of buffaloes.

37. A gramaphone with 15 records costs Rs. 180. The same gramaphone with 20 records costs Rs. 210. Find the cost of the gramaphone and the average price of a record.

38. A boat goes upstream $5\frac{1}{2}$ miles in $5\frac{1}{2}$ hours and downstream 15 miles in 3 hours. Find the rate of the boat and the stream.

39. A number of two digits is such that thrice the digit in the units' place is 4 times the digit in the tens' place diminished by 3. If the sum of the digits is multiplied by 5 and the product increased by 3, the result is the original number. Find the number.

40. If a rectangular field were 4 yds. less long and 7 yds. less broad, the area would have been 875 sq. yds. less. If on the other hand the field were 18 yds. longer and 8 yds. broader, the area would have been 2,160 sq. yds. more. Find the dimensions of the rectangular field.

41. 1 kg. $=$ 2·2 lbs. Draw a conversion graph to enable you to read off the number of lbs. equal to a given number of kilograms and *vice versa*. Find, from the graph, the equivalent of 15 lbs. in kilograms and the equivalent of 8 kilograms in lbs.

42. Assuming 1 m. $=$ 1·1 yd., roughly, draw a graph which would express a length in yards in terms of metres up to 50 yds. Find from the graph how many metres are equal to 45 yds. and how many yards are equal to 35 metres.

43. 1 viss $= 3\frac{1}{8}$ lbs. Draw a graph to convert lbs. into

viss and viss into lbs. From your graph, find how many viss are equal to 2 qrs. and how many lbs. are equal to half a maund.

44. Assuming 1 km. = ·621 of a mile, find graphically the number of miles in 20 km. and the number of km. in 16 miles.

45. Assuming that 1 cm. = 0·394 in., construct a graph to read off cms. in terms of inches. Find how many inches there are in 3·5 cms.

46. A speed of one mile per hour is equal $\frac{2}{1}\frac{2}{5}$ ft. per second. Represent this relation by means of a graph. From the graph read off how many feet per second are equal to (*a*) 20 miles per hour, (*b*) 35 miles per hour and (*c*) 43 miles per hour. Also how many miles per hour are equal to (*a*) 35 ft. per second and (*b*) 47 ft. per second?

47. 100 degrees of centigrade are equal to 80° of Reaumer scale. Draw a graph to represent this relation. From it, find how many degrees of R = 75°C and how many degrees C = 35°R.

48. 100°C = 212°F; 0°C = 32°F. Construct a graph to illustrate the relation between the two scales. From the graph, find the number of degrees F corresponding to 75°C, 32°C, 45°C. Also find the number of degrees C corresponding to 135°F, 95°F, and 112°F.

49. Supposing $F = mc + p$, find m and p graphically from the graph drawn in Question 48.

50. The expenses of a family are partly constant and partly proportional to the number of its members. They amount to Rs. 133 when there are 8 persons and Rs. 148 when there are 9 persons. Draw the graph and from the graph find what the expenses would be when the family consists of (*a*) 12 persons, (*b*) 4 persons and (*c*) 20 persons.

51. The expenses of a hotel keeper are partly constant and partly dependent upon the number of customers. If there are 50 customers the expenses per month amount to Rs. 520. If there are 45 customers they amount to Rs. 480.

Draw a graph and from the graph find the amount of the expenses when there are (*a*) 25 customers, (*b*) 73 and (*c*) 86.

52. The income of a touring officer consists of a fixed income and a batta depending upon the number of days he is outside his head-quarters. If he is out for 15 days he gets Rs. 375 and if for 17 days Rs. 385. Draw a graph and find from it what he would get if he is out for (*a*) 20 days and (*b*) 25 days.

53. A bottle half full of mercury weighs 248·4 gm. and 122·4 gm. when $\frac{1}{7}$ full. Find what fraction of the bottle is filled when it weighs (*a*) 163·2 gm. and (*b*) 272 gm.

54. A bullock cart driver demands 2 annas for the first mile and $1\frac{1}{2}$ anna for every subsequent mile. Draw a graph to illustrate the relation between the number of miles and the charge and read off the charges for travelling (*a*) 8 miles, (*b*) 12 miles and (*c*) 23 miles.

55. The speed of a ball thrown up with a velocity of 644 ft. per second is retarded by 32 ft. per second every second. Find graphically what would be its velocity at the end of (*a*) 5th second, (*b*) 8th second and (*c*) 25th second. Find when its velocity would be zero.

56. In a hostel students have to pay a caution fee of Rs. 30 for the first month and Rs. 15 for every subsequent month. Find graphically how much a student will have to pay for (*a*) 8 months, (*b*) $10\frac{1}{2}$ months and (*c*) $6\frac{1}{2}$ months.

57. A cistern $\frac{3}{4}$ full has an outlet which empties $\frac{1}{8}$ of the cistern every hour. Find how full the cistern would be at the end of (*a*) 2 hours, (*b*) 3 hours 35 minutes and (*c*) 4 hours 45 minutes. Find when the cistern would be completely empty.

58. It is known that the mercurial column in a barometer diminishes by one inch for a height of about 1,000 ft. ascended above the sea-level. What is the height of a place where the barometer reads (*a*) 27·5, (*b*) 27·8 and (*c*) 28·3 in. assuming the barometrical reading at sea-level is 30 inches.

59. A client agrees to pay a barrister Rs. 500 and Rs. 50 for each day's engagement at court. Draw a graph to illustrate this and find how much the barrister would get if he were engaged for (*a*) 20 days and (*b*) 25 days. Find for how many days he has been engaged if he gets Rs. 1,650.

60. A surgeon charges a fixed sum for performing an operation on a patient and an additional sum for every day he dresses him. The surgeon received Rs. 910 after dressing for 40 days. He would have received Rs. 1,210 had the dressing been continued for another 30 days. Find how many days he dressed if he received Rs. 810.

61. A person who takes 12 shares in the Mylapore Fund incurs an initial expenditure of Rs. 4-12-6. Another subscriber taking 8 shares in the same fund spends Rs. 3-4-6 initially. Assuming the charges to consist of 2 parts one a fixed sum and the other depending upon the number of shares taken, draw a graph and from the graph find the initial expenses of the person taking 50 shares in the fund.

62. It is found that the cost of a marriage in a certain Hindu family is partly constant and partly variable being proportional to the number of guests invited and fed. When 500 persons were fed, the cost of a marriage came to Rs. 3,500 ; while the cost of another marriage celebrated by the same family was Rs. 5,500 when 900 guests were invited and fed. Draw a graph and from it find the number of persons invited and fed when the family celebrated a third marriage the cost of which came to Rs. 4,700.

63. A rickshaw driver has to pay to the owner of the rickshaw a fixed sum daily for the use he makes of the rickshaw out of his earnings which are proportional to the number of miles he runs pulling the rickshaw. One day his net earnings were Re. 1-3-6 for a total journey of 7 miles. Another day they were Re. 1-8-0 for a total journey of 10 miles. Draw a graph and from it find how many miles he pulled the rickshaw on a day on which he earned a net sum of Re. 1-5-0.

64. Assuming that the cost of a building is estimated roughly to consist of two parts, one the price of the ground which is constant and the other the price of the superstructure depending on the number of stories built. Find the cost of a house built on 1 ground of land with 3 stories high, it being given that a building 2 stories high built on 1 ground costs Rs. 7,000 and another building 1 story high built on the same area costs Rs. 5,000.

65. To telegraph an urgent message of 50 words it costs Rs. 6-4-0 ; another of 8 words Re. 1-8-0 ; another of 11 words Re. 1-8-0 ; a fourth of 12 words Re. 1-8-0 and a fifth of 13 words Re. 1-10-0. Draw a graph and from it find the number of words in a message for which Rs. 2-8-0 is charged.

66. It is found that a school spends a certain amount every year in playing matches with other schools in the same locality, being partly constant (consisting of entrance fees) and partly variable (depending on the number of matches played). The school spent Rs. 65 in a year when 20 matches were played and Rs. 90 in a subsequent year when 30 matches were played. Draw a graph and from it find how many matches were played when the expenditure was Rs. 75.

67. Bricks are supplied from a brick-kiln by a contractor to a place where a building is erected on condition that a fixed sum should be paid for the cartage in addition to the cost of the bricks supplied. When 2,000 bricks were supplied the cost including cartage came to Rs. 12 and when 750 bricks were supplied the cost came to Rs. 9-8-0. Find graphically what amount is charged for the cartage ?

68. In an examination paper the maximum for which is 85, three boys get 45, 32 and 16 marks. If the maximum is increased to 100, find what their marks will be (by means of a graph) to the nearest unit.

69. In a paper which carries $37\frac{1}{2}$ marks 2 boys get 13 and $17\frac{1}{2}$. Find what they will get, if the maximum is raised to 50, to the nearest unit.

70. In an examination 4 boys get 75, 63, 49 and 27 marks. If the last boy's marks are raised to 35, find by how many the marks of the others should be increased to the nearest unit.

71. In an examination the first boy got 75 marks and the last boy 27 marks. If the marks of the first boy are raised to 85 and those of the last boy to 35 marks, find the revised marks of a boy who got 68 marks graphically.

72. In a paper the first boy got 65 marks and the last 43 marks. If the marks of the first boy are increased to 80 and those of the last reduced to 36, find what marks in the revision a boy would get if his marks were originally 49.

73. Draw a graph to find the simple interest for 9 months at $4\frac{1}{2}$ per cent. per annum on sums ranging from Rs. 300 to Rs. 325. Read off the interest on Rs. 311 and Rs. 318-8-0 to the nearest pie. Also find to the nearest rupee what sums will produce an interest of Rs. 10-2-8, and Rs. 10-3-4.

74. The price of twill varies from Re. 1-6-0 per yd. to Re. 0-8-0 per yd. according to the quality. If the dearest quality is sold at a reduced price of Re. 1-2-0 and the cheapest at Re. 0-4-0, find for how much a yard of the quality costing Re. 0-12-0 will now be sold.

75. In an examination, the maxima for English, Mathematics, History and Vernacular were 75, 50, 35 and $37\frac{1}{2}$ marks. A got 35, 28, 17 and 28 in these subjects respectively. If the maxima of all the subjects were raised to 100, find what he would get in each subject.

76. In the previous question the marks of the last boy in the several subjects were 17, 13, 12 and 18. If he were to get 35 per cent. in each subject find how much the boy in Question 75 would have got in each subject.

77. The following table gives the radii of different circles and their circumferences in inches :—

Radii	...	1	2	2·5	3	4·5	10
Circumferences	...	6·28	12·56	15·70	18·84	28·26	62·8

From it determine the circumference of a circle whose radius is (*a*) 8″, (*b*) 5·6″ and (*c*) 7·5″.

78. The following table gives the sides of equilateral triangles and their perpendiculars in inches :—

Sides	...	1	2	3	3·7	4·5
Perpendiculars		·870	1·73	2·60	3·20	3·90

Plot the points and construct a graph. From the graph find the perpendicular of an equilateral triangle of (*a*) 8″ side, (*b*) 9·5″ side and (*c*) 7·6″ side.

79. The following table gives the sides of several squares and their diagonals in inches :—

Sides	...	3	5	6	7·5	9·8
Diagonals	...	4·2	7·0	8·4	10·5	13·7

Plot the graph and from it find the diagonal of a square whose side is (*a*) 15″, (*b*) 10·5″ and (*c*) 8·6″.

80. Compare the fractions $\frac{3}{7}$, $\frac{4}{5}$, $\frac{11}{20}$ and $\frac{11}{15}$ graphically and arrange them in ascending order of magnitude.

81. Convert the above fractions into percentages graphically.

82. A railway company charges $2\frac{1}{2}$ pies per mile for third class for passenger trains and $3\frac{1}{2}$ pies per mile for third class for mail trains. Draw graphs (with reference to the same origin and axes of co-ordinates) which would show the charges for distances up to 100 miles for each kind of train and find the distance which would make a difference of 4 as. between the two fares. Also find how many miles more could be travelled in the passenger train for a sum of Re. 1-1-6.

83. In the accompanying figure the straight lines through A, C and E represent the graphs of 3 trains in motion. Assuming that OX represents the distance axis, scale 1″=10 miles and OY represents the time axis scale 1″=1 hour; find graphically the rates of the trains.

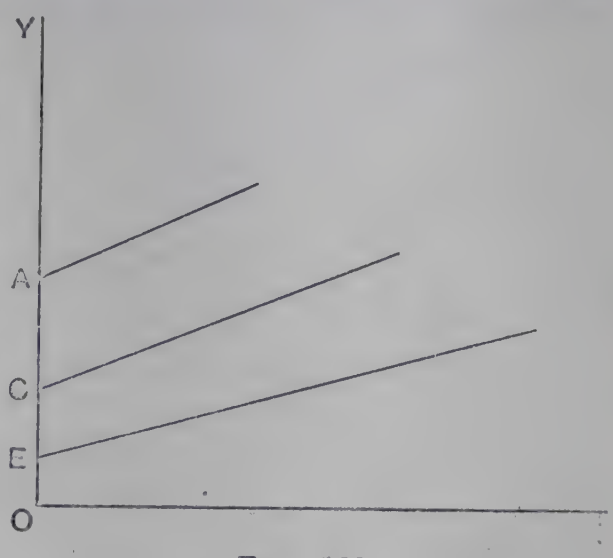


FIG. 180.

84. Find graphically which of the following investments yields the highest rate of profit :—

At the beginning of the year.	At the end of the year.
(1) Rs. 300.	Rs. 375.
(2) ,, 450.	,, 550.
(3) ,, 650.	,, 750.

85. If an aeroplane is moving with a maximum velocity of u ft. per second and diminishes its velocity at a uniform rate of 'f' ft. per second for every second, show that its velocity v after t seconds is given by the formula $v = u -$ ft.

If $u = 110$, $f = 10$, draw a graph showing the relation between v and t. Find (a) from the graph its velocity after the sixth second, and (b) when it comes to rest.

86. In a clearance sale, a shop-keeper alters an original price of Rs. x into the reduced price Rs. $(mx - \frac{1}{4})$. A silk saree which originally cost Rs. 32 was obtained for Rs. 23-12-0. Draw a graph showing the relation between the two prices and from it find the reduced price of a Dhariwal shawl originally costing Rs. 8.

87. A railway company was obliged to enhance its fares and altered an original fare of Rs. x into the increased

fare Rs. $(mx + c)$. The original fares to two stations were Rs. 2 and Rs. 3 and the altered fares were Rs. 2-12-0 and Rs. 4 respectively. Draw a graph to show the relation between the original and altered fares and from the graph find the original price if the altered fare was Rs. 10-4-0.

88. Soap nut sells at Rs. 6 a maund, the cost of powdering it in a mill is Rs. 2 per maund and the price of a gunny to hold the powder is 4 as. Draw a graph showing the relation between the quantity of soap nut bought and powdered and the amount paid including the price of the gunny. Find from the graph how much of soap nut powder could be had for Rs. 4-4-0; also the price of 14 viss of soap nut powder. (A gunny will ordinarily hold 2 mds. of powder.)

89. Test if the following points are in a straight line:—

(*a*) (1, 11); (4, 26); (— 4, — 14); (— 8, — 34)

(*b*) (1, $-\frac{1}{4}$); (10, $-13\frac{3}{4}$); (— 5, $8\frac{3}{4}$); (— 9, $14\frac{3}{4}$)

and find the equation of the lines passing through each set.

90. Two quantities x and y are connected by the relation $y = mx$. $y = 10$, when $x = 5$. Draw a graph and from it find the value of m. What is the inclination of the graph to the axis of x?

91. Two variables x and y are connected by the relation $y = mx$. When $x = -4$, $y = 8$. Draw the graph and from it find the value of m. What is the inclination of the graph to the axis of x?

92. Draw the graph of $2x + 3 = y$. Test if (— 6, — 3), (9, 7) are on the line.

93. Draw the graph of $3x - 5y = 1$. Test if (7, 4), (8, $4\frac{3}{4}$) are on the line.

94. Assuming that a relation of the form $y = mx + c$ exists between the values of x and y in the following tables, find graphically the values of m and c.

(*a*)

$x =$	2	4	10	20	−30	
$y =$	7	17	47	97	−153	

(*b*)

$x =$	1·5	6·5	10·5	15·5	18	
$y =$	2	$15\frac{1}{3}$	26	$39\frac{1}{3}$	46	

95. Examine if the following sets of points are collinear :—

(*a*) (10, 33) ; (— 5, — 27) ; (8, 25).

(*b*) (5, 5) ; (10, 11) ; ($7\frac{1}{2}$, 8).

Read the equations of the straight line graphs passing through them.

96. Find graphically the equation of a straight line passing through the origin parallel to each of the graphs in Question 95.

97. Find graphically the equation of a straight line passing through the origin perpendicular to each of the graphs in Question 95.

98. Find graphically the equation of a straight line passing through the point (5, 10) parallel to each of the graphs in Question 95.

99. Find graphically the equation of a straight line passing through the point (5, 10) perpendicular to each of the graphs in Question 95.

100. Examine graphically if the following sets of straight lines are concurrent :—

(*a*) $5x + 6y = 10$ (1) ; $4x - 8y = 7$ (2) ; $3x - 18y - 4 = 0$ (3).

(*b*) $6x - 7y = 6$ (1) ; $8x + 3y = 52$ (2) ; $x + 5y - 23 = 0$ (3).

How would you prove without drawing graphs that in each set there are concurrent lines ? Measure the angles between (1) and (3) in (*a*) ; and those between (1) and (2) in (*b*).

101. Draw the locus in each of the following cases :—

AB and AC are two straight lines at right angles to each other.

(*a*) A point moves so that its distance from AB is equal to (i) its distance and (ii) thrice its distance from AC.

(*b*) A point moves so that the sum of its distances from AB and AC = 2 inches.

(*c*) A point moves so that the difference of its distances from AB and AC = 1·5 inches.

(*d*) A point moves so that twice its distance from AB increased by thrice its distance from AC = 5 inches.

102. No. 15 Bangalore Passenger train leaves Central Station at 5 and reaches Bangalore at 19.40. The distance between Bangalore and Madras is 240 miles. No. 16 Passenger train leaves Bangalore at 7.20 and arrives at Central Station at 19.20. Find when and where they meet and their average rates of travelling.

103. No. 7 Bangalore Mail leaves Central Station at 20.40 and reaches Bangalore 240 miles distant from Madras at 6.30. No. 8 Bangalore Mail leaves Bangalore at 20.40 and reaches Central Station at 6.30. Find when and where these two trains meet and their average rates of running.

104. Draw the graphs in Questions 102 and 103 with reference to the same axes. Find when and where (*a*) No. 15 passenger and No. 8 mail meet and (*b*) No. 16 passenger and No. 7 mail meet.

105. Draw the graph of the following train (No. 21 Fast Passenger) with the help of the data furnished below :—

No. of Miles from Madras	Station	Time of Arrival	Time of Departure
0	Madras ...	—	7.15
43	Arkonam ...	8.55	9.15
84	Renigunta ...	10.55	11.5
162	Cudappah ...	14.25	14.35
276	Guntakkal ...	20.15	21.5
351	Raichur ...	0.40	Nil.

From the gradients of the several courses of the train find the greatest speed of the train in miles per hour. (The distance may be expressed to the nearest 5 miles.)

106. In the figure in Question 105, with reference to the same axes and origin, draw the graph of the following train running from Raichur to Madras (No. 22 Bombay fast passenger).

No. of Miles from Raichur	Station	Time of Arrival	Time of Departure
0	Raichur ...	—	4.25
75	Guntakkal ...	7.25	7.55
189	Cudappah ...	13.10	13.20
267	Renigunta ...	16.50	17.5
308	Arkonam ...	18.50	19.15
351	Madras ...	21.10	Nil.

From the gradients of the several courses of the train find (*a*) the greatest speed of the train in miles per hour and (*b*) when and where trains No. 21 and 22 meet. Find from the graph the distance between the two trains at 14.0 and when they will be 10 miles distant.

107. Draw the graphs of the two following trains with reference to the same axes :—

No. 5 Bombay Mail running from Madras to Raichur.

Distance in miles from Madras	Station	Time of Arrival	Time of Departure
0	Madras ...	—	21.15
43	Arkonam ...	22.20	22.35
84	Renigunta ...	23.50	0.5
162	Cudappah ...	2.50	3.0
276	Guntakkal ...	7.45	8.15
351	Raichur ...	11.20	—

No. 6 Bombay Mail running from Raichur to Madras.

Distance in Miles from Raichur	Station	Time of Arrival	Time of Departure
0	Raichur ...	—	17.0
75	Guntakkal ...	19.40	20.20
189	Cudappah ...	0.50	0.55
267	Renigunta ...	3.55	4.20
308	Arkonam ...	5.45	6.5
351	Madras ...	7.30	—

Find when and where the two mails meet. Compare the maximum speeds of the two trains respectively. State between what two stations the trains run with maximum speeds.

108. Draw the graph of the following train :—

No. 31 Waltair Passenger running from Madras to Waltair.

Distance in Miles from Madras	Station	Time of Arrival	Time of Departure
0	Central Station ...	—	8.10
86	Gudur ...	13.0	13.25
268	Bezwada ...	23.20	23.40
392	Samalkot ...	6.25	6.40
485	Waltair ...	11.10	—

[Let 10 miles be represented by $\frac{1}{10}$" and 10 minutes by $\frac{1}{10}$" and let the distance be expressed correct to the nearest 10 miles]. From the graph find when it goes to (*a*) Bitragunta and (*b*) Tenali their distances from Madras being 131 miles and 249 miles. Find also the distances from Madras to (*a*) Rajamundry and (*b*) Tuni if the train arrives at these two stations at 4.50 and 8.0 respectively.

109. Draw the graph of the following train, with reference to the same axes as in Question 108 :—

No. 10 Fast Passenger running from Waltair to Madras.

Miles from Waltair	Stations	Time of Arrival	Time of Departure
0	Waltair ...	—	16·0
93	Samalkot ...	20.10	20.40
217	Bezwada ...	4.45	5.35
399	Gudur ...	13.20	13.40
485	Central Station.	17.30	—

Find (*a*) at what time the train arrives at (i) Tuni and (ii) Ellore 60 and 180 miles from Waltair respectively, (*b*) the distances of (i) Ongole and (ii) Ponneri from Waltair supposing the train reaches these stations at 9.15 and 16.20 respectively and (*c*) when and where the two passenger trains Nos. 31 and 10 meet.

110. Draw the graphs of the following trains with reference to the same axes and in the same figure :—

(A) Calcutta mail running from Madras to Waltair.

Distance from Madras	Station	Time of Arrival	Time of Departure
0	Central Station ...	—	20.0
131	Bitragunta ...	0.35	0.45
268	Bezwada ...	5.30	5.50
305	Ellore ...	7.5	7.20
392	Samalkot ...	10.50	11.5
485	Waltair ...	14.35	—

(B) Calcutta mail running from Waltair to Madras.

Distance from Waltair	Station	Time of Arrival	Time of Departure
0	Waltair ...	—	13.20
93	Samalkot ...	16.50	17.5
180	Ellore ...	20.35	20.50
217	Bezwada ...	22.5	22.25
354	Bitragunta ...	3.0	3.10
485	Central Station Madras.	8.0	—

Find (*a*) when and where the two mail trains meet, (*b*) what the distance between the two trains at 4.0 and 2.0, is and (*c*) what the interval of time is between the instants when the two trains pass (i) a station 200 miles from Madras and (ii) a station 310 miles from Waltair.

111. A motor car starts at 5 a.m. from Madras and goes to Chingleput 37 miles from Madras at the rate of 20 miles per hour. A Vellore jutka starts from Chingleput at 6 a.m. for Madras at the rate of 10 miles an hour.

Find graphically (*a*) when and where the car and jutka meet (*b*) at what distance the jutka will be from Madras when the car reaches Chingleput.

112. If in the preceding question, the car meets with an accident at 6 a.m. which detains it for 30′, find when and where the jutka will meet the car.

113. A can do a piece of work in 10 days, B can do a piece of work in 15 days. Draw the graph of A's work, B's work and of the combined work of A and B. Find from the graph in how many days A and B will finish the work. (Note the L.C.M. of 10 and 15 will be of some use in deciding the scale you choose).

114. If in the preceding question A works for 2 days, and then B joins him, find when the work will be finished by both A and B working together.

115. A, B and C can do a piece of work in 3, 4 and 5 days respectively. A works at it for the first two days and

then leaves off. B and C work at it until it is finished. Find graphically when the work was finished. Draw the graphs of the work of (1) A, (2) B, (3) C, (4) A and B, (5) B and C, (6) A and C and (7) A, B and C.

116. If in the preceding question A worked for the first day, B for the second day and A, B and C from the third day, find graphically when the work was finished.

117. A cistern is filled by two taps A and B in 4 and 5 hours respectively. Draw the graphs of the quantity of water supplied by A and B and of the combined supply of A and B. Find from the graph in how many hours the cistern would be filled if both the taps were opened together.

118. If, in the preceding question, the tap A alone were kept open for 1 hour and then B also opened, find graphically in what time the cistern would have been filled.

119. A cistern is filled by a tap A in 5 hours and emptied by a tap B in 4 hours. Draw the graphs of the quantity of water supplied by A, that emptied by B and of the combined emptying action of A and B. From the graph find when the cistern will be emptied if both the taps were opened when the cistern is full.

120. A cistern is filled by two taps A and B and emptied by a third tap in 3, 4 and 5 hours respectively. If all the taps were opened simultaneously find graphically when the cistern will be full.

121. A cistern contains water to a depth of 25 ft. There is a leak in the bottom through which water flows so that the surface is lowered by 4 inches in every 5 minutes. Draw a graph to show the escape of water through the leak and from it find (*a*) when the depth of water would be 16 feet 8 inches, (*b*) what the depth would be after 6 hours and (*c*) when the cistern would be empty.

122. If in the preceding question, there is a supply of water which raises its surface by 3 inches in 10 minutes, find graphically when the cistern will be (*a*) empty and (*b*) the water will be 10 feet deep.

123. Find graphically when the minute and hour hands of a watch will be together between (*a*) 1 and 2, (*b*) 2 and 3, (*c*) 11 and 12 and (*d*) 8 and 9.

124. Find graphically when the minute and hour hands of a watch are at right angles to one another between (*a*) 1 and 2, (*b*) 2 and 3, (*c*) 11 and 12 and (*d*) 8 and 9.

125. Find graphically when the minute and hour hands of a watch are opposite to one another between (*a*) 1 and 2, (*b*) 2 and 3, (*c*) 11 and 12 and (*d*) 8 and 9.

126. Find graphically the interest on Rs. 350 at 5 per cent. for 3 years.

[The interest on 100 for 3 years is 15; the interest on 0 is 0. Hence plot (100, 15); (0, 0). Join.]

127. Find graphically the interest on Rs. 600 at 9 per cent. for 5 years. Hence find (*a*) on what sum Rs. 112-8-0 is the interest and (*b*) what the interest on Rs. 250 is.

128. Rs. 500 is lent at 5 per cent. for 6 years. Find graphically (*a*) what Rs. 450 amounts to in the same time at the same rate and (*b*) what sum amounts to Rs. 455 in the same time at the same rate.

129. Rs. 750 is lent at 12 per cent. for 1 year and 8 months. (*a*) What sum amounts to Rs. 1,200 in the same time at the same rate? (*b*) What is the amount of Rs. 270 at the same rate in the same time?

130. A merchant sells all his articles so as to gain 20 per cent. Find the C.P. of an article whose selling price is Rs. 54 and the S.P. of an article whose C.P. is Rs. 100.

CHAPTER XXVII

Graphs Chiefly Statistical

159. Two variables are generally connected by a certain relation. For a particular value of one variable, there is some value of the other variable. In certain cases it is possible to find the value of one variable corresponding to a given value of the other variable provided the relation connecting the two variables is known or the mathematical law governing their interdependence is stated. For example, in the relation $y = 4x + 5$, if $x = 3$, $y = 17$ or if

$y = 6$ then $x = \frac{1}{4}$; or in the relation $y = x^2$, if $x = 3$, $y = 9$ or if $y = 16$, $x = \pm 4$. In other cases it is not possible to find the value of one variable corresponding to a given value of the other variable. For example, what the population will be in a particular year in a town, or what the temperature will be at a certain place on a certain day cannot be determined according to any definite law known but may be obtained by observation and ascertaining the result. In all such cases figures representing different pairs of the corresponding values of the two variables are got from returns, reports prepared by Government in consultation with the various departments collecting information for purposes of investigation. Such figures are called **statistics**. To convey a good impression to the mind of the changes in the values of the variables, a diagrammatic representation in the form of a curve is generally made. The following examples will show what is meant :—

Model No. (1) The average attendance per week for some successive six weeks in a class of 40 boys is as follows :—

First week	Second week	Third week	Fourth week	Fifth week	Sixth week
30	35	30	37	28	32

Represent the above graphically.

(1) Supposing 25 to be the average just prior to the first week.

(2) Supposing 25 itself to be the average in the first week and not 30.

(1) *Solution.*—Along OX, $\frac{2}{10}$" represents a week ; along OY, $\frac{1}{10}$" represents 1 pupil and O itself along the y axis represents 25 pupils. (This device is adopted to diminish the space required for the diagram.) The graph is not a continuous line but a series of broken lines. These are not graphs but there is some advantage in the graphical representation. The

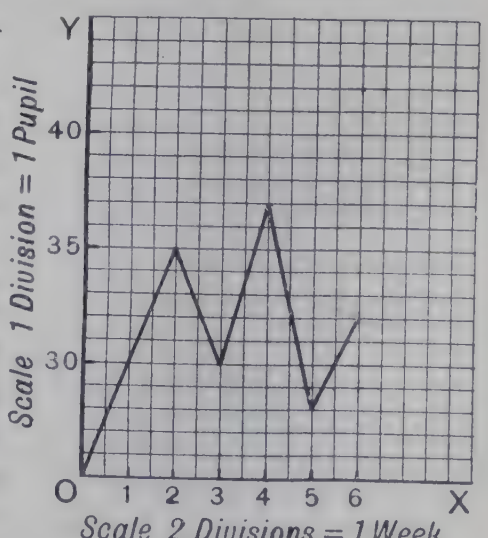


FIG. 181.

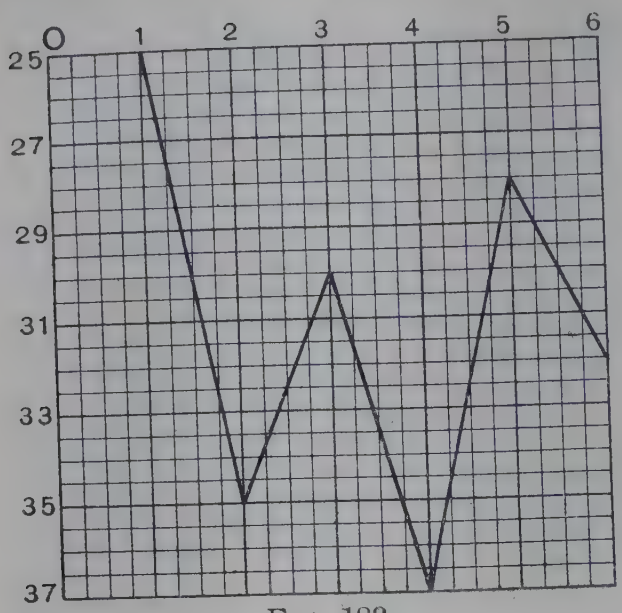


FIG. 182.

graph vividly tells us that the average is lowest just prior to the first week and highest in the fourth week. It also shows how it rises and falls. Sometimes in newspapers a diagram is drawn as in Fig. 182.

(2) The II part of this model is done as represented in newspapers.

Model No. (2) The daily temperature of Madras for some days in June, 1923 is given in the following table :--

Date in June 1923	14	15	16	18	19	20	21
Temperature in degrees	99·1	99·8	100·4	99·3	101·3	105·0	104·8

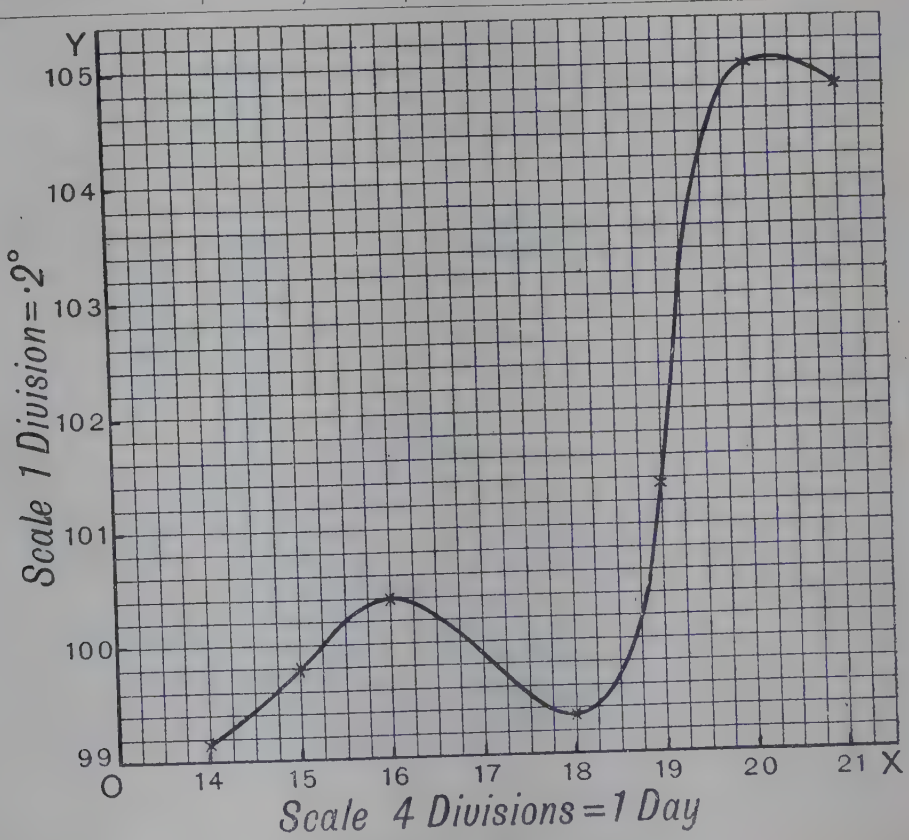


FIG. 183.

Represent the above graphically and find the probable temperature on the 17th June.

Along OX, $\frac{1}{10}$" represents 1 day ; along OY, $\frac{1}{10}$" represents ·2° of the temperature. The origin O represents 99° and 13th of June. The graph is a continuous curve.

From the curve it is easily seen that the daily temperature increases from the 14th to the 16th June, there is a fall from the 16th to the 18th, again there is a rise from the 18th to the 20th and again a fall from the 20th. The probable temperature on the 17th is 99·8. When we read from the graph the value of any one variable corresponding to any value lying between two consecutive values of the other variable, we are said to find the required quantity by **interpolation**.

In drawing the continuous curve, it is supposed that there is no knowing whether between two days, say the 16th and the 18th, there was a continuous fall in temperature or there were rises and falls not shown by the curve but it may be taken that in all probability there was a gradual fall. If the readings are to be accurate, the interval between two successive readings taken should be very small. If the readings were taken every two days, the graph then drawn would give different readings of temperature for days in the interval. The smooth curve suggests that there are no abrupt changes in temperature whereas a broken line indicates sudden changes.

Exercise XXVII.

1. The recorded monsoon rainfall at Nagpur for the years from 1854 to 1868 is given below :—

Year	Inches	Year	Inches
1854	48·40	1862	43·29
1855	24·04	1863	37·46
1856	44·33	1864	28·96
1858	31·87	1865	38·16
1859	29·48	1867	53·72
1861	40·89	1868	19·28

The records for 1857, 1860 and 1866 were lost. Draw

a graph and supply the probable missing figures. Do you get a continuous curve or a broken line? (The rainfall may for the purposes of plotting be corrected to the nearest half inch.)

2. The following table shows the educational expenditure for public institutions in certain years in India :—

Year	1910–11	1911–12	1912–13	1913–14	1914–15	1915–16
Amount in crores of Rs.	4·8	5·2	6·0	6·7	7·3	7·4

Draw a graph to show the variations in the expenditure.

3. The following table gives the number of prisoners under trial in India in the years given below :—

Year	1910	1911	1912	1913	1914	1915
No. of Prisoners...	1885	1960	2133	2140	2120	2083

Draw a graph to show the variations in the number of prisoners. (For purposes of plotting, the number of prisoners may be corrected to the nearest five.)

4. The army expenditure in India in recent years is given in the following table :—

Year	1911	1912	1913	1914	1915	1916	1917	1918
Amount in millions of £	19·1	19·5	19·6	19·8	20·3	21·9	24·3	22·2

Draw a graph to show the increase.

5. The total daily sales in Liverpool market for a

successive number of days in June, 1923, are given below :—

Date in June 1923	12	13	14	15	16	18	19
Amount in £ ...	8,000	7,000	5,000	4,000	4,150	5,000	5,000

Draw a graph to indicate the fluctuations and from it find the probable amount of sales on the 17th.

6. The sales of copies of a certain text-book for a number of days commencing from the date of the reopening of a school, in its boys' co-operative branch are given below :—

Day from the reopening	1	2	3	4	5	6	8
No. of books sold...	45	33	56	29	32	25	28

Draw a graph to show the variations and find the probable number of books sold on the 7th. Is the curve continuous or a broken line ?

7. The temperature of a patient suffering from typhoid fever recorded hourly on a certain day is as follows :—

6.30 a.m. 103·2	11.30 a.m. 100·9	4.30 p.m. 103·4
7.30 ,, 103·1	12.30 p.m. 99·5	5.30 ,, 102·9
8.30 ,, 102·9	1.30 ,, 101·3	6.30 ,, 102·3
9.30 ,, 100·5	2.30 ,, 102·7	
10.30 ,, 101·0	3.30 ,, 103·1	

Plot the readings, construct a graph and interpolate so as to get his temperature at 11 a.m., 1 p.m. and 2.45 p.m.

8. The following table gives the rate of exchange of German marks into £ sterling :—

Date in June 23	14	15	16	18	19	20	21
No. of marks per £	443,000	485,000	470,000	515,000	680,000	666,000	540,000

Draw a graph and interpolate the rate of exchange on the 17th.

9. In the following table the atmospheric pressure as corrected and recorded in the Madras Observatory is given for a number of days :—

Date in June 1923	11	13	15	17	19	21	
Pressure in inches	29·611	29·646	29·684	29·630	29·703	29·747	

Plot the graph and interpolate so as to get the pressure on the 20th.

10. The soundings taken by a surveyor in the Menambakkam tank in January 1922 along a cross-section are as follows :—

Distance in ft.	0	10	20	30	40	50	60	70	80	90	100
Depth in inches.	6	1·1	1·5	1·7	2·3	3·5	2 9	3·7	4·0	3·8	4·2

Plot the bed of the tank on that cross-section and estimate what would be the depth of water at the 75th foot and 93rd foot.

11. The solubility of Potassum Alum in water in grams

per 100 gm. of water at different temperatures is given below :—

0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°
6·5	7·5	8·5	11	12·5	14·5	17	19·5	23	27	31·5	40	57·5	80	112·5	140

Draw a graph and find the solubility at $62\frac{1}{2}°$ and at 18°. Also find at what temperature 85 gms. will be the solubility.

12. The results of an experiment to verify Hooke's law are given below :—

Weight in the pan in gm.	0	50	100	150	200	250	300	350	400
Reading of the pointer in cms.	55·9	58·7	61·6	64·5	67·5	70·4	73·3	76·2	79·1

Draw a smooth curve to pass through all these points. Find from the graph the relation between the load and the elongation of the spring. Find what load will produce an elongation of (*a*) 3·8 cm. and (*b*) 18·9 cms. Also find what elongation a load of 225 gm. will cause.

13. The following table gives the results obtained in a pendulum experiment :—

Length of the pendulum in cms.	83·13	76·53	68·13	107·23	99·93	129·53	117·93	122·53
Time of 1 oscillation.	1·84	1·74	1·64	2·1	2·0	2·26	2·16	2·22

Plot the points and draw a smooth curve to pass through them. Find the length of a pendulum whose time of oscillation is 1·95 seconds. Also find the time of oscillation of a pendulum 105 cms. long.

14. The following table gives the lengths of a degree of longitude at mean sea-level at different latitudes :—

Degree of latitude	0°	6°	12°	18°	24°	30°	36°
1° of longitude in statute miles.	69·16	68·78	67·66	65·80	63·21	59·94	56·01
Degree of latitude	42°	48°	54°	60°	66°	72°	78°
1° of longitude in statute miles.	51·47	46·96	40·74	34·67	28·21	21·43	14·42

Draw a smooth curve and find the length of a degree of longitude at (*a*) 14° lat., (*b*) 22°, (*c*) 35°, (*d*) 65° and (*e*) 75°. Also find the latitude of the place where the length of a degree of longitude is (*a*) 35·50, (*b*) 12·05 and (*c*) 62·20 miles.

15. The following table gives the boiling point of water at several heights above the sea-level in Fahrenheit scale :—

Scale ...	200°	201°	203°	204°	205°	206°	207°	208°	209°	210°	211°	212°	213°	214°
Feet ...	6366	5825	4749	4215	3682	3151	2622	2094	1568	1044	521	0	—518	—1015

Draw a graph. Find the boiling point of water at (*a*) 4,000 ft., (*b*) 3,550 ft., (*c*) 2,870 ft. above sea-level, (*d*) 675 ft. and (*e*) 800 ft. below sea-level.

16. The following table gives the number of persons attacked with plague and also the number of deaths :—

Year	1898-99	1899-1900	1900-1	1901-2	1902-3	1903-4	1904-5	1905-6	1906-7
Seizures ...	17	2	11	7	14	12	32	75	3
Deaths ...	11	0	7	4	10	10	19	57	2

Draw two graphs with reference to the same axis and find if there could have been a time when the number of seizures was exactly equal to the number of deaths.

17. The following table gives the value of paper produced in Indian mills :—

Year	1915	1916	1917	1918	1919	
Value in lakhs of rupees.	89·6	124·9	187·8	211·2	244·5	

Indicate the expansion of the manufacture by means of a graph.

18. The following table shows the quantity of imported beer into British India :—

Year	1914	1915	1916	1918	1919	
Gallons in millions.	4·2	2·8	2·9	2·0	1·9	

Draw a graph and from it find what quantity was imported in 1917.

19. The following table gives the quantity in lbs. of yarn produced in cotton mills of India :—

Year	Millions of lbs.	Year	Millions of lbs.
1910–11 ...	41·1	1916–17 ...	44·2
1911–12 ...	42·8	1917–18 ...	43·1
1912–13 ...	45·0	1918–19 ...	42·8
1913–14 ...	44·7	1919–20 ...	44·3
1914–15 ...	43·0		

Draw a graph to represent the variations and from the graph find what quantity of yarn was produced in 1915–16.

20. The following table shows the value of salt manufactured in India :—

Year	Value in lakhs of rupees	Year	Value in lakhs of rupees
1910	84·8	1915	99·0
1911	70·3	1917	149·4
1912	76·5	1918	246·8
1913	81·2	1919	182·4
1914	72·5		

Draw a graph to represent these variations correcting the values to the nearest lakh and from the graph find the value of salt manufactured in 1916.

21. The following table shows the quantity of diamonds in carats dug out from Indian mines and their value in thousands of rupees :—

Year	Quantity in carats	Value in thousands of rupees
1910	77	9,000
1911	53	7,000
1912	28	6,000
1913	116	27,000
1914	55	12,000
1916	20	5,000
1917	18	7,000
1918	73	39,000

Draw two graphs with reference to the same axes to indicate the variations in the quantity and also in value. (The quantities may be corrected to the nearest ten for purposes of plotting.)

From the graphs find the quantity dug out in 1915 and also its value.

22. The following table shows the production of kerosene in the Punjab in gallons :—

Year	Production in gallons	Year	Production in gallons
1900-01 ...	824	1905–6 ...	655
1901–2 ...	797	1907-8 ...	536
1902–3 ...	852	1908-9 ...	223
1903–4 ...	789	1909–10 ...	382
1904–5 ...	729		

Draw a graph to indicate the variations in the production and from your graph find the probable production in 1906–7.

The quantity of production may be corrected to the nearest 10 gallons.

23. The following table shows the percentage of Indian kerosene oil consumed :—

Year		Percentage	Year		Percentage
1900–01	...	10·7	1905–6	...	50
1901–2	...	13·4	1907–8	...	51·2
1902–3	...	18·3	1908–9	...	43·9
1903–4	...	34·5	1909–10	...	57
1904–5	...	38			

Draw a graph to show the fluctuations in the percentage and from the graph find the probable percentage for the year 1906–1907.

24. The following table shows the quantity of coal consumed on the Indian railways :—

Year		Quantity in millions of tons	Year		Quantity in millions of tons
1911–12	...	4·3	1915–16	...	5·2
1912–13	...	4·7	1917–18	...	5·6
1913–14	...	5·0	1918–19	...	5·9
1914–15	...	5·0	1919–20	...	6·1

Draw a graph to show the variations and from the graph find the probable quantity of coal consumed in the year 1916–17.

25. The following table shows the quantity of wood consumed on Indian railways :—

Year		Quantity in tens of thousands of tons	Year		Quantity in tens of thousands of tons
1911–12	...	13	1916–17	...	12
1912–13	...	12	1917–18	...	24
1913–14	...	11	1918–19	...	34
1915–16	...	11	1919–20	...	32

Draw a graph to show the variations and from the graph find the probable quantity consumed in the year 1914–15.

26. The following table shows the number of Imperial Post Offices open at the close of each year mentioned below :—

Year	No. of Post Offices open	Year	No. of Post Offices open
1910–11 ...	18813	1916–17 ...	19409
1911–12 ...	18801	1917–18 ...	19410
1912–13 ...	18789	1918–19 ...	19445
1913–14 ...	18946	1919–20 ...	19439
1914–15 ...	19158		

Correct the number of Post Offices in each case to the nearest ten. Plot the graph and from it find the probable number of Post Offices open in the year 1915–16.

27. The following table shows the Post Office revenue for 10 years from 1910–11 to 1919–20 :—

Year	Revenue in lakhs of rupees	Year	Revenue in lakhs of rupees
1910–11 ...	72·3	1916–17 ...	95·5
1911–12 ...	77·1	1917–18 ...	100·0
1912–13 ...	82·4	1918–19 ...	114·7
1913–14 ...	87·6	1919–20 ...	128·4
1914–15 ...	87·1		

Each amount of revenue may be corrected to the nearest ·5 of a lakh. Draw a graph to show the gradual increase in the revenue and from it find the probable revenue in the year 1915–16.

28. The following table shows the Post Office expenditure for ten years from 1910–11 to 1919–20 :—

Year	Expenditure in lakhs of rupees	Year	Expenditure in lakhs of rupees
1910–11 ...	29·7	1916–17 ...	32·8
1911–12 ...	30·7	1917–18 ...	36·5
1912–13 ...	31·2	1918–19 ...	40·4
1913–14 ...	32·0	1919–20 ...	46·5
1915–16 ...	32·4		

Correct the amount of expenditure in each case to the nearest ·5 of a lakh and draw the graph. From the

graph, find the probable amount of expenditure in the year 1914–15.

29. The following table shows the area of the irrigable land in India in each of the years from 1910–11 to 1919–20.

Year	Area in millions of acres	Year	Area in millions of acres
1910–11 ...	14·2	1915–16 ...	15·6
1911–12 ...	15·3	1916–17 ...	16·5
1912–13 ...	14·6	1917–18 ...	16·3
1913–14 ...	15·2	1919–20 ...	18·8
1914–15 ...	16·1		

Draw a smooth curve to show the variations and from it find the probable irrigable area in the year 1918–19.

30. The following table shows the total gross receipts from irrigation and navigation :—

Year	Gross receipts in crores of rupees	Year	Gross receipts in crores of rupees
1910–11 ...	5·8	1915–16 ...	7·4
1911–12 ...	6·2	1916–17 ...	7·9
1912–13 ...	6·8	1917–18 ...	7·8
1913–14 ...	7·3	1918–19 ...	8·3
1914–15 ...	—	1919–20 ...	9·0

Plot the graph and from it find the blank in the above table.

31. The following table gives the expenditure under the heading irrigation and navigation for the years from 1910–11 to 1919–20 including major and minor works :—

Year	Expenditure in lakhs of rupees	Year	Expenditure in lakhs of rupees
1910–11 ...	204	1915–16 ...	251
1911–12 ...	209	1916–17 ...	243
1912–13 ...	217	1917–18 ...	253
1913–14 ...	—	1918–19 ...	258
1914–15 ...	236	1919–20 ...	286

Plot the graph and from it find the blank in the above table.

32. The following table gives the depth of water in a tank as observed on the first of every month:—

Month	Depth	Month	Depth
January ...	39′ 3″	April ...	31′ 0″
February ...	36′ 6″	May ...	26′ 3″
March ...	32′ 9″	June ...	24′ 6″

Draw a graph to show the variation in depths and find when the depth was 30 ft. 6 in.

33. The following table shows the height of the barometer in centimetres corresponding to heights above sea-level expressed in kilometres :—

Height above sea-level in km.	0	1	2	3	4	6	7	8	9	10
Height of barometer in cms.	76	70	59	52	47	37	34	24	23	22

Represent the variations graphically and find the reading of the barometer at a place 5 km. high above the sea-level.

34. The following is a set of observations, the volumes being measured in cubic centimetres and the pressure in centimetres of mercury to which a gas is subjected :—

Pressure	54	58·5	63·4	60	76	84·5	95
Volume	70	65	63·4	55	50	45	40

Represent the changes by means of a graph.

35. The daily velocity of the wind by observation has been recorded as follows :—

Date in June	18	19	20	21	22	23	25	26	27	28	30
Velocity in miles per hour	149	206	224	240	221	214	214	197	215	193	204

Represent the variations graphically and find the velocity on the 24th and 29th.

36. The following table gives the average heights of boys between 6 and 16 :—

Age in years	6	7	8	9	10	11	12	13	14	15	16
Height of boys in inches	40·5	42·3	44·1	45·5	47·9	49·9	50·5	52·8	54·2	55·9	58

Represent the variations graphically.

37. The following table gives the weight of iron square sheets of different sizes of the same thickness :—

Length of side in ft. ...	1	2	3	4	5
Weight in lbs. ...	39	156	351	624	975

Plot a graph to denote these variations and find the weight of a square sheet of the same thickness whose side is (*a*) $3\frac{1}{2}$ ft. and (*b*) $4\frac{1}{2}$ ft.

38. The following is the table of annual premiums payable throughout life to assure Rs. 1,000 with profit at death :—

Age next birthday	Annual premium RS A	Age next birthday	Annual premium RS A
20 ...	21 10	27 ...	25 5
21 ...	22 1	28 ...	25 15
23 ...	22 9	29 ...	26 9
24 ...	23 9	30 ...	27 4
25 ...	24 2	32 ...	28 11
26 ...	24 11		

Draw a graph to show the increase in the rate of premium and find the premium for the age 31 next birthday.

39. The following table below gives the expectation of life for males at different ages :—

Age	Expectation of life	Age	Expectation of life
40	25·30	51	18·31
41	24·65	55	15·95
48	20·18	56	15·37
49	19·55	60	13·14
50	18·93		

Draw a graph and from it find what the expectation of life is at 45 and at 58.

40. The interior angle of a regular polygon $= \frac{2(n-2)}{n} \times 90°$. Give to n values from 3 to 12. Plot a graph to show the variations in the magnitude of the interior angle of a regular polygon as the number of sides is increased from 3 to 12.

41. The following table shows the quantity of rainfall in the Thames basin during the period 1883–1902 :—

Year	Rainfall in inches	Year	Rainfall in inches
1883	28·4	1893	22·1
1884	22·9	1894	32·3
1885	29·1	1895	26·3
1886	31·1	1896	25·8
1887	21·3	1897	27·8
1888	28·4	1898	22·1
1889	25·6	1899	24·8
1890	22·8	1900	27·9
1891	33·3	1901	23·5
1892	25·0	1902	21·9

Draw a graph to show these variations.

42. The formula $h = 16t^2$ gives the number of feet a body falls through in t seconds.

t in seconds	1	2	3	6	8	10	
h in feet ...	...	...	...	...	...	...	...

Fill up the above blanks and draw a graph showing the variations in h and t. From the graph find h when $t = 5$, and find t when $h = 300$.

43. $v^2 = 2gh$ (where g=32). This formula gives the height in feet to which a body projected with a velocity of v ft. per second will rise before it begins to fall.

$v =$	32	48	64	80	96
$h =$	...	...	...	...	...

Draw a graph to show the relation between v and h and from the graph find v when $h = 50$ and h when $v = 60$.

CHAPTER XXVIII.

Construction of Quadrilaterals.

160. A quadrilateral may be divided into two triangles by drawing a diagonal. Hence a quadrilateral can be constructed, if the two triangles of which it is made up are constructed from the data. Three elements of a triangle must be given to construct it. If a quadrilateral consists of two triangles, six elements must be given but it will be found generally that five elements are enough on account of the contiguous position of the triangles forming quadrilateral.

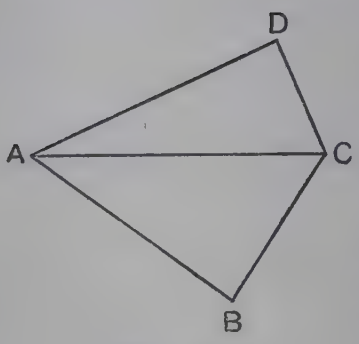


FIG. 184.

Model No. (1) **5 Lengths.**

Construct a quadrilateral, given its sides and a diagonal.

AB = 4·5 cm., BC = 3·2 cm., AC = 5·3 cm., CD = 2·3 cm. and DA = 4·9 cm.

Solution.—Construct a triangle ABC with sides AB, BC and AC = 4·5 cm., 3·2 cm. and 5·3 cm. respectively. With centres A and C describe circles of radii 4·9 cm. and 2·3 cm., intersecting at D. Then ABCD is the required quadrilateral.

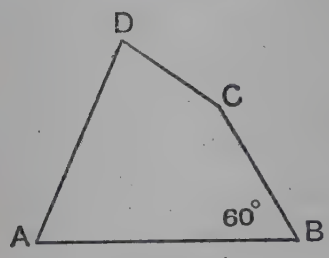


FIG. 185.

Model No. (2) **4 lengths and one angle.**

Given AB, BC, CD and DA = 4·5 cm., 3·2 cm., 2·3 cm. and 4 cm., respectively and angle B = 60°, construct the quadrilateral ABCD.

Solution.—Proceed as in constructing the triangle ABC, from the data AB = 4·5 cm., ∠B = 60° and BC = 3·2 cm.

With A and C as centres and radii 4 cm. and 2·3 cm., describe circles cutting each other at D. Then ABCD is the required quadrilateral.

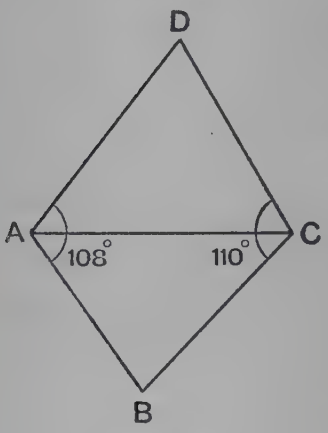


FIG. 186.

Model No. (3) **3 lengths and 2 angles.**

Given AB, BC, CA, ∠A and ∠C to be 4 cm., 5 cm., 6 cm., 108° and 110°. Construct the quadrilateral ABCD.

Solution.—Construct a triangle ABC, having its sides AB, BC, CA, equal to 4 cm., 5 cm. and 6 cm. respectively. At A and C make the angles BAD and BCD equal to 108° and 110° respectively, meeting at D. Then ABCD is the required quadrilateral.

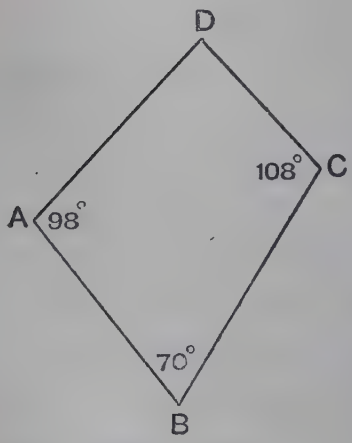


FIG. 187.

Model No. (4) **2 lengths and 3 angles.**

Given AB = 4 cm., BC = 5 cm., ∠A, ∠B and ∠C = 98°, 70° and 108° respectively, construct the quadrilateral ABCD.

Solution.—Draw an angle ABC equal to 70°. Make BA = 4 cm., BC = 5 cm. At A and C make angles BAD and BCD equal to 98° and 108° respectively, meeting at D. Then ABCD is the required quadrilateral.

Model No. (5) **1 length and 4 angles.**

Given BC, ∠ DBC, ∠ DCB, ∠ ABC and ∠ ACB to be 2 in., 30°, 90°, 105° and 35°.

Solution.—Draw a line BC equal to 2 in. At B draw ∠ DBC and ∠ ABC equal to 30° and 105° respectively as in the figure. At C draw angles DCB and ACB equal to 90° and 35° respectively Let BA and CA meet at A; let BD and CD meet at D as in the figure. Then ABDC is the required quadrilateral.

It is not possible to exhaust all the cases. A few have been given, the rest may be similarly constructed.

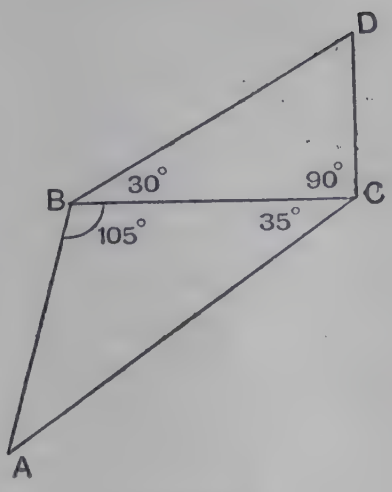


FIG. 188.

Exercise XXVIII.

1. Construct a quadrilateral ABCD in which AB = 1 in., BC = 1·5 in., CA = 2 in., CD = 1·8 in. and DA = 1·9 in. Supposing it represents a field drawn to the scale of 1 inch to 500 yards, find its area.

2. Construct a quadrilateral ABCD in which AB = 1 in., BC = 1·5 in., BD = 2·8 in., CD = 2·4 in., and AD = 1·9 in. Supposing it represents a field drawn to the scale of 1 inch to a furlong, find its area.

3. Through the angular points of the quadrilateral ABCD draw parallels to the diagonals. What fraction is the area of the quadrilateral of that of the parallelogram? Hence check your result in question (1) by finding the area of the parallelogram.

4. Draw the plan of a field ABCD in which AB = 400 yds., BC = 200 yds., AC = 400 yds., CD = 500 yds. and ∠ BAD = 120°, and find its area. Verify your result by drawing parallels to the diagonals through the angular points.

5. Draw the plan of a field ABCD in which AB = 800 yds., BC = 400 yds., AC = 750 yds., AD = 450 yds. and ∠ BCD = 108°, and find its area. Verify your results by drawing parallels to the diagonals through the angular points.

6. Construct a quadrilateral ABCD in which AB = 2·3 cm., BD = 4·5 cm., DC = 3·2 cm., ∠ ABD = 18° and ∠ BDC = 45°. Measure the other sides and angles in the quadrilateral.

7. Construct a quadrilateral ABCD in which AB = 50 km., BC = 60 km., CD = 75 km., ∠ ABC = 75° and ∠ BCD = 98°. Measure the other sides and angles in the quadrilateral and find the area in hectares.

8. Construct a quadrilateral in which AB = 1·2 in., AC = 2·3 in., BD = 2·3 in., ∠ ABC = 80° and ∠ BAD = 104°. Measure the other sides and angles.

9. Construct a quadrilateral ABCD in which AB = 1 in., BC = 2 in., ∠ B = 45°, ∠ C = 90° and ∠ D = 102°. Measure the other sides and angles in the quadrilateral.

10. Construct a quadrilateral in which BC = 4 cm., CD = 5 cm., ∠ B = 60°, ∠ C = 90° and ∠ D = 102°. Measure the other sides and angles.

11. Supposing A and B to be two trees on a sea-shore and C and D to be two buoys on the sea, find the distance between the two buoys on the sea supposing that 1 in. represents 50 yds. and making use of the following observations :—

AB = 1 in., ∠ ABD = 40°, ∠ BAD = 100°, ∠ BAC = 50° and ∠ ABC = 80°.

12. B and D are two inaccessible objects. To find the distance between them the following observations are made :—

Two places C and A are chosen and the distance between them is found to be 150 yds., ∠ CAD = 42°, ∠ CAB = 53°, ∠ ACD = 63° and ∠ ACB = 72°. Construct the quadrilateral ABCD and find the distance between the inaccessible objects. Find also the distance of each of these inaccessible objects from the places of observation.

13. Construct a quadrilateral ABCD, given BD = 4 in., ∠ A and ∠ C are right angles, ∠ ABC = 75°, ∠ ABD = 30°. Measure the sides BC, CD, DA and AB. Find its area

and verify by calculation. If one inch represents 100 yds. express the area in acres and square yards.

14. One side AB of a field ABCD runs from east to west (A east of B) and is 150 yds. long. AD runs north-west and is 40 yds. long. BC runs north-east and is 90 yds. long. Draw an accurate plan of the field (scale 20 yds. = 1 cm.). Measure the length of CD and verify by calculation. Also find the area of the field in acres and square yards.

15. Construct a quadrilateral ABCD of which two sides AB, and BC 1·1 and 2·4 in. long respectively contain an angle of 130° and the other two sides CD and DA are of lengths 2·5 and 3 in. Find the area of the figure.

16. Construct a quadrilateral ABCD given AB = 440 yds., BC = 550 yds., BD = 550 yds., $\angle A = 90°$ and $\angle C = 60°$. Measure the sides AD and CD. Give construction. Find its area in acres to the nearest integer (scale 1 cm. = 110 yds.).

17. The diagonals of a rhombus are 8 cm. and 6 cm. Construct the rhombus. What is the length of a side of the rhombus and what is its area? Why are 2 measurements alone sufficient in this case?

18. Construct a quadrilateral ABCD having AC = 2·5″, $\angle B = 90°$, AB = 1·5″. D is equidistant from B and C and $\angle CAD = 45°$. Measure the remaining sides and angles and find the area. Verify by calculation.

19. Two sides of a field ABCD are parallel and the other two are not. The parallel sides AD and BC are 175 yds. and 55 yds. respectively, $\angle BAD = 75°$ and AB = 60 yds. Measure CD and find the area of the field in acres. Why are 4 measurements enough in this case?

20. Construct a quadrilateral ABCD given AC = 10 cm., $\angle B = 90°$, sides AB and CD are respectively equal to 6 cm. and 5·4 cm. and $\angle ACD = 67·5°$. Find the area of the field.

N.B.—For more examples see the section on quadrilaterals (*vide* Miscellaneous Examples).

CHAPTER XXIX.

Symmetry in Algebra and Geometry. Properties of Circles Chords and Distances from Centre, Equal Chords, Equal Arcs, Equal Angles at the Centre.

161. Symmetry in Algebra. An expression in a and b is said to be symmetrical when it remains unaltered if a is put for b and b for a, i.e., if they are interchanged. Thus $a + b$ becomes $b + a$ when a and b are interchanged; because $b + a$ is the same as $a + b$, $a + b$ is said to be symmetrical with respect to a and b. $2a + 3b$ becomes $2b + 3a$ when a and b are interchanged but these two are different. Therefore $2a + 3b$ is not symmetrical with respect to a and b. What has spoiled the symmetry in this particular example is the inequality of the co-efficients of a and b. If a and b had equal co-efficients there would be symmetry. $2a + 2b$ and $3a + 3b$ are both symmetrical with respect to a and b.

162. Symmetry with respect to three quantities a, b and c. The expression $2a + 2b + 5c$ is symmetrical with respect to a and b for it changes into $2b + 2a + 5c$, when a and b are interchanged, which is not different from the original expression. But it is not symmetrical with respect to a and c for if these are interchanged the resulting expression is $2c + 2b + 5a$ which is quite different from $2a + 2b + 5c$. What has spoiled the complete symmetry is the difference in the co-efficients of a and c or b and c. If all the co-efficients are equal, the expression will be symmetrical with respect to any pair, i.e., either a and b, or b and c, or c and a in which case it is said to be symmetrical with respect to all the quantities a, b and c.

Exercise XXIX (A).

1. Examine if the following expressions are symmetrical or state with respect to what quantities each is not

symmetrical. Make the necessary changes to make each perfectly symmetrical :—

1. $5a + 3b + 5c$.
2. $2a^2 + 2b^2 - 3c^2$.
3. $7ab + 7ac + 3bc$.
4. $2ab + 2ac - bc$.
5. $3a^2 + 3b^2 - 5ab$.
6. $7a^2 + 7b^2 + 8ab$.
7. $a^2 + ab + ac + bc$.
8. $3a^2 - 4ab - 4ac - 4bc$.
9. $2x^2 + 2y^2 + 2z^2 - 6xy$.
10. $5a^2 + 5b^3 + 5c^3 - 7ab - 7ac - 7bc$.
11. $2a^3 + 2b^3 - 3a^2b - 4ab^2$.
12. $a + b^2 + c^3$.

2. What should be the value of k in each of the following expressions to make the expression perfectly symmetrical ? :—

1. $5a^3 + 5c^3 + kb^3$.
2. $-3b^2 - 3c^2 + ka^2$.
3. $7a^2 + 7b^2 + kab$.
4. $9a^2 + 9b^2 + 9c^2 + k(ab + bc + ca)$.
5. $pa^2 + pb^2 + kc^2$.
6. $3a^3 + 3b^3 + 9a^2b + kab^2$.
7. $4a^3 + 4c^3 - 7a^2c + kac^2$.
8. $3a^3 + 3b^3 - k(c^3 + d^3)$.
9. $2abd + 2acd + k(bcd + abc)$.
10. $4(a^3 + b^3 + c^3) - kabc$.

163. Symmetry in Geometry. It has been already stated that a circle is symmetrical about a diameter which means that if a circle is folded about a diameter, the two halves coincide. Folding a figure about a line and the coincidence of corresponding parts are the fundamental tests of symmetry in geometry. Fold a sheet of paper along a line AB and scissor off the double sheet in any way you like or in the manner shown in the figure. If the paper be unfolded and spread evenly there will be two parts lying on either side of the crease. Since these two parts resemble one another in every respect the figure is said to be symmetrical about the line AB. The line about which a figure when folded gives two exactly similar parts is called **the axis**

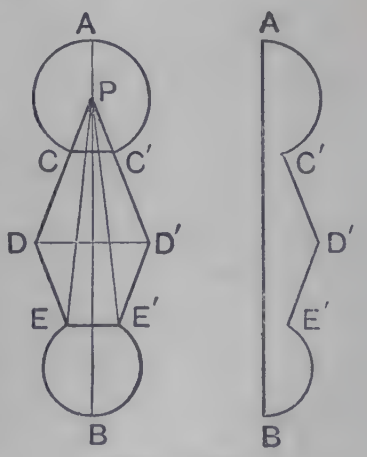


FIG. 189.

of symmetry. Points such as (C, C′), (D, D′) and (E, E′) which coincide when the figure is folded about AB are said to be corresponding points or one point is said to be the image of the other in the axis of symmetry. C′ is the image of C in AB (as the image of an object is as far behind a plane mirror as the object is in front of it). It is clear that

(1) every point on one side of the axis has a corresponding point or image on the other side of the axis; and the line joining the corresponding points is bisected at right angles by the axis;

(2) the corresponding point of any point in the axis is itself;

(3) the line joining any two points on one side of the axis is equal to the line joining their corresponding points on the other side of the axis;

(4) any point on the axis is equidistant from a pair of corresponding points, i.e., PC = PC′, PE = PE′; and

(5) the area of any figure formed by joining a number of points on one side of the axis is equal to the area of the figure formed by joining the corresponding points on the other side of the axis, i.e., is equal to its image.

Exercise XXIX (B).

1. Show by folding and considerations of symmetry that (*a*) the bisector of the vertical angle of an isosceles or equilateral triangle bisects the base at right angles. (Draw an isosceles or equilateral triangle. Scissor it off and fold it about the vertex so that one of the equal sides falls on the other), (*b*) every point on the bisector is equidistant from the extremities of the base or all points equidistant from the extremities of the base lie on the perpendicular bisector of the base and (*c*) that the base angles of an isosceles triangle are equal.

2. Is an isosceles triangle symmetrical about a perpendicular drawn from one extremity of the base to the opposite

side? Is an equilateral triangle symmetrical about the perpendicular drawn from one extremity of the base to the opposite side? How many lines of symmetry are there in an equilateral triangle? Are they concurrent?

3. Draw a square ABCD. Bisect AB, BC, CD and DA at E, F, G and H respectively. Examine if there is symmetry about the following lines:—(1) AC, (2) BD, (3) EG and (4) FH.

4. Draw a rectangle ABCD. Bisect AB, BC, CD and DA at E, F, G and H, respectively. Examine if there is symmetry about the following lines:—(1) AC (2) BD (3) EG (4) FH.

5. Draw a parallelogram ABCD. Bisect AB, BC, CD and DA at E, F, G and H, respectively. Examine if there is symmetry about the following lines:—(1) AC, (2) BD, (3) EG and (4) FH.

6. Draw a rhombus ABCD. Bisect AB, BC, CD and DA at E, F, G and H respectively. Examine if there is symmetry about the following lines:—(1) AC, (2) BD, (3) EG and (4) FH.

7. Two bungalows A and B are 20 yards and 25 yards distant from a main road on one side of it. Mark the position of two other bungalows C and D so that the bungalows may be symmetrically situated about the main road.

8. If the distance between A and B in the preceding question is 50 yards draw a figure to show the positions of the bungalows A, B, C and D.

9. AB is a mirror, P, Q, R denote the positions of three chairs in front of the mirror such that PQ = 4 ft., QR = 3 ft. and PR = 5 ft. and the distances of P and R from the mirror are 10 ft. and 12 ft. respectively. Indicate the positions of the chairs and of the images of the chairs also in the mirror.

10. There are two wells in a field 20 yards apart. There are three lamp posts equidistant from the wells. The distances of the three lamps from either well are 100 yards,

120 yards and 150 yards. Find the distances between the lamp posts.

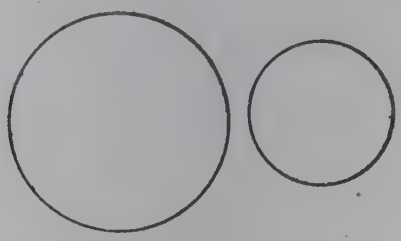


FIG. 190.

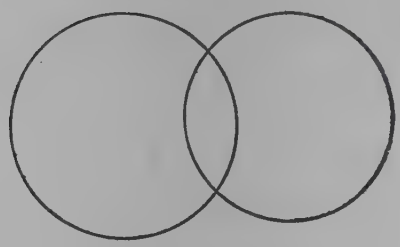


FIG. 191.

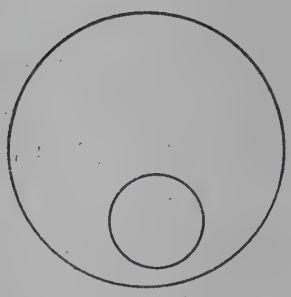


FIG. 192.

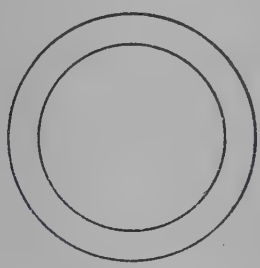


FIG. 193.

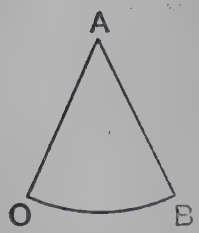


FIG. 194.

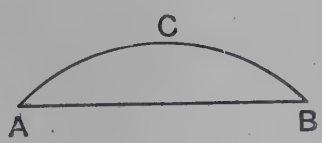


FIG. 195.

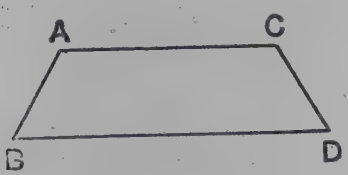


FIG. 196.

11. Find the lines of symmetry in the following :—

(*a*) In the case of two circles one lying without the other. (Fig. 190).

(*b*) In the case of two intersecting circles. (Fig. 191).

(*c*) In the case of two circles one lying within the other. (Figs. 192 and 193).

(*d*) In the case of a sector AOB bounded by two radii and an arc. (What construction can you give for bisecting an arc ?) (Fig. 194).

(*e*) In the case of a segment (a figure bounded by a chord and an arc). (Fig. 195).

(*f*) In the case of an isosceles trapezium in which AC is parallel to BD and the non-parallel sides AB and CD are equal. (Fig. 196).

12. Show from Question 11 (*b*) that the common chord is bisected at right angles by the line of centres.

13. From Question 12 deduce a construction for

(*a*) bisecting a straight line and (*b*) for drawing a straight line perpendicular to a given straight line.

14. Show that the bisector of an angle of a triangle is an axis of symmetry with respect to the arms of the angle and that any point on the bisector is equidistant from the arms of the angle.

15. Two roads intersect at an angle of 60° and enclose rice fields. A scare crow in the middle of the fields is at a distance of 100 yards from either road. Mark the position of the scare crow and find its distance from the intersection of the roads.

16. Find a point equidistant from the sides AB, BC and CA of a triangle ABC 3·5, 4·2 and 5·3 cm. long respectively. Find its distance from each of the sides.

17. Find a point equidistant from the side AB and the sides BC and CA both produced in Question 16.

18. Find a point equidistant from the side BC and the sides AB and CA produced in Question 16.

19. Find a point equidistant from the side CA and the sides AB and BC produced in Question 16.

20. Construct a triangle ABC so that the sides AB, BC and CA are equal to 4, 5 and 6 cm., respectively. Find a point equidistant from A, B and C.

[Draw the perpendicular bisectors of AB and BC. If they intersect at S, S is equidistant from A, B and C.]

21. In Question 20 with S as centre, and SA as radius describe a circle, then it will pass through B and C. This circle is called the **circumscribed circle of the triangle.** Its circle is called the **circum-centre.**

22. Draw an acute-angled triangle, a right-angled triangle and an obtused-angled triangle. Find the circum-centre in each. What do you note about the position of the circumcentre with respect to the triangle ?

23. The sides of a triangular field are 500 yds., 350 yds. and 400 yds. There is a haystack in the field at a place

equidistant from the corners. Mark its position and find its distance from each corner of the field.

24. Three roads intersect and form a triangle enclosing a meadow within. The portions of the roads intercepted by one another are 1 m. $1\frac{1}{2}$ m. and 2 miles. In the meadow there is a small temple equidistant from the roads. Find the distance of the temple from any of the roads.

25. How many circles can be described so as to pass through two given points A and B? Give reasons for your answer.

26. Describe a circle passing through two points A and B 5 cm. apart and having a radius of 4·5 cm. How many circles will there be? What is the reason?

27. Describe two circles passing through two points A and B 2·4 in. apart and having their radii 1·8 and 2·6 in. respectively. Calculate the distance between their centres.

28. Describe a circle passing through three points which are in a straight line. Why is this impossible?

29. Describe a circle passing through four given points A, B, C and D. (Consider the perpendicular bisector of AB, BC, also of CD.) When will the construction fail?

30. State the possibility or impossibility of the following constructions :—

(*a*) Describe a circle passing through the corners of a square.

(*b*) Describe a circle passing through the corners of an oblong.

(*c*) Describe a circle passing through the corners of a rhombus.

(*d*) Describe a circle passing through the corners of a rhomboid (a parallelogram whose sides are unequal and whose angles are not right angles).

164. To find the centre of a circle.

1. Let ABC be a circle. Take any two points A and B on the circumference. Join AB. Fold the circle along AB so that B falls on A. The crease or the axis of symmetry is a diameter. Similarly take two other points C and D. Join CD. Fold the circle along CD so that C falls on D. The crease is another diameter. The point common to these two creases is the centre of the circle. Without folding, the centre is the point of intersection of the perpendicular bisectors of any two chords (not parallel).

FIG. 197.

2. Let AB be a diameter, CC′, DD′ and EE′ be drawn at right angles to AB cutting it at M, N and K. Fold the circle about AB. Then it will be seen that C′ coincides with C, D′ with D and E′ with E. ∴ MC′ = MC, ND′ = ND and KE′ = KE. ∴ M, N and K are the middle points of CC′, DD′ and EE′ respectively.

This may be verified by measurement.

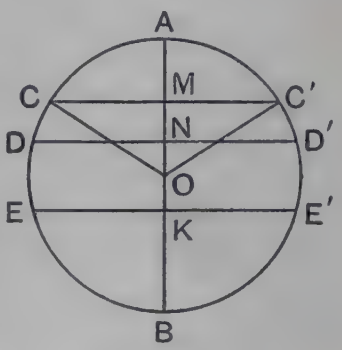


FIG. 198.

Hence learn that **if a diameter cuts a chord or chords at right angles it must bisect it or them.**

3. Let a chord CC′ cut AB a diameter at M so that M is the middle point of CC′. Now the two triangles CMO and MC′O are congruent. For CM = C′M, MO is common and CO = OC′ (three sides of the one are respectively equal to three sides of the other) or the triangle CMO may be shown to be equal to the triangle C′MO by superposition.

∴ ∠CMO = ∠C′MO. These being adjacent angles, each must be a right angle.

This may also be verified by measurement.

Hence learn that **if a diameter bisects a chord or chords it must cut it or them at right angles.**

4. It is also clear that the middle points of a system of parallel chords of a circle lie on a diameter at right angles to the chords. Hence learn that **the locus of the middle points of a system of parallel chords is a diameter at right angles to them.**

5. It is also clear that **parallel chords cut off equal arcs** as the arc CD = the arc C′D′ and the arc DE = the arc D′E′ ; for, C, D and E being the corresponding points of C′, D′ and E′, coincide with them when folded about the axial diameter AB and also any points on the arcs CD, DE coincide with the corresponding points, on the arcs C′ D′ and D′ E′.

6. It is also clear that the chord CD = the chord C′D′, that the chord DE = the chord D′E′ ; similarly AC = AC′, AD, = AD′ and so on. Hence learn that

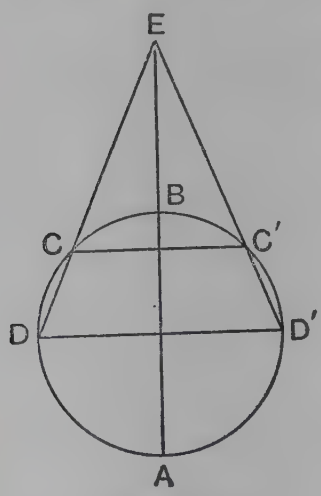


FIG. 199.

(*a*) **a chord joining two points on one side of a diameter = the chord joining the two corresponding points on the other side of the diameter ;**

(*b*) **if two chords of a circle are equal the arcs they cut off are also equal, the minor arc equal to the minor, the major arc equal to the major ;**

(*c*) **and conversely if two arcs are equal in a circle, the chords they cut off are also equal** (for when folded about the axis of symmetry the arcs and chords coincide).

7. Suppose C′, D′ are the images of C, D respectively in AB. Produce DC to meet AB produced in E. (Fig. 199). Then D′C′ also when produced will meet AB produced at E. This may be verified by actual drawing ; or by folding, the line DCE completely coincides with D′C′E. Hence learn that

(*a*) **if two straight lines are equal one being the image of the other in the axis of symmetry, then they meet on the axis of symmetry ;**

(*b*) **and they are equally inclined to the axis of symmetry for ∠ CEB = ∠ C′EB.**

8. Let AB and CD be equal chords of a circle whose centre is O and let their middle points be E and F respectively. Then OE and OF are perpendicular to AB and CD by example 2. Produce BA and DC to meet at X. Join XO. Then fold the figure about XO.

Then F coincides with E. ∴ OE = EF (or it may be verified by measurement).

Hence learn that **equal chords of a circle are equally distant from the centre.**

9. Let AB and CD be two chords whose perpendicular distances OE and OF from the centre are equal. E and F are also the middle points of AB and CD (by example 3). Produce BA and DC to meet at X. Join OX. Fold the figure about OX. Then it will be seen that F coincides with E and C with A. ∴ FC = EA. ∴ 2 FC or DC = 2 EA or BA (or it may be verified by measurement).

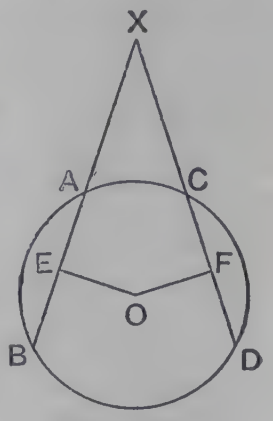


FIG. 200.

Hence learn that **if two chords are equally distant from the centre they are equal.**

10. In the figure (201) AOB and COD are two equal angles at the centre O of the circle. Bisect the angle BOD by OK since ∠ AOB = ∠ DOC then OK is also the bisector of the ∠ AOC. Fold the figure about OK. Then C coincides with A and D with B. ∴ CD coincides with AB. ∴ the arc AB coincides with the arc DC. (This may be verified by measuring the arcs AB and CD with a string) or by congruence of triangles.

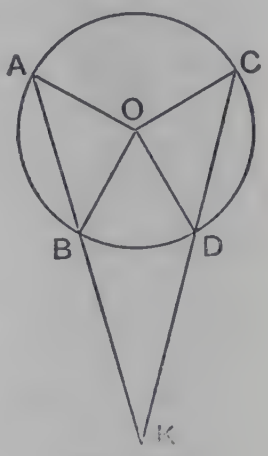


FIG. 201.

Hence learn that **in a circle or in equal circles equal angles at the centre or centres subtend equal arcs.**

11. Conversely let the arcs AB and DC be equal, then ∠ AOB must be equal to the ∠ DOC. Join AB and CD. Produce them to meet at K. Fold the figure about K. Then OC and OA coincide; and OD and OB coincide. ∴ ∠ AOB coincides with ∠ DOC or (by measurement of the angles AOB and COD or by congruence of triangles it may be shown that ∠ AOB = ∠ COD).

Hence learn that **in a circle or in equal circles equal arcs are subtended by equal angles at the centre or centres.**

Exercise XXIX (C).

1. The arc in the accompanying figure represents an arch in a building over which a superstructure is raised. How

FIG. 202.

would you draw the whole circumference of which it is the given arc?

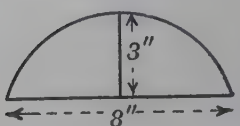


FIG. 203.

2. The base of the segment of a circle is 8 in. and the height of the segment is 3 in. Construct the segment.

3. The chord of an arc is 2 ft. and the chord of half the arc is 1 ft. 1 in. Draw the arc.

4. To measure the extent of a circular lake, a surveyor throws a long pole 24 ft. in length into the water and asks his luscars to hold the two extremities of the pole firmly so that each end of it rests on the circumference. Then he measures the arc with a tape and finds the middle point of the arc by taking half the measured length. He then finds the distance between one end of the pole and the middle point of the arc to be 13 ft. Show how he was able to find the diameter of the circle. What was its length?

5. Describe a circle of radius 1·8 in. Place in it a chord 2·4 in. long. Find its distance from the centre of the circle by measurement and also by calculation.

6. In a kitchen there is a big arch in a wall and a plank is fixed in the wall underneath the arch, on which things are placed. The length of the plank is 8 ft. and the highest point of the arch is 2·5 ft. above the plank. At what distance from the centre has the plank been fixed?

FIG. 204.

7. A room is circular in shape and its diameter is 20 ft. Two nails are driven into the wall at the same height above the ground and a straight rope is tied to the nails for hanging cloths from it. Find the length of the rope if its distance from the centre of the circular section of the room at the same height above the ground as the nails is 6 ft.

8. (*a*) The lid of a cask is circular in shape its diameter being 3 ft. The portion of the lid AKB turns about AB being hinged to the other portion at C and D which are equally distant from A and B. If AB = 2 ft., and AC = 4 in., find the distance of C from the centre of the lid. Verify your result by calculation.

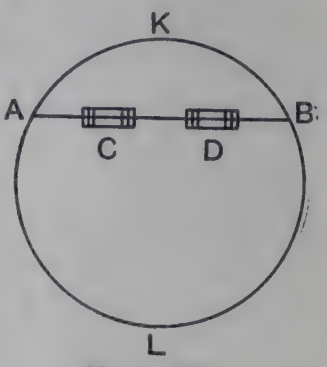


FIG. 205.

(*b*) The top of a circular well is covered by a lid of wooden frame fitted with iron bars. The diameter of the well is 4 ft. The moveable portion of the lid is hinged to the immoveable portion and the common boundary between the two portions is 2 ft. 6 in. How far is the line of hinges from the centre?

9. If, in Question 6, in the kitchen two planks 8 ft. and 10 ft. are fixed in the arch at a vertical distance of 2 ft. from one another, what is the radius of the arch? Verify your result by calculation.

10. If, in Question 7 another rope 9 ft. long is fastened to two pegs driven into the wall, how much farther will it be from the centre of the room than the other?

11. The broad gauge of the M. & S. M. Railway is 5 ft. 6 in. The distance between the centres of the first and last wheels of a locomotive engine is 20 ft. What must be the minimum radius of a circular disc used for reversing the engine when the coal tender is to the front of a train.

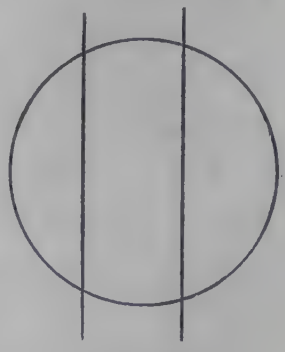


FIG. 206.

12. The metre gauge of the M. & S. M. Railway is 3 ft. $3\frac{3}{8}$ in. The distance between the centres of the first and last wheels of a steam engine used on this railway is 12 ft. What must be the minimum radius of a circular disc used for reversing the engine when the coal tender is to the front of the train.

13. The figure represents the terraced roof of a portico which is greater than a semi-circle. AB is a cross beam. CD, FE, etc., are joists running at right angles to the beam, CD passing through the centre. If CD = 12 ft., the radius of the circle = 8 ft. and CF = 2 ft., find the length of EF and AB by measurement. Verify by calculation.

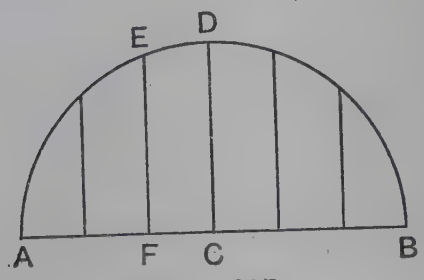


FIG. 207.

14. ABC is a circle of radius 2 inches. At the centre O draw three angles AOB, BOC and COA each equal to 120°. Join AB, BC and CA. What triangle is ABC? Give reasons for your answer.

15. ABC is a circle of radius 6 cms. Inscribe an equilateral triangle in it.

16. Draw two diameters AB and CD at right angles in a circle ACBD whose centre is O. Join AC, CB, BD and DA. What figure is ACBD? Give reasons for your answer. Show by considerations of symmetry also that it is a regular figure.

17. How many equilateral triangles can be inscribed in a circle? Give reasons for your answer.

18. How many squares can be inscribed in a circle? Give reasons for your answer.

19. Inscribe an equilateral triangle ABC in a circle. Bisect the arcs subtended by the sides, i.e., the arcs AB, BC and CA at D, E and F. (Draw the perpendicular bisectors of the chords). Join AD, DB, BE, EC, CF and FA. What figure is formed? Are the sides equal? If so, why? Are the angles equal? If so why?

20. Draw six angles at the centre O of a circle ABCDEF such that $\angle AOB = \angle BOC = \angle COD = \angle DOE = \angle EOF = 60°$. Join AB, BC, CD, DE, EF and FA. What figure is ABCDEF? Give reasons for your answer.

21. Inscribe a square ABCH in a given circle. Bisect the arcs subtended by the sides, i.e., the arcs AB, BC, CH and HA at D, E, F and G. (Draw the perpendicular bisectors.) Join AD, DB, BE, EC, CF, FH, HG and GA. What figure is formed? Are the sides equal? If so why? Are the angles equal? If so why?

22. Draw eight angles at the centre O of a circle ABCDEFGH such that $\angle AOB = \angle BOC = \angle COD = \angle DOE = \angle EOF = \angle FOG = \angle GOH = 45°$. Join AB, BC, CD, DE, EF, FG, GH and HA. What figure is ABCDEFGH? Give reasons for your answer.

23. Draw four equal angles at the centre of a circle each equal to 72°. Then the fifth must also be 72°. Why? Join the points where the arms of the angles intersect the circumference, in succession. What figure is got? Give reasons for your answer.

24. In the above figure bisect the arcs. Join the angular points and the middle points of the arcs in succession. What figure is formed? Are the sides and angles equal?

25. Instead of adopting the method in Question 24, draw 9 equal angles at the centre each equal to 36°. Then the tenth must also be 36°. Why? Join the points where the arms of the angles intersect the circumference, in succession. Do you get the same figure as in Question 24?

26. Inscribe a dodecagon in a circle by taking an inscribed hexagon and bisecting the arcs.

165. Inscribing a regular polygon in a given circle. From the foregoing examples it must be clear to the student how a regular polygon of n sides can be inscribed in a circle. The aim is to get equal chords in a circle which means getting equal arcs which is finally a question of getting equal angles at the centre. Hence the general method is (1) to get n equal angles at the centre each equal to

$\frac{360°}{n}$ and join in succession the points where the arms of the angles intersect the circumference or (2) to construct an angle AOB at the centre O equal to $\frac{360°}{n}$, to join AB and to step off the chord BC equal to the chord AB. This process may be repeated and it will be found that the extremity of the n^{th} chord falls on A the point from which we started. The figure thus formed is a regular polygon. The figure is thus divided into n equal triangles.

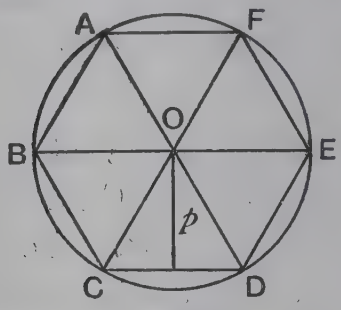


FIG. 208.

The area of each triangle $= \frac{1}{2}\ pa$ where a is a side of the triangle and p the perpendicular from the centre on a side. (How do you know that the perpendiculars or the altitudes of all the triangles are equal?)

$\therefore$ the area of n triangles $= \frac{1}{2}\ pa \times n$

$= \frac{1}{2}\ pna$

$= \frac{1}{2}\ p \times$ the perimeter of the polygon.

In a hexagon $p = a\frac{\sqrt{3}}{2}$, (Pythagoras).

In an octagon $p = a\frac{(\sqrt{2}+1)}{2}$.

Let OX be at right angles to CD in the triangle COD. Make the $\angle$ OCY equal to the $\angle$ COX. Then CY = YO and CX = XY for $\angle$ COX $= 22\frac{1}{2}°$, $\angle$ OCX $= 67\frac{1}{2}°$, whence $\angle$ CYX $= 45°$. $p =$ OY + XY $=$ CY + XY $=$ CY + CX $= \frac{a}{\sqrt{2}} + \frac{a}{2}$.

(3) The following construction for inscribing a regular polygon in a given circle is very interesting and applicable when $\frac{360°}{n}$ gives us a fractional number of degrees as when $n = 7, 11$, etc.

Let AB be a diameter of a circle. Suppose we are required to inscribe a regular polygon of n sides in it. With A and B

as centres and a radius equal to AB describe two arcs cutting at C. Divide AB into n equal parts. (In the figure, AB is divided into 7 equal parts) and number the parts 1, 2, 3, etc. Join C to the point of division in AB numbered 2 (always to the second point of division whatever may be the number of sides of the polygon) and produce C2 to meet the circle at D. Join AD. The AD is a side of the regular polygon. Step off chords each equal to AD and complete the polygon.

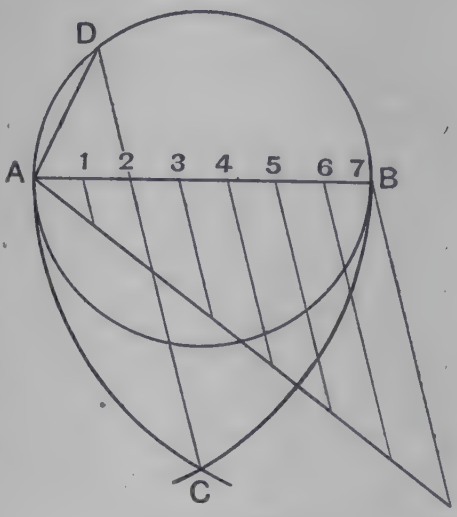


FIG. 209.

166. Construction of a regular polygon on a given base. Suppose a regular polygon of n sides has to be constructed on a given base AB. It is possible to construct the polygon (a) without describing a circle. At A in AB make an angle BAC equal to $180° - \frac{360°}{n}$ (for this is the magnitude of an interior angle of a regular polygon of n sides).

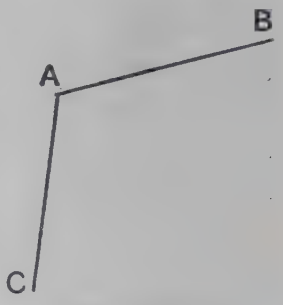


FIG. 210.

Make AC equal to AB. Similarly at C in CA make an angle ACD equal to $180° - \frac{360°}{n}$ and so on until you complete the figure.

(b) We can construct the polygon by describing the circle that will pass through its angular points and then completing the polygon. At A make an angle equal to $180° - \frac{360°}{n}$. Make AC equal to AB. Draw the perpendicular bisectors of AB and AC meeting at O. Then with O as centre and OA as radius describe a circle. Place chords in it successively each equal to AB. A regular polygon of n sides is thus obtained.

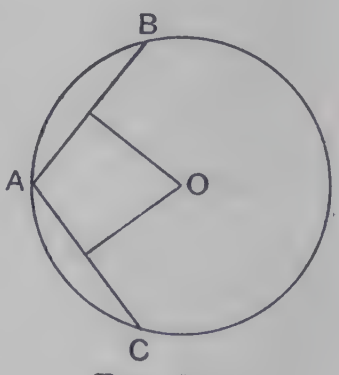


FIG. 211.

Exercise XXIX (D).

1. Describe a circle of radius 2·3 in. Inscribe an equilateral triangle in it. Measure a side of the triangle and find its area.

2. Describe a circle of radius 6 cm. Inscribe an equilateral triangle in it. Measure a side of the triangle and find its area.

3. Describe a circle of radius r units. Inscribe an equilateral triangle in it. Express a side of the triangle in terms of r and find the area of the triangle. Apply these results to check your answers in 1 and 2.

4. Describe a circle of radius 2·1 in. Inscribe a square in it. Measure a side of the square and find its area.

5. Describe a circle of radius 5 cm. Inscribe a square in it. Measure a side of the square and find its area.

6. Describe a circle of radius r units. Inscribe a square in it. Express a side of the square in terms of r and find its area. Apply these results to check your answers in 4 and 5.

7. Describe a circle of radius 2·5 inches. Inscribe in it a regular hexagon. Measure a side of the hexagon and find its area.

8. Describe a circle of radius 7 cm. Inscribe in it a regular hexagon. Measure a side of the hexagon and find its area.

9. Describe a circle of radius r units. Inscribe in it a regular hexagon. Express a side of the hexagon in terms of r and find its area.

Hence learn that **a side of a regular hexagon inscribed in a circle is equal to a radius of the circle.**

10. From the above property, what special construction will you suggest for inscribing a hexagon in a given circle?

11. Describe a circle of radius 1 in. Inscribe a regular octagon in it. Measure a side and find its area.

12. Describe a circle of radius 2 in. Inscribe a regular octagon in it. Measure a side and find its area.

13. From the results of 11 and 12, can you guess what the side of a regular octagon inscribed in a circle is in terms of its radius.

14. Construct a regular hexagon ABCDEF on a base AB 1·5 in. long. Examine if the figure is symmetrical about (1) AD, (2) BE and (3) CF. Examine also if the figure is symmetrical about the line joining the middle points of

(1) AB and CD, (2) BC and FE and (3) CD and AF. How many lines of symmetry are there? Where do all these meet?

15. Construct a regular octagon ABCDEFGH on a base 2 in. long. Examine if the figure is symmetrical about AE, BF, CG and DH; also about the lines joining the middle of (1) AB and EF, (2) BC and FG, (3) CD and GH and (4) DE and AH. How many lines of symmetry are there? Where do all these meet?

16. Construct a square ABCD. Draw the diagonals interesting at O. Measure DF along DA equal to OD and AE along AD equal to OA. In AB take points G and H, in BC, points K and L and in CD, points M and N such that AG = HB = BK = CL = CM = DN = DE or AF. Show by measurement that the figure FGHKLMNE is a regular octagon.

17. ABC is an equilateral triangle. Trisect AB at D and E, BC at F and G and AC at H and K. Show that the figure DEFGHK is a regular polygon.

18. A bungalow consists of two hexagonal rooms at each of its extremities in its front. If each side of the room is 8 ft., find the cost of matting the rooms at Re. 1-4-0 per sq. yd.

19. Divide a regular hexagon of side a into two parts, the shape of each part being a trapezium, by an axis of symmetry. Given that the area of a trapezium $= \frac{1}{2}$ the sum of the parallel sides $\times$ the altitude, find the area of the hexagon.

20. Find the area of a regular octagon of side a by dividing it into a rectangle and two trapezia.

CHAPTER XXX.

Some other Properties of Circles not based on Symmetry and Congruence. Tangents, etc.

167. Let ACB be a circle whose centre is O. Take any three points A, C and B on the circumference. Join AC, CB and BA. Then each of the triangles AOB, BOC and COA is an isosceles triangle. The base angles of each of the triangles are equal.

Let each of the base angles OCB and OBC of the triangle OBC be denoted by x.

Let each of the base angles OAC, and OCA of the triangle OAC be denoted by y.

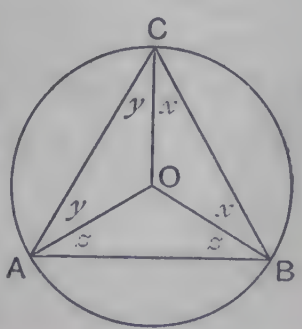


Fig. 212.

And let the base angles OAB and OBA of the triangle OAB be denoted by z.

Then $\angle A + \angle B + \angle C = 2$ right angles.

i.e., $y + z + z + x + x + y = 2$ right angles.

i.e., $2x + 2y + 2z = 2$ right angles.

$\therefore \angle AOB = 2$ right angles $- \angle OAB - \angle OBA = 2$ right angles $- 2z$.

$= 2x + 2y + 2z - 2z = 2x + 2y$.

But $\angle ACB = x + y$.

$\therefore \angle AOB = 2 \angle ACB$.

Or the angles AOB and ACB can be measured and it will be found that the angle AOB is double the angle ACB.

Hence learn that **the angle at the centre of a circle is double the angle at the circumference.**

168. AOB and COD are angles at the centre standing on equal arcs AB and CD of the same circle or equal circles. APB and CQD are angles at the circumference standing on these arcs.

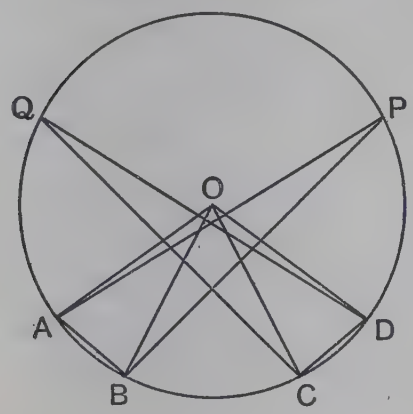


FIG. 213.

But $\angle APB = \frac{1}{2} \angle AOB$.

And $\angle CQD = \frac{1}{2} \angle COD$.

Now $\angle AOB = \angle COD$ because they stand on equal arcs.

$\therefore \angle APB = \angle CQD$ being halves of equals.

Or the angles APB and CQD may be shown to be equal by measurement, in the same circle or equal circles.

Hence learn that **angles at the circumference of a circle or at the circumferences of equal circles standing on equal arcs are equal.**

Also if $\angle CQD = \angle APB$, then the $\angle AOB$ must be equal to the $\angle COD$; for these are doubles of equal angles, viz., APB and CQD respectively.

$\therefore \angle AOB = \angle COD \therefore$ arc AB = arc CD.

Hence learn **if angles at the circumference of a circle or at the circumferences of equal circles are equal, the arcs on which they stand must also be equal.**

Exercise XXX (A).

1. ABC is an equilateral triangle inscribed in a circle whose centre is O. What is the magnitude of each of the

angles AOB, BOC and COA? Hence what must be the magnitude of each of the angles ABC, ACB and BCA?

2. ABC is an isosceles triangle having each of the angles B and C double the angle A. It is inscribed in a circle whose centre is O. Calculate the magnitude of the angles AOB, BOC and COA and verify by measurement.

3. ABC is an equilateral triangle inscribed in a circle whose centre is O. P, Q and R are any points on the arcs AB, BC and CA. What fraction is the $\angle$ APC of the $\angle$ AOC? What fraction is the $\angle$ BPC of the $\angle$ BOC? Hence what is the magnitude of the $\angle$ BPA? Similarly what is the magnitude of the $\angle$ CRA and of the $\angle$ BQC?

4. ABCD is a square inscribed in a circle whose centre is O. P is any point on the arc AB. What fraction is the $\angle$ CPD of the $\angle$ COD? Hence what is the magnitude of the $\angle$ CPD? Also find the magnitude of the angles APD and BPC. Hence find the magnitude of the whole $\angle$ APB.

5. Draw a segment of a circle greater than a semi-circle. Let its centre be O. Take any points P, Q, R and S on the arc of the segment. Measure the angles APB, AQB, ARB and ASB. Are they equal? What fraction of the $\angle$ AOB is each of these angles. Hence what is your inference?

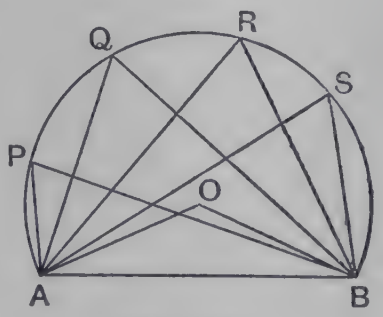


FIG. 214.

6. In the adjoining figure, APQRSB is a segment less than a semi-circle. Consequently the centre O is outside the segment. Measure the $\angle$ APB and the reflex $\angle$ AOB. What fraction is the $\angle$ APB of the reflex $\angle$ AOB? Similarly what fraction is each of the angles AQB, ARB and ASB of the reflex angle AOB? Hence what can you say

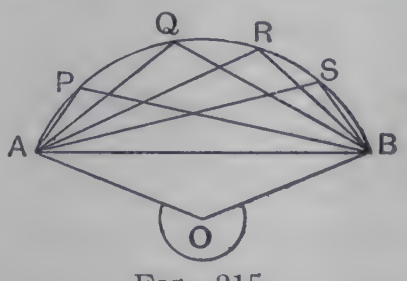


FIG. 215.

with regard to all the angles APB, AQB, ARB and ASB?

Hence learn from Questions 5 and 6 that **angles in the same segment of a circle are equal to one another.**

7. APQRSB is a semi-circle described on AB as diameter. What do you know about the angles APB, AQB, ARB and ASB? What is the magnitude of the angle AOB marked in the figure? (It is called a straight angle. It is 180°). Hence what is the magnitude of each of the angles APB, AQB, ARB and ASB? Measure also the same angles.

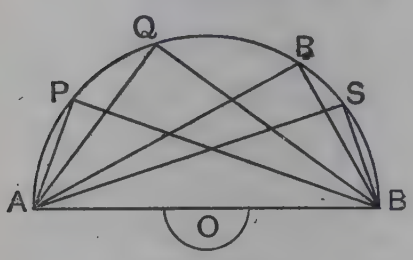


FIG. 216.

Hence learn that **the angle in a semi-circle is a right angle.**

8. Describe a circle ABC. Draw any chord AB. At B in AB make an angle ABC equal to 40°. Join AC. Then the segment ABC contains an angle of 40°. Take any points P, Q and R on the arc ABC. What is the magnitude of each of the angles APC, AQC and ARC (without measuring). Why? Is every angle in that segment 40°? It may be called the 40° segment.

9. Suppose then you have a number of triangles ABP, ABQ, ABR and ABS standing on AB and on the same side of it and having the vertical angle in each 40°. What is the locus of the points P, Q, R and S?

10. AB is a pole 30 ft. long fixed on the ground. P, Q, R and S are pegs to each of which strings passing through the extremities of the pole are tied so that each of the angles subtended by the pole at the pegs P, Q, R and S is 60°. Find the locus of the pegs P, Q, R and S.

11. Draw the locus in the above question thus:—Since the angle at P is 60°, the sum of the angles PAB and PBA must be 120°. Hence different pairs of angles PAB, PBA may be got so that their sum is 120°, e.g., ∠ PAB, and

∠ PBA may be 80° and 40° or 70° and 50° or 60° and 60° or 90° and 30° and so on. Hence different triangles can be constructed. Then describe a circle passing through the angular points of one of the triangles. The same circle will also pass through the vertices of the other triangles.

12. Construct a segment of a circle on a base 5 cm. containing an angle of (1) 80°, (2) 100°, (3) 120° and (4) 90°.

13. For the fourth part in Question 12, give a construction using the property that the angle in a semi-circle is a right angle.

14. Draw any triangle ABC right-angled at C. Bisect AB at O. With O as centre and OB or OA as radius describe a circle. It must pass through C. Why? (Complete the rectangle ACBD. What do you know about the diagonals AB and CD, besides intersection at O? They are equal. Hence $CO = \frac{1}{2} CD = \frac{1}{2} AB$. i.e. $CO = AO = OB$. ∴ the circle passes through C).

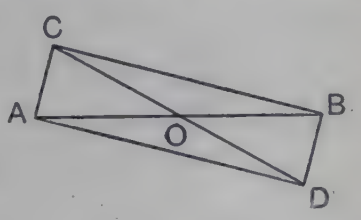


FIG. 217.

15. In the previous question, what will happen if the semi-circle described upon AB as diameter does not pass through C. It will cut AC or AC produced at D. Then what do you know about the angles ADB and ACB? What is the absurdity?

16. Describe a circle ABCD. In it inscribe any quadrilateral ABCD. Measure the two opposite angles BAD and BCD; similarly measure the two opposite angles ABC and ADC.

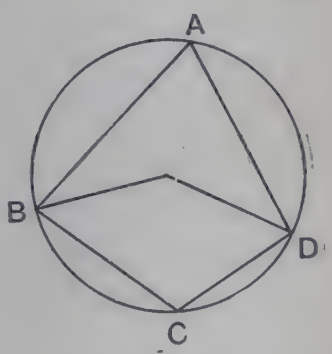


FIG. 218.

17. Repeat the experiment in Question 16 with any two quadrilaterals inscribed in a circle.

18. If in Question 16, O is the centre of the circle, what fraction is angle BAD of the angle BOD? Why? What fraction is the angle BCD of the reflex angle BOD? Why? What fraction is ∠ BAD + ∠ BCD of the sum of th

angle BOD and the reflex angle BOD? Hence what is the magnitude of $\angle BAD + \angle BCD$? To how many right-angles are all the angles at O equal? Hence learn that **the opposite angles of a cyclic quadrilateral (i.e. a quadrilateral inscribed in a circle) are together equal to 2 right angles.**

19. Construct a cyclic quadrilateral ABCD in each of the following cases :—

(*a*) AB = 2 in., BC = 1·5 in., $\angle BAD = 40°$, $\angle ABC = 85°$. (Explain why the above four elements are enough in the case of cyclic quadrilaterals.)

(*b*) AC = 2·3 in., $\angle BAC = 40°$, $\angle CAD = 45°$ and $\angle BCA = 50°$. (Explain why four elements are enough.)

(*c*) BA = 4 cm., AD = 5 cm., $\angle BAD = 90°$ and BC = 3·5 cm.

(*d*) BA = 4 cm., AD = 5 cm., $\angle BAD = 80°$ and BC = 3·5 cm.

(*e*) AB = 2·1 in., AD = 2·3 in., BD = 3·2 in. and $\angle ABC = 96°$.

(*f*) AB = 4·6 cm., AD = 3·4 cm., BD = 3·2 cm. and CD = 2·5 cm.

20. ABCDE is an equilateral pentagon inscribed in a circle. Show that its angles must be equal from a consideration of the arcs on which they stand.

21. Similarly show that an equilateral hexagon inscribed in a circle is also equiangular from a consideration of the arcs on which they stand.

22. Similarly show that an equilateral octagon inscribed in a circle is also equiangular from a consideration of the arcs on which they stand.

23. P and Q are the bottoms of two tall trees 5 yds. apart. An observer lying on the ground at A finds that PQ subtends an angle of 40° at his eye. R and S are the bottoms of two other trees so situated that RS subtends an angle 40° at the eye of another observer at B lying on the ground. If all the trees and the eyes of observers are on the circumference of a circular lake, find the distance between R and S.

24. A and B are the extremities (lying on the ground) of a doorway 6 ft. wide in the wall of a circular room. C is a point on the circular edge of the ground equidistant from A and B. What angle is subtended by AB at C if the diameter of the room is 12 ft.?

25. If in the previous question the $\angle ACB = 72°$, find the diameter of the circular room.

169. You have already seen that the middle points of a system of parallel chords of a circle lie on a diameter at right angles to the chords. Imagine that the line CD moves parallel to itself and occupies several positions. As the line approaches the centre, the length of the chord increases; as it recedes from the centre its length decreases until at last it becomes quite inappreciable at B. The two points of intersection tend to coincide and the chord meets the circle only in one point and in no other. In such a case BX is said to **touch the circle or be a tangent**. Similarly AY is a tangent at A.

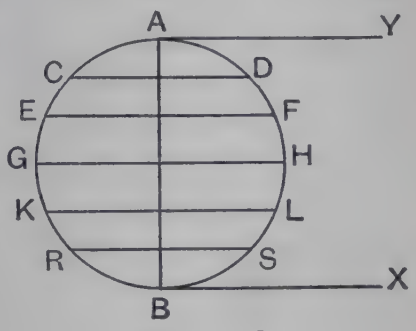


FIG. 219.

It is clear that (1) the tangent at any point is always at right angles to the diameter or radius through the point, (2) the foot of the perpendicular from the centre of the circle on the tangent is the point of contact, (3) the distance of the tangent from the centre of the circle = the radius and (4) the tangents at the extremities of a diameter are parallel.

Property (1) suggests the following construction for drawing a tangent to a circle at a point on its circumference. Draw a circle whose centre is O. Let A be a point on the circumference. Join OA. From A draw AX at right angles to OA. AX is a tangent.

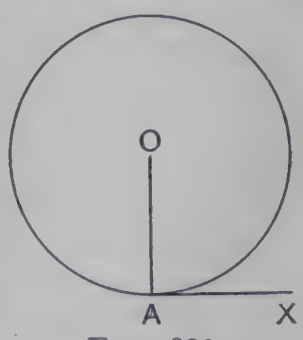


FIG. 220.

170. Tangents from an external point to a circle.

Let BAC be any angle and AD the bisector of the vertical angle. (In the figure AD is not drawn. AD may be drawn by pupils). Take any point D on AD and from it draw DE and DF perpendicular to AB and AC respectively. Now DE = DF for AD is an axis of symmetry with respect to the arms of the angle (or the two triangles AED and AFD are congruent because two angles of the one are equal to two angles of the other and one side is common). With D as centre and DE as radius describe a circle. It must also pass through the point F for DE = DF. Since AE and AF are at right angles to the radii DE and DF, AE and AF are tangents and are equal. This equality may be verified by measurement.

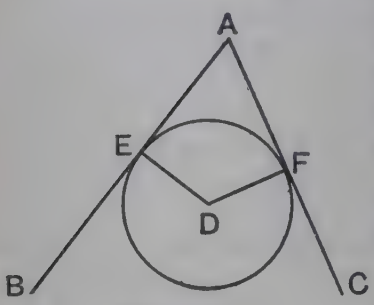


FIG. 221.

Hence learn that **(1) the tangents drawn to a circle from an external point are equal, (2) that the tangents are equally inclined to the straight line joining the external point and the centre, (3) the radii to the points of contact are also equally inclined to the straight line passing through the centre and the external point and (4) the centre of a circle touching two cutting straight lines is on the bisector of the angle between them.**

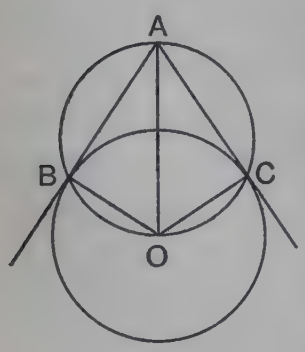


FIG. 222.

This property of the tangents being at right angles to the radii at the points of contact gives us a construction for drawing tangents to a given circle from an external point. From A it is required to draw a tangent to a circle BC of which O is the centre. On AO as diameter describe a semi-circle cutting the circle at B and C. Join AB, BO, AC and CO. The angle ABO in a semi-circle is a right angle. So AB is at right angles to BO a radius.

∴ AB is a tangent. Similarly AC is a tangent.

Exercise XXX (B).

1. Describe a circle of diameter 3 in. Place in it a chord PQ 2 in. in length. At P and Q draw tangents PR

and QR to meet at R. If O is the centre of the circle, measure the distance OR.

2. PQR is an angle of 80°. Describe a circle so as to touch the arms PQ, QR. How many solutions will there be? Suppose the radius of the circle is 4·5 cm., then how many solutions will you have? Find the distance of the centre of the circle from Q.

3. In the previous question, you are given the option of producing PQ and RQ beyond the vertex Q. Then if the radius is 2·5 inches, how many circles can be described? Find the distance of Q from the centre of each of the circles.

4. Construct a triangle whose sides are 4 cm., 5 cm. and 6 cm. Inscribe a circle in the triangle, i.e., describe a circle touching the three sides. [Consider where its centre should be (*a*) if it touches 1 pair of sides (say 4 cm. and 5 cm.), (*b*) if it touches another pair of sides (say 5 cm. and 6 cm.) and (*c*) if it touches all the three sides.]

5. In the figure of question 4 describe a circle so as to touch the side 4 cm. and the other two sides 5 cm. and 6 cm. produced [consider where its centre should be (*a*) if it touches the side 4 cm. and the side 5 cm. produced, (*b*) if it touches the side 4 cm. and the side 6 cm. produced and (*c*) if it touches the side 4 cm. and the sides 5 cm. and 6 cm. produced.]

6. In the figure of Question 4, describe a circle so as to touch the side 5 cm. and the sides 4 cm. and 6 cm. produced. (Proceed as in Question 5).

7. In the figure of Question 5, describe a circle so as to touch the side 6 cm. and the sides 4 cm. and 5 cm. produced.

8. Describe two circles whose centres are 8 cm. apart and whose radii are 5 cm. and 3 cm. Do the circles touch? If so internally or externally? Draw a tangent to one of the circles at their point of meeting. Show that it is also a tangent to the other circle.

9. Describe two circles whose centres are 3 cm. apart and whose radii are 7 cm. and 4 cm. Do the circles touch? If so whether internally or externally? Draw a tangent to one of the circles at their point of meeting. Show that it is also a tangent to the other circle.

Hence learn that **two circles touching one another externally or internally have a common tangent at the point of contact and the distance between the centres is equal to the sum or difference of the radii.**

10. Let ABC be an equilateral triangle inscribed in a circle. At A, B and C draw tangents to the circle. Let these tangents meet one another in D, E and F. Examine what kind of a triangle DEF is.

11. From Question 10, give a construction for circumscribing an equilateral triangle about a given circle.

12. ABCD is a square. Circumscribe a circle about a given square. (Which point will be the centre and which line will be the radius?)

13. ABCD is a square inscribed in a circle. At A, B, C and D draw tangents intersecting one another at E, F, G and H. What kind of a four-sided figure is EFGH? Give reasons for your answer.

14. From Question 13, give a construction for circumscribing a square about a given circle.

15. ABCD is a square of side 1·5 in. Inscribe a circle in the square ABCD, i.e., describe a circle which will touch the sides AB, BC, CD and DA (you can guess the position of the centre from considerations of symmetry). What is the radius of the circle?

16. Circumscribe a regular pentagon about a given circle. [Inscribe a regular pentagon in it; through the angular points draw tangents whose points of intersection will give another regular pentagon.]

17. Circumscribe a regular hexagon about a given circle. [Inscribe a regular hexagon in it; through the

angular points draw tangents whose points of intersection will give another regular hexagon.]

18. Circumscribe a regular octagon about a given circle. [Inscribe a reguar octagon in it; through the angular points draw tangents whose points of intersection will give a regular octagon.]

19. Inscribe a circle in a regular hexagon of side 4 cm. [Use the theorem that the centre of a circle lies on the bisector of the angle between two intersecting tangents.] Find the length of the radius.

20. Inscribe a circle in a regular octagon of side 1·4 in. Proceed as in Question 19. Find the length of the radius.

21. AB, CD and EF denote the lengths and directions of three houses situated in a village. There is a banyan tree equidistant from AB, CD and EF. If the figure is drawn to scale 1 in. = 40 ft., find the distance of the banyan tree from any one of the houses.

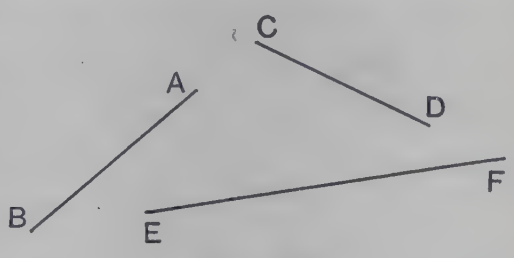


FIG. 223.

22. Two circles of radii 4 cm. and 3 cm. with their centres 5 cm. apart intersect one another at the points A and B. Join AB. In AB produced take any point P. From T draw TP and TR tangents to the two circles. Measure TP and TQ. What peculiarity do you notice?

23. There is a bundle of scout staves each of which is 5 ft. 6 in. These are scattered in a circular room so that the extremities of each staff rest on the wall of the room. What is the locus of their middle points? Supposing the diameter of the room is 12 ft., draw the locus.

24. If a quadrilateral is circumscribed about a circle show, by measurement, that the sum of one pair of opposite sides = the sum of the other pair.

25. AB is a straight fence of a garden, consisting of trees. O is a peg 10 ft. from AB to which a cow is tethered

by a rope. What must be the length of the rope (*a*) if the cow should have no access to the trees in the fence, (*b*) if the cow should just reach the fence and (*c*) if some trees in the fence are accessible to the cow?

26. Construct a triangle ABC in which AC = 2·3 in., $\angle$ A = 75° and CB = x inches. What must be the value of x (*a*) if there is to be no triangle, (*b*) if there is to be only one triangle and (*c*) if there are to be two triangles. (*vide* Fig. 224).

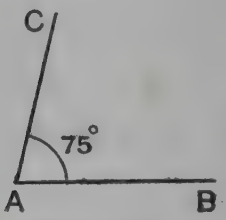


FIG. 224

27. L is a bright luminous body. APB is a spherical wooden body lying in the path of the rays. Find the extreme rays intercepted by the body. (*vide* Fig. 225).

L

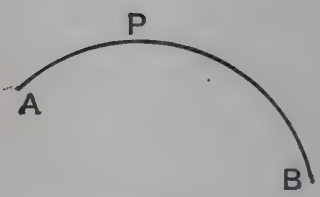


FIG. 225.

28. Draw a tangent to a given circle parallel to a given straight line. How many such tangents can be drawn?

29. Draw a tangent to a given circle perpendicular to a given straight line. How many such tangents can be drawn?

30. Two equal circles touch one another. How many common tangents can be drawn? Draw them. What is the length of each tangent?

31. Three equal circles each of radius 1 in. touch one another. Find a point equidistant from their centres.

32. In the preceding question circumscribe a circle about the triangle formed by joining the three centres. What triangle will that be?

33. Describe a circle touching all the three circles in Question 31. What is the diameter of the circle? How many circles can be drawn?

34. Two pipes of diameters $2a$ and $2b$ inches rest on the ground touching each other. The distance between their points of contact on the ground is d inches. Express d in terms of a and b.

35. Figure (227) represents a sphere placed in contact with two equal wooden cubes which is a device for measuring the diameter of a sphere. Show that it can be done and prove that the diameter of the sphere is equal to the sum of the heights of the points of contact of the sphere with the cubes, above the base plane on which these stand.

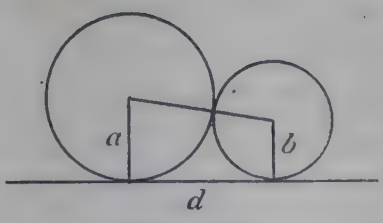


FIG. 226.

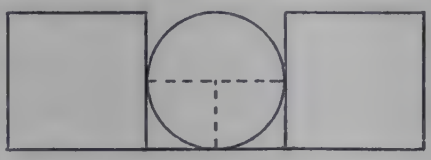


FIG. 227.

36. From a point A outside a circle, two secants ABC and ADE are drawn cutting the convex portion at B and D and the concave portion at C and E. Find two points on the circumference to which lines drawn from A form the boundaries between the convex and concave portions of the circumference. (A secant is a straight line drawn from a point outside a circle cutting it in two points. **Concave** means hollow and **convex** means bulging.

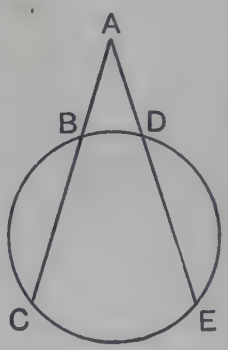


FIG. 228

37. AE is a circular public urinal built by the Corporation of a city in a street, the entrance to which is shown in the figure at E. A man stands at B. Show what external portion of it is visible to him and where he should stand to obtain a glimpse of the entrance?

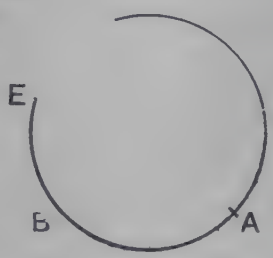


FIG. 229.

38. QCT is a tangent to the circle CBAD and CA a chord is drawn to the circle. Show that the angle TCA = the angle in the alternate segment CBA and the angle QCA = the angle in the alternate segment ADC, by measurement.

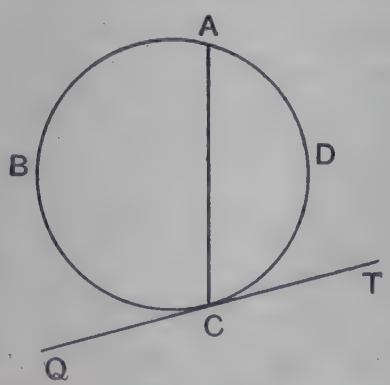


FIG. 230.

39. Describe a circle CBAD of radius 1·5 inch. At any point C on the circumference draw a tangent QCT. At C draw a chord CA making an angle of 60° with the tangent. Measure the angle in the alternate segment CBA.

Hence learn that **the angles made by a tangent to a circle with a chord drawn from the point of contact are equal to the angles in the alternate segments of the circle.**

40. On AB 5 cm. long describe a segment of a circle containing an angle of 35°. (Proceed as directed below.)

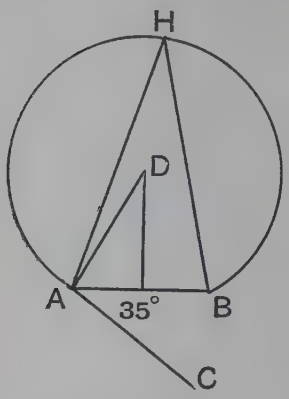


FIG. 231.

[The preceding theorem gives us the following construction:—At A in AB, make an angle BAC of 35°. Draw the perpendicular bisector of AB (for the circle should pass through A and B and its centre must be on the perpendicular bisector of AB). From A draw AD at right angles to AC. (For the centre is on a line at right angles to the tangent drawn from the point of contact). Let them meet at D. Then with D as centre and DA as radius describe a circle AHB. Measure any angle in the segment AHB. It will be 35°.]

41. Without measuring in the preceding question, how can you say that $\angle BAC =$ any angle in the segment AHB?

42. On a billiard table the position of a ball was noted and it was found that the shorter side of the table 4 ft. in length subtended an angle of 48° at the ball. Is it possible to fix the position of the ball with the help of this observation? If not how will you fix the position of the ball making use of any one of the following additional observations:—

(*a*) Its distance from the longer side of the table was 1·5 ft.

(*b*) Its distance from the shorter side of the table was 2·7 ft.

(*c*) Its distance from one of the extremities of the shorter side was 2·5 ft.

(*d*) A right angle was subtended by the longer side of 6 ft. at it.

(*e*) It was found to be equidistant from the other two corners of the table.

(*f*) It was found to be equidistant from a pair of shorter and longer sides.

43. Describe a circle ACBD and draw two chords in it AB and CD intersecting in E. Measure AE, CE, BE and DE and show that the rectangle AE.EB = the rectangle CE.ED. (Rectangle AE.EB means rectangle whose two adjacent sides are AE and EB.)

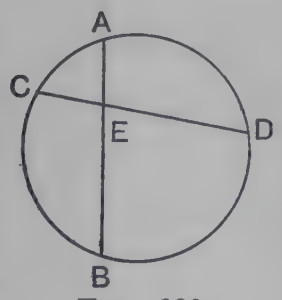


FIG. 232.

44. In the above question, draw PQ and RS through E. Measure PE, QE, RE and SE and show that the rectangle PE.EQ = the rectangle RE.ES.

45. Draw chords AB and CD in a circle. Produce BA and DC to meet in E. Measure EA, EB, EC and ED and show that the rectangle EA.EB = the rectangle EC.ED.

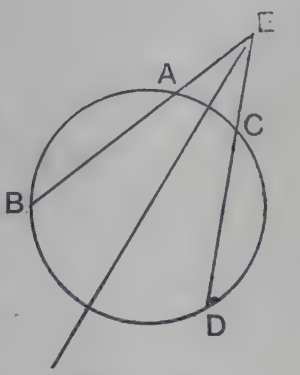


Fig. 233.

46. Describe another circle and repeat the experiment in Question 45. Hence learn that **if two chords of a circle intersect or chords produced intersect, the rectangle contained by the segments of the one = the rectangle contained by the segments of the other.**

47. Describe a circle ABC. From a point T outside the circle draw a tangent TC and a secant TAB. Measure TC, TA and TB and show that the rectangle TA.TB = TC^2.

48. Repeat the experiment with any other circle.

Hence learn that **if a secant and a tangent are drawn to a circle from a point without it, the rectangle contained by the segments of the secant = the square on the tangent.**

49. Describe a circle of radius $1\frac{1}{2}$ inch. Take a point $3\frac{1}{2}$ inches from the centre and from it draw a tangent to the circle. Measure the length of the tangent. Find by calculation, what its length should be.

50. Describe a circle of radius 2 inches. Produce any radius OA to a point T such that the two tangents drawn from T to the circle are inclined to one another at an angle of 80°.

[What is the magnitude of the angle between the normals or the radii at the points of contact? Remember that OT is the line of symmetry.]

51. Describe two circles of radii 6 cm. and 8 cm. which have a common chord of length 3 cm. Measure the distance between their centres. At what angle do the tangents at one of the common points of the two circles, intersect?

52. Draw a scalene triangle ABC, having its sides AB, BC and CA respectively equal to 3·5, 5·5 and 6 cm. Describe a circle so as to touch AB at A and pass through the point C.

53. Describe a circle of radius 6 cm. Draw tangents each 4 cm. long from different points on the circumference. What is the locus of the extremities of the tangents?

54. AB is a straight line and C a point without it. With C as centre and any convenient radius, describe a circle cutting AB or AB produced at D. On CD describe a semi-circle CED cutting AB at E. Then CE is perpendicular to AD. Why? Note this construction for drawing the perpendicular from C to AD.

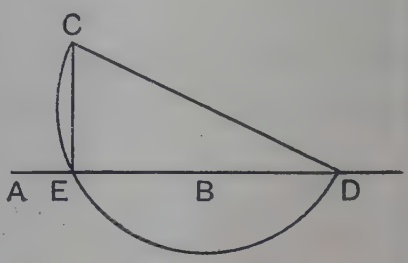


FIG. 234.

55. KN is a straight line in which L is a point. With centre L and any radius LM describe a circle cutting KN at M. Step off equal arcs MO, OP each equal to LM along the circle. With centres O and P and a radius each equal to LM describe arcs cutting at Q. Join QL. QL is perpendicular to KN. Why? Note this construction for drawing a straight line at right angles to a given straight line from a given point in it. (In the figure M has been omitted which may be supplied by the pupil.)

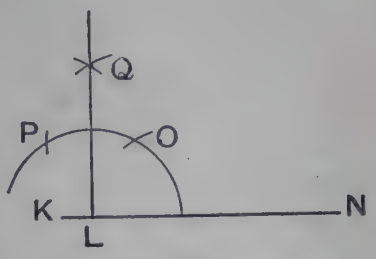


FIG. 235.

56. A, B, C, and D are the feet of four trees in a straight line such that AB = 5 ft., BC = 2 ft. and CD = 6 ft. A treasure at P is hidden and it is noted that angles APC and BPD are right angles. Find the position of P.

57. A and B are two ports 3 miles apart along a straight strip of coast. Mines are spread near the coast. No ship should sail up to where AB subtends an angle of 72°. Represent in a figure drawn to scale the danger area.

58. A steamer finds, in the previous question, that at a certain place where it is, the angle subtended by AB is 83°. Is it within the danger zone or outside it?

59. Plot the points (8, 0), (— 6, 0) and (0, 12). Describe a circle passing through the three points. Find by calculation the co-ordinates of the other point where the circle intersects the y axis.

60. ACB is a circle of a radius of 4 in. From a point D $5\frac{1}{2}$ inches from the centre, a tangent is drawn. What must be the length of the tangent? Calculate it in two different ways.

CHAPTER XXXI.

Interesting Graphical Solutions of Certain Arithmetical Problems

Model No. (1) A certain wall under construction was 2 ft. high. From the next day the work was entrusted to a set of workmen and the following progress was noted :—The wall was 5 ft. high at the end of the first day, 8 ft. high at the end of the second day, 13 ft. high at the end of the third day, 20 ft. high at the end of the fourth day and 27 ft. high at the end of the fifth day. The rapid progress in the latter part of the construction was due to the employment of an additional set of workmen. Find graphically when the latter set was employed.

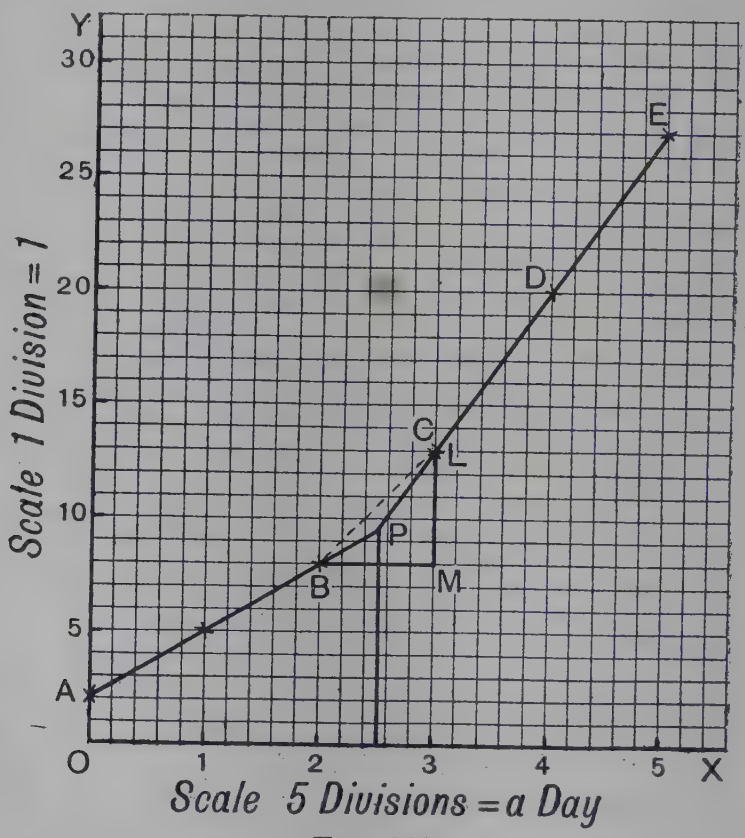


Fig. 236.

Solution.—Along OX $\frac{5}{10}$″ represents a day; along OY $\frac{1}{10}$″ represents a height of 1 foot of the wall. Plot (0, 2), (1, 5), (2, 8), (3, 13), (4, 20) and (5, 27) and join them in order. In the figure the graph consists of 3 parts. AB represents the graph of the work of the first set of workmen ; CE represents the graph of the combined work of both sets of workmen ; the part BC represents the work done by the first set of workmen, from the end of the second day to the end of the

third day and that done by the second set for a portion of the third day. Produce AB the first part of the graph (i.e. of the work of the first set of workmen) and the last part of the graph EC (of the combined work of both sets) to meet at P. The abscissa of P denotes when the second set joined the first set, i.e. in the afternoon on the third working day.

The graph tells us many things. The work done on the third working day is indicated (*vide* BC the second part of the graph) by MC consisting of 5 vertical divisions in the figure. Produce BP to meet CM in (L must be 2 divisions lower than it is on the line CM). On the third working day the first set did work indicated by ML containing 3 divisions. Therefore LC denotes the work done by the second set in half a day.

Model No. (2) A cistern 25 inches deep is filled by a pipe A. At 4 a.m., when there are 4 inches of water in the cistern the pipe A is turned on. At 5 a.m. there are 8 inches; at 6 a.m. 12 inches; at 7 a.m. 13 inches; at 8 a.m. 11 inches; at 9 a.m. 9 inches when it was noticed that there had been a leak in the cistern. It is required to find when it became leaky.

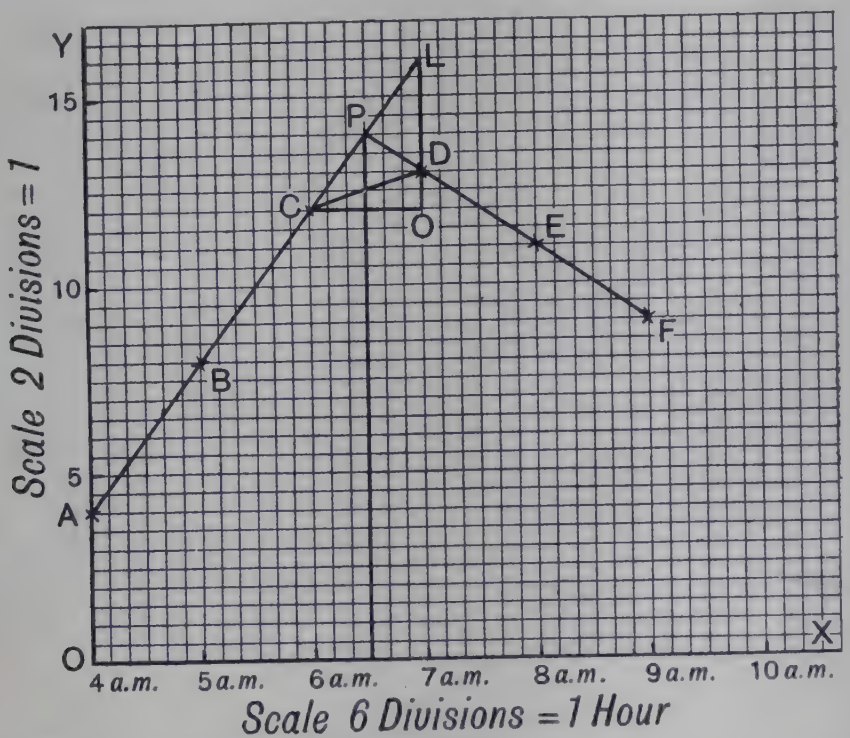


FIG. 237.

Solution.—Along OX, $\frac{6}{10}'' = 1$ hour; along OY $\frac{2}{10}'' = 1$ inch. Plot the points (4 a.m., 4 in.), (5 a.m., 8 in.), (6 a.m., 12 in.), 7 a.m., 13 in.), (8 a.m., 11 in.), (9 a.m., 9 in.). The figure shows

the graph of the stock of water in the cistern at different times. The graph consists of three parts (1) ABC, (2) CD and (3) DEF. Part (1) is the graph of the supply of water by pipe A; part (3) of the net supply of water by pipe A after the springing of the leak; part (2) represents the pipe running the whole hour and the leak letting out water only for a part of that time. If we produce AC and FD to meet at P, the abscissa of P denotes the time when the cistern became leaky. The figure also shows the pipe was at work from 6 to 7 a.m., the quantity of supply by it alone being OL (8 vertical divisions). The leak was at work for half an hour from 6.30 to 7; and in this time there was a fall in the level denoted by LD measuring 6 divisions or 3 inches. Hence the rate of discharge is 6 inches per hour.

Model No. (3) A motor-car overtakes at 7 a.m. a cyclist ridingat the rate of 15 miles per hour and after 20

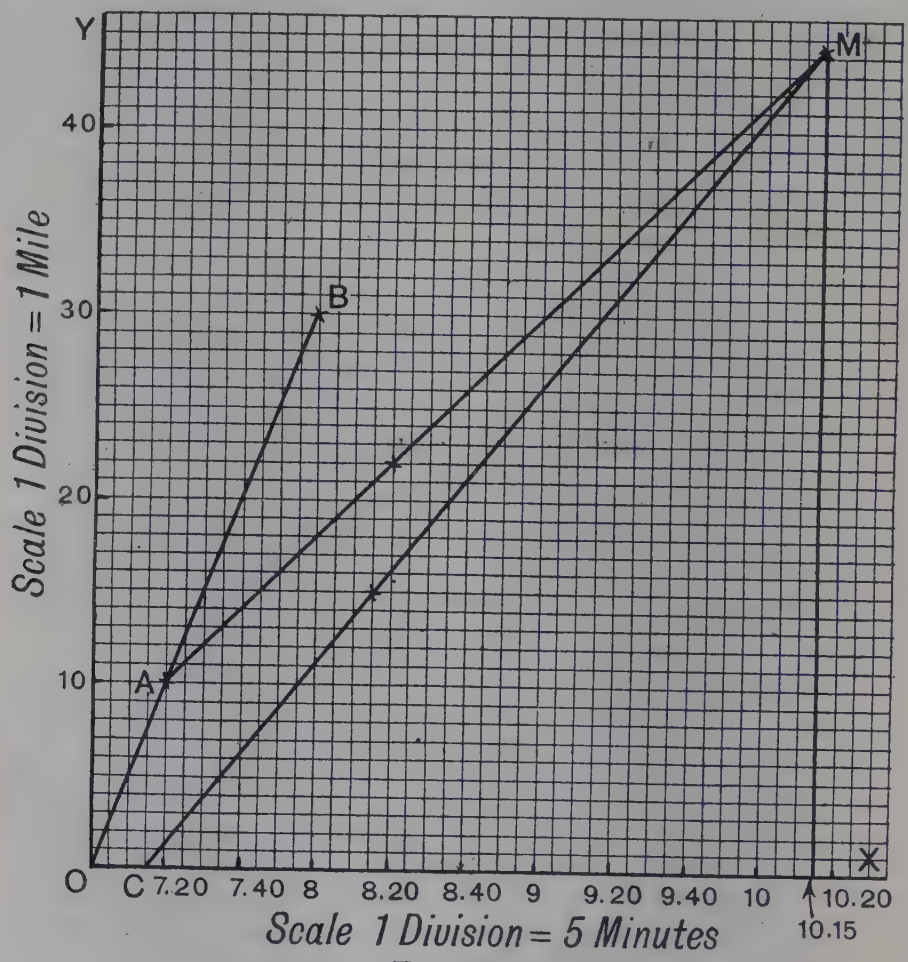


FIG. 238.

minutes overtakes another cyclist riding at the rate of 10 miles an hour. If the motor runs at 30 miles per hour, find when the former cyclist overtakes the latter supposing the former takes rest for 15 minutes as soon as the motor

overtook him and the latter increased his speed by 2 miles an hour when he was overtaken by the motor.

Solution.—Along OX, $\frac{1}{10}'' = 5$ minutes; on OY, $\frac{1}{10}'' = 1$ mile. OAB represents the graph of the motor the gradient indicating a speed of 30 miles an hour. AM represents the graph of the second cyclist (starting at 7.20 and travelling at the rate of 12 miles an hour). CM represents the graph of the first cyclist (starting at 7.15 and travelling at the rate of 15 miles an hour). It is found that these two graphs meet at M whose abscissa is 10.15 a.m. The first cyclist overtakes the second at 10.15 a.m.

Model No. (4) The Boat Mail No. (1) leaves Egmore at 20.0, reaches Chingleput 39 miles from Egmore at 21.24, halts there for 12 minutes and travelling at the same rate as before reaches Acharavakkam at 22.30. Find the distance of Acharavakkam from Madras. If it travels at the same rate and reaches Villupuram 102 miles from Madras, find its time of arrival at Villupuram.

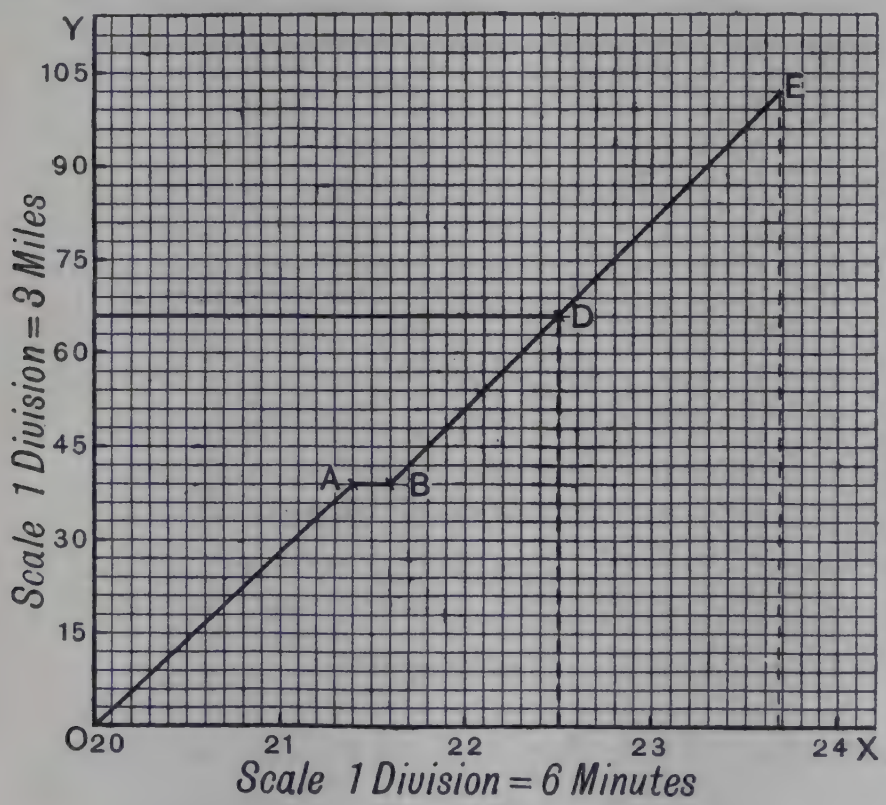


FIG. 239.

Solution.—Along OX, $\frac{1}{10}'' = 6$ minutes; along OY, $\frac{1}{10}'' = 3$ miles. OA represents the graph of the Boat Mail the co-ordinates being (21–24. 39). It halts for 12 minutes indicated by the level AB. Then it proceeds at its former rate. Draw BDE parallel to OA (as the gradient will be the same). Find the ordinate corresponding to the abscissa 22.30, the distance lies between 63 and 66 miles. Now read off the abscissa corresponding to 102 miles, the reading is about 23-42.

Model No. (5) Multiply one number by another say 9 by 8.

Method I. The student himself will supply the figure.

Along OX, $\frac{1}{10}'' = 1$; along OY, $\frac{1}{10}'' = 1$. Mark off numbers along both the axes thus (1, 5, 10, 15), etc. Plot the point 1, 8 (say A). Join OA. Plot the point (9, 0) call it B. Through B draw an ordinate meeting OA produced, if necessary, at C. Measure BC. It will be found to be 72.

Method II. Plot the point 1, 9. Through the point (8, 0) an ordinate may be drawn as before. This ordinate also will measure 72.

Model No. (6) Divide 64 by 4 graphically.

Along OX, $\frac{1}{10}'' = 1$; along OY, $\frac{1}{10}'' = 1$. Mark off numbers along OX and OY thus (1, 5, 10, 15), etc. Plot the point (4, 1) call it A. Join OA. Plot the point (64, 0) call it B. Through B draw an ordinate meeting OA produced if necessary at C. Then BC is 16 which is the required answer.

Exercise XXXI.

N.B.—All sums in this chapter to be worked graphically.

1. A and B can do a piece of work in 3 and 4 days. A works for 1 day. Then B joins him. In how many days more will the work be finished by both of them? Give a graphical solution. Check by calculation.

2. A and B can do a piece of work in 4 and 5 days. A works for 2 days. Then he is assisted by B. In how many days more will the work be finished? Do the question graphically and check by calculation.

3. A and B can do a piece of work in 10 days and 25 days respectively. A and B finish it in 8 days, A being assisted by B for the last few days. Find graphically when B joined A. Check your result by calculation.

4. A and B can do a piece of work in 6 and 9 days respectively. Both of them finish the work in 4 days, B working only for the last few days. Find graphically when B joined A. Check your result by calculation.

5. A, B and C can do a piece of work in 6, 8 and 10 days. A works for the first day ; then A and B together work for the next two days. Then they are assisted by C. In how many days more will the work be finished ?

6. A, B and C can do a piece of work in 8, 10 and 12 days. A and B work for some days when they are assisted by C and the work is finished in $3\frac{2}{3}\frac{9}{7}$ days from the beginning. Find when C joined them.

7. A cistern is filled by two pipes A and B in 4 hrs. and 12 hrs. respectively. A is opened at 7 a.m. B was accidentally opened some time after and it was found that the cistern was full at 10.15 a.m. Find when the pipe B was opened.

8. A cistern is filled by two pipes A and B in 4 and $2\frac{2}{3}$ hrs. respectively. At 12 noon the pipe A is opened. After some time B is also opened. It was found that the cistern was full at 2.30 p.m. Find when B was opened.

9. A cistern is filled by a pipe A in 3 hrs. and emptied by a pipe B in 5 hrs. A is opened at 6 a.m. Some time after, B also is opened. It was found that the cistern was full at 12 noon. When was B opened ?

10. A cistern is filled by a pipe A in 5 hrs. and emptied by a pipe B in 3 hrs. A is opened at 8 a.m. Some time after, B is also opened. It was found that there was no water in the cistern at 11 a.m. Find when the pipe was opened.

11. A cistern can be filled by a pipe A in 5 hrs. It is opened at 6 a.m. but is closed for the last 5 minutes in every hour. Find graphically when the cistern will be filled.

12. A cistern can be filled by a pipe B in $6\frac{1}{2}$ hrs. It is opened at 9 a.m., but is closed for 5 minutes at the end of every hour. Find graphically when the cistern will be filled.

13. A cistern is filled by a pipe A in the first 30 minutes and emptied by a pipe E in the last 30 minutes every hour. If the pipe A is opened at 12 noon and their rates of filling

and emptying the cistern are 4 hrs. and 6 hrs. respectively, find graphically when the cistern will be full.

14. A snail climbs up a pole 25 ft. high at the rate of 13 ft. in the day time but slips down 12 ft. during the night time. Find graphically when the snail will reach the top.

15. Two tanks contain water to a depth of 30 ft. and 10 ft. respectively. Water flows from one to the other so that the surface of the water in the first tank falls 3 ft. every hour and that in the second rises 2 ft. every hour. Draw a graph showing the depth of water in each tank at any time from the commencement of the flow of water and find when the level of water was the same in both.

16. A gentleman invests Rs. 20,000 and Rs. 8,000 in the two branches A and B of a bank respectively. He withdraws money from A at the rate of Rs. 2,250 every 3 months and invests Rs. 1,500 every 6 months in B. Draw a graph showing his amounts in A and B and find when he had equal amounts in both the branches.

17. Multiply graphically (*a*) 1·2 by 3·4, (*b*) 4·6 by 1·8, (*c*) 3·6 by 1·8 and (*d*) 0·3 by 4·6.

18. Divide graphically (*a*) 3·4 by 1·2, (*b*) 4·6 by 2·4, (*c*) 5·4 by 2·3 and (*d*) 1·2 by ·5.

19. Evaluate graphically (*a*) $\frac{1\cdot2 \times 2\cdot3}{1\cdot3}$, (*b*) $\frac{2\cdot3 \times 2\cdot5}{3\cdot2}$, (*c*) $\frac{1\cdot6 \times 2\cdot9}{1\cdot3}$ and (*d*) $\frac{3\cdot4 \times 4\cdot6}{4\cdot8}$.

20. (*a*) No. 1 Ceylon Boat Mail is timed as follows :—

Distance from Madras Beach in miles	Station	H.M.
3	Egmore ...	20.0
38	Chingleput *a.* ...	21.30

(*b*) No. 25 Madura Express is timed as follows :—

Distance from Madras Beach in miles	Station	H.M.
3	Egmore ...	19.30
38	Chingleput ...	22.15

(*c*) No 5 Trivandrum Express is timed as follows :—

Distance from Madras Beach in miles	Station	H.M.
3	Egmore ...	21.10
38	Chingleput ...	23.5

Supposing the trains travel uniformly find (*a*) when and where the Ceylon Boat Mail will overtake the Madura Express, (*b*) when and where the Trivandrum Express will overtake the Madura Express and (*c*) from the following table of distances, where the Madura Express should remain at the siding to keep the main line clear for the passage of (*a*) Ceylon Boat Mail and (*b*) Trivandrum Express :—

Miles from Madras Beach	Station	Miles from Madras Beach	Station
8	Saidapet.	57	Pakkam.
10	Guindy.	61	Acharapakkam.
11	St. Thomas Mount.	65	Perambair.
13	Minambakkam.	71	Olakur.
15	Pallavaram.	78	Tindivanam.

21. No. 4 Rameswaram Express is timed as follows :—

Miles from Beach	Station	H.M.
252	Trichy ...	5.20
221	Tanjore ...	7.25

Find (*a*) when and where No. (4) meets No. 1 train, (*b*) when and where it meets No. 25 train (*c*) when and where it meets No. 5 train mentioned in the preceding question and (*d*) also the times when No. 4 will meet the next Madura Express No. 25 and the next Boat Mail No. 1.

22. The down Boat Mail is timed as follows :—

From Beach	Station	H.M.
38	Chingleput ...	5.52
3	Egmore ...	7.20

The Arkonam Branch down train reaching Chingleput is timed as follows :—

Distance from Beach	Station	H.M.
77	Arkonam ...	6.25
38	Chingleput ...	8.25

It is proposed to start a train from Conjeeveram (on the Arkonam-Chingleput Branch line) so as to reach Chingleput at 5.50 to catch the down Boat Mail. Assuming that the speed of all the trains working on the branch line is the same, find graphically when the train from Conjeeveram (59 miles from Beach) should start.

23. The down train Rameswaram Express No. 4 is timed as follows :—

Miles from Beach	Station	H.M.
177	Mayavaram *d.* ...	10.37
128	Cuddalore New Town *a.*	13.14
102	Villupuram *a.* ...	14.37

A train leaves Pondicherry (125 miles from Beach) arrives at Chinnasamudram (115 miles from Beach) the boundary between the French and English territory where all luggage without a permit is examined by customs authorities, stays there for 45 minutes and then proceeds to Villupuram reaching the latter station exactly at the same time as No. 4 Rameswaram Express. Supposing the speed of all trains working on the branch line from Pondicherry to Villupuram is on an average 15 miles per hour, find the time of departure of the train from Pondicherry.

24. Divide Rs. 100 among A, B and C so that their shares will be in the proportion of 8, 7, 5.

25. Divide the total profits, in a concern, of Rs. 1,000 in the ratio of the capitals contributed by three partners which are Rs. 2,500, Rs. 1,500, and Rs. 1,000 respectively.

26. Two ghees worth Rs. 2-12-0 and Rs. 4 per viss are mixed in a certain proportion and the mixture is worth Rs. 3 per viss. Find graphically the ratio of mixture.

27. Nellore rice costs 5 as. 4 ps. per measure. Tenali rice 4 as. per measure. A mixture of these rices is worth 5 as. per measure. Find graphically the ratio of the mixture.

28. A labourer is hired for 16 days on condition that he will be paid 12 as. for each day he works and he should lose

4 as. for each day he was absent. He received on the whole Rs. 6. Find graphically how many days he was absent.

29. During the time of harvest, a farmer found that for every day he was present in the farm he got 10 markals of paddy and for every day he did not go to the farm his stock of grain was diminished by 2 markals. At the end of 15 days he found his stock of paddy consisted of 90 markals. Find graphically how many days he was absent from the farm.

30. A student started at 6 a.m. from Madras walking at the rate of 4 miles an hour to a place P. At 6.15 a cyclist left P to go to Madras at $7\frac{1}{2}$ miles an hour and was followed by a motor cyclist riding at 10 miles an hour. All the three met exactly at 8.15 a.m. Find (*a*) the distance from Madras to P, (*b*) the distance of the place of meeting from P and (*c*) when the motor cyclist and the cyclist reached Madras.

31. Two students S_1 and S_2 start at 6 a.m. from a village to bathe in the sea at a place P some miles distant from it. S_1 rides on a bicycle at 10 miles an hour, arrives at P where he spends a quarter of an hour, leaves the bicycle for S_2 and walks back to the village by the same road at 2 miles an hour and arrives at 7 a.m. S_2 also starts at 6 a.m. walking at 2 miles an hour meets S_1 returning, bathes in the sea for $12\frac{1}{2}$ minutes and cycles back to the village reaching it at 7 a.m. Find (*a*) the distance between the village and P, (*b*) where and when S_1 and S_2 meet and (*c*) the rate at which S_2 cycled back from P to the village.

32. Two persons A and B set out together on a journey. They walked at the rate of 3 miles an hour ; and after they proceeded $\frac{3}{4}$ of a mile, B returned walking at the same rate to the place of starting. Here he was detained $\frac{3}{4}$ of an hour. Setting out again he overtook A who had been walking all the time, at the end of $2\frac{1}{2}$ hours from the second time of starting. At what rate did he walk ?

33. A peon walks from A to B at the rate of 3 miles an hour and after transacting some business at B which takes

him an hour returns to A by tram at the rate of 5 miles an hour. He then finds he has been absent 2 hrs. 20 minutes. Find the distance from A to B.

34. A and B start at the same time from P and Q and travel towards each other at the rate of 9 and 3 miles an hour respectively. A reaches Q and returning immediately overtakes B at a certain place. Find the distance of the place from P.

35. A person sets out to walk from A to B at the rate of 3 miles an hour. After walking $1\frac{1}{2}$ mile he is overtaken by a bandy which started from A 15 minutes after him. At a distance of $3\frac{1}{2}$ miles from A he meets the bandy returning from B where it remained 5 minutes. Find the rate of the bandy and the distance between the two places.

36. A, B and C walk at the rate of 4, 6 and 9 miles an hour. They start from Salem at 6, 7 and 8 o'clock respectively in the morning and go along the same road in the same direction. When B catches A he sends him back with a message to C. When will C get the message?

37. A and B start from P and Q the opposite extremities of a trunk road 210 miles long and travel at 10 and 15 miles an hour. They meet at R and continuing their journey to Q and P return immediately and meet at S. Find the length of RS.

38. A and B run a mile race. A gives B a start of 20 seconds and beats him by 80 yds. If A's rate is 8 miles an hour what is the rate of B?

39. In a mile race A can give B a start of 80 yds. and B can give C a start of 40 yds. How many yards' start can A give C in a mile race?

40. A and B run a race. A runs at the rate of 600 yds. per minute, but in every successive minute increases his pace by a yard a minute. B diminishes his pace by the same and is overtaken by A in four minutes. What was B's pace at starting?

41. A can beat B by 40 yds. in a mile race. B can beat C by 40 yds. in a mile race. How many yards' start must A give C that there may be a dead heat?

42. At what times will the hour and minute hands of a clock be 5 minutes apart between any two given hours? Solve graphically.

43. In the same diagram find at what time the hour and minute hands of a clock will be 10 minutes apart.

44. A watch that gains 10 minutes in 2 hrs. is set right. In how many hours will it be exactly right again?

45. A clock which indicated correct time at noon shows 10 minutes to 5 when it is 5 o'clock. What is the time when the clock indicates 5 o'clock?

CHAPTER XXXII.

AREAS OF CIRCLES SECTORS AND SEGMENTS.

171. Relation between the circumference of a circle and its diameter.

In the chapter on ratio and proportion, the ratio of the circumference of a circle to its diameter was found to be a constant quantity very nearly equal to $3\frac{1}{7}$ or the Greek letter π is used to denote this constant ratio. Hence if C denotes the circumference of a circle, r its radius, and d the diameter, (1) $C = \pi d$ or (2) $C = \pi . 2r$ or $2\pi r$.

Also (3) $d = \frac{C}{\pi}$ and (4) $r = \frac{C}{2\pi}$.

Exercise XXXII (A).

In the following exercises, $\pi = \frac{22}{7}$ unless otherwise stated.

1. The diameter of a fly-wheel of an engine is 24 ft. Find its circumference.

2. Explain how you would measure the circumference and diameter of a rupee or a half-anna.

3. The diameter of the circular rim of a bucket is $8\frac{1}{4}$ in. Find the circumference.

4. The diameter of a bull's eye window in a tower is $4\frac{1}{6}$ in. Find the curved length of the outer rim of the frame.

5. What is the diameter of a circular mile race course?

6. What is the diameter of a circular race course 2 miles in circumference? Can it be deduced from the answer to Question 5 without calculation?

7. The hindwheel of a coach makes 714 revolutions in going a distance of 1·275 miles. Find the diameter of the wheel.

8. The inner rim of the circular brick wall of a well measures 15 ft. $7\frac{1}{2}$ in. Find the radius of the well.

9. A circle and a square have the same perimeter. The area of the square is 132·25 sq. in. Find the radius of the circle.

10. The outer circumference of a cycle tyre after full inflation is 7 ft. 4 in. If the thickness of the tyre be $1\frac{1}{2}$ in., find the radius of the inner rim.

11. A flower-bed is arranged in the space enclosed by the circumferences of two concentric circles. The circumferences are 198 ft. and 154 ft. Find the width of the flower-bed.

12. The equatorial circumference of the earth is 65,759,885 ft. and its polar circumference is 65,540,065 ft. Find the difference between the equatorial and polar radii of the earth.

13. The equatorial circumference of the earth is 25,000 miles. The diameter of a small circle of the earth parallel to the equator at a latitude of $60° = \frac{1}{2}$ the equatorial diameter. Find the circumference of the small circle otherwise called the parallel of latitude at 60°.

14. A rupee has a diameter of 1·6 in. and a pie of ·64 in. On the rims of both, two marks are made with a chalk piece. The rupee and the pie are made to touch each other at the marks. The rupee is held fixed and the pie is rotated round the rupee. Find after how many revolutions the chalk marks will again coincide.

15. A train is running at the rate of 40 miles per hour. Find how many revolutions a big wheel of the engine 5 ft. 6 in. in diameter makes in a minute.

16. Calculate the speed of a train a wheel of whose engine makes 6 revolutions in 5 seconds, the diameter of the wheel being 2 ft.

17. A curved line is measured by rolling a quarter anna coin over it. The coin made $6\frac{1}{2}$ revolutions. If the diameter of the coin be ·75 in., find the length of the curved line.

18. A road-levelling machine has a roller whose diameter is 4 ft. 6 in. and in traversing a road it makes 50 revolutions. Find the length of the road.

19. The outer circumference of a test tube = 3·2 in. and its inner circumference = 2·6 in. Find its thickness.

20. The outer circumference of a chimney = 4·5 in. and the thickness of the glass = ·025 in. Find its internal diameter.

21. Two concentric rings are connected by a straight rod knitted together with a wire at its 4 points of intersection with the circumferences. The radii are 8 in. and 5 in. and the length of the rod is 14 in. Find the length of the rod intercepted between the rings.

22. The lengths of the second hand, hour hand and minute hand of a watch are ·56 in., ·76 in. and 1·24 in. respectively. Find the distances travelled by their ends in a day.

23. A circular ring made of wire has a diameter of 9·6 in. The diameter of a rupee is 1·6 in. A number of rupees are placed inside the ring laid flat on a table, so as to touch one another and the circumference of the ring. How many rupees will be required? If they are placed outside the ring in a similar manner, how many more rupees will be required?

24. What is the total length of the path on which the centres of (*a*) all the internal rupees lie and (*b*) all the external rupees lie, in Question 23?

FIG. 240.

25. Two spectators in a festival sit on a wooden horse and elephant suspended from the circular top of a merry-go-round. If the distances of the horse and the elephant are 10 ft. 8 in. and 9 ft. 6 in. from the central post, calculate the distance each spectator has to go round in 56 revolutions.

26. ABCD is a rectangle whose length is 12 in. and breadth 5 in. The rectangle is held in a vertical position resting on a table so that the diagonal AC is vertical. The rectangle is rotated about the diagonal AC. Find the distance of the paths traversed by B and D.

FIG. 241.

27. In Question 23 show that the circumference of the ring is the average of the paths on which the centres of the rupees lie.

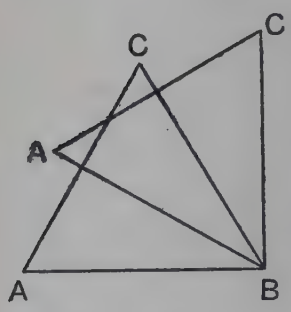


FIG. 242.

28. ABC is an equilateral triangle with its base AB resting on a table. AB = 6 in. The triangle is in a vertical plane and rotated about B until BC is vertical. Find the height of the vertex A above the table. What fraction of a complete revolution has it undergone and what distance has it traversed?

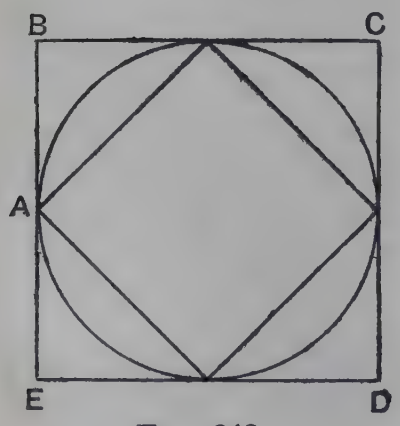


FIG. 243.

29. Two persons start from A and walk, one along the sides of a square field and the other along the circumference of the inscribed circle and return to A. If the figure is drawn to scale 1 in. = 100 yds. find what distance is saved by travelling along the circle.

30. What further distance could be saved by travelling along the square inscribed in the circle?

172. The area of a circle can be obtained from that of an inscribed regular polygon.

Let ABCDEFGH be any regular polygon of n sides inscribed in the circle whose centre is O. Then the polygon can be divided into a number of equal triangles having a common vertex at O and resting upon the sides of the polygon. From O draw OM perpendicular to BC.

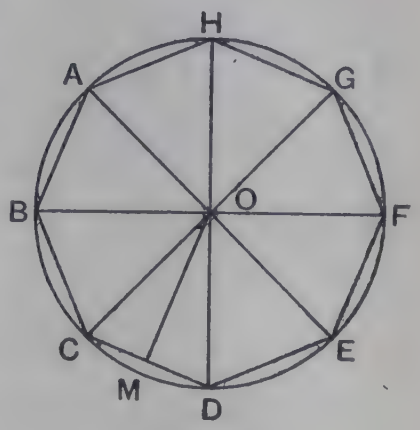


FIG. 244.

Then the area of the polygon
$= n$ times triangle DOC.
$= n . \frac{1}{2}$ CD. OM.
$= \frac{1}{2}$ OM nCD.
$= \frac{1}{2}$ OM $\times$ perimeter of the polygon.

If the number of sides of the polygon is indefinitely increased the perimeter of the polygon tends to become the circumference and the polygon itself coincides with the circle. OM becomes ultimately the radius. Therefore the area of the circle $= \frac{1}{2} r \times$ the circumference.

$$= \tfrac{1}{2} r \times 2\pi r = \pi r^2.$$

The area may also be found by dividing the circle into a number of sectors. Cut out the sectors and place them alternately as shown in figure (246). They would be very nearly equal to a parallelogram. If the number of sectors is very large a rectangle would be formed whose two sides are the semi-circumference of the circle and the radius. (Each sector may ultimately be regarded as a triangle.) Hence **the area of a circle** $= \pi r \times r = \pi r^2$.

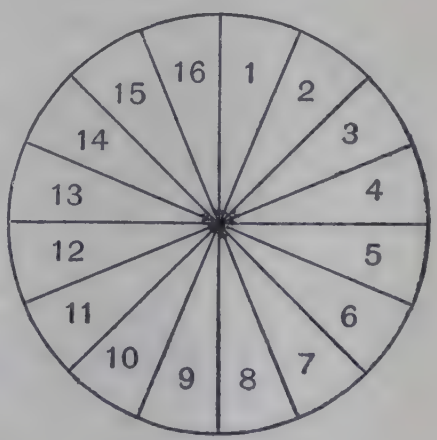


FIG. 245.

An **annulus** is the space enclosed between the circumferences of two concentric circles. Its area $= \pi R^2 - \pi r^2$ where R and r are the radii of the outer and inner circles.

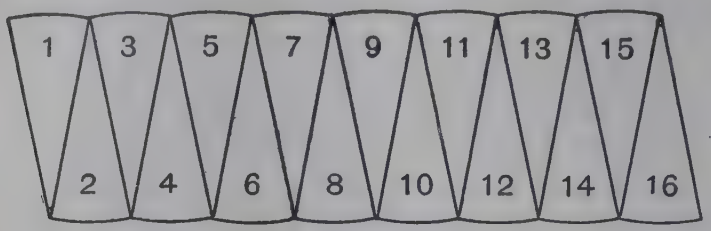


FIG. 246.

Exercise XXXII (B).

1. Describe a circle of diameter one inch on squared paper. Measure the area by counting squares neglecting those less than half and counting bigger fractions as complete units. Find the area by formula. What is the percentage of error in your measurement?

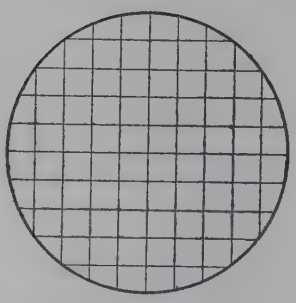


FIG. 247.

2. Find what surface is covered by water in a pond whose diameter is 82 ft.

3. If the circumference of the lake Manasarowar is 49 miles, find its area in sq. miles.

4. A square is inscribed in a circle of diameter 3·5 in. Find the area of each of the spaces outside the square.

5. A circle is inscribed in a square of side 8 in. Find the area of one of the spaces outside the circle but inside the square.

6. Four equal circles are described about the four corners of a square of side 4·75 in. so that each circle touches two of the others. Find the area of the space enclosed between the circumferences of the circles.

7. A mile race course circular in form has a width of 20 ft. on either side of the course. Find the area of the running space left for the competitors.

8. The circular reflector of a Ditmar lamp is $15\frac{1}{2}$ in. in

diameter and has a circular hole in the centre whose radius is $1\frac{1}{2}$ in. Find the area of the reflecting space.

9. A cow and a calf are tethered to a pole in a green grassy field the lengths of the ropes being 15 yds. and 10 yds. respectively. Find the surface of the grass available for the cow alone.

10. If a and b are the radii of two circles, find the radius R of the circle whose area is equal to the sum of the areas of these two circles. If $a = 12$ ft., and $b = 18$ ft., find R.

11. In the previous question, find the radius r of the circle whose area is equal to the difference of the areas of two circles. If $a = 18$ ft., $b = 24$ ft., find r.

12. A tin rectangular plate is $9\frac{1}{2}$ in. square. If circular pieces each of diameter 2 in. are to be cut out of it, find how many such pieces can be had. Find also the remaining area of the tin. Draw a square and actually show the circles that can be cut out. How do you explain the disparity between theory and practice ?

13. There is a semi-circular arch over a window 3 ft. 6 in. $\times$ 5 ft. 0 in. in a wall of a room. Find through what area light is allowed to enter the room.

14. Two men purchase a grindstone 24 in. in diameter which has a central hole of diameter 9 in. for the axle. If one man pays Rs. 7 and the other Rs. 5, how many inches of stone may the former grind before he allows the other to use it ?

15. Two shepherds buy a circular farm of diameter 300 yds., which has a circular pond in the centre of radius of 60 yds. If one of them pays Rs. 400 and the other Rs. 600 where should a circular fence be erected so that each may get the worth of his money for grazing his sheep ? The portion next the pond should be owned by the man who paid more money.

16. A class-room is in the form of a rectangle 24 ft. × 16 ft. having a semi-circular space at each end as shown in the figure. Find the area of the class-room.

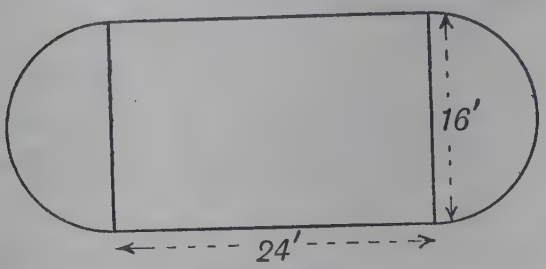


FIG. 248.

17. What is the area of the largest circle that can be cut from a rectangular sheet of metal 10 in. by 8 in.? If the circle touches three of the sides of the rectangle, what is the area of the largest square that can be cut from the remaining piece? Draw the rectangle to scale and show by dotted lines exactly where the cuts are to be made.

18. Prove the following statement:—The area of the space between two concentric circles is equal to the area of a circle which has for its diameter a chord of the outer circle which touches the inner.

19. In a circus a carriage goes round the central circular place intended for the display of feats. The outer wheel makes $15\frac{5}{7}$ revolutions and the inner 14 revolutions. The distance between the wheels is 6 ft. Find the area of the circular plot and the diameter of the wheels.

20. In cutting 4 equal circles, the largest possible, out of a piece of cardboard 12 in. square, how many square inches of cardboard have necessarily to be wasted?

21. The well of a winding stair case is 6 ft. in diameter; its height is 36 ft. The hand rail makes $2\frac{1}{4}$ revolutions. What is the length of the rail?

22. Construct an equilateral triangle of side 1·2 in. Show that a circle whose circumference is equal to the perimeter of the triangle is greater than the triangle. What are the areas of the triangle and the circle?

23. If a circle and a square have the same perimeter, which has the greater area?

24. A cow is tethered by a rope fastened to a ring which slides on a rod 7 yds. long fixed to the ground. The

length of the rope is 20 yds. Find the area in which the cow is allowed to graze. (The ring cannot slip off the rod).

25. In three equal circles of diameter 9 in., an equilateral triangle, a square and a regular hexagon are inscribed one in each. Find the area of the space between the circle and the inscribed figure in each case.

26. Two circles are inscribed in and circumscribed about a regular hexagon whose side is 3 ft. Find the space enclosed between the circles.

27. ABCD is a square of 10 ft. side. The middle points of AB, BC, CD and DA are E, F, G and H. Join EG and HF. K is the middle point of BF, and L that of GC. KL is joined to cut HF in N. EM = NF = GO = HP. Semi-circles are described on PM, MN, NO and OP as diameters. Find the area of the space enclosed between the square and the curved figure PMNO.

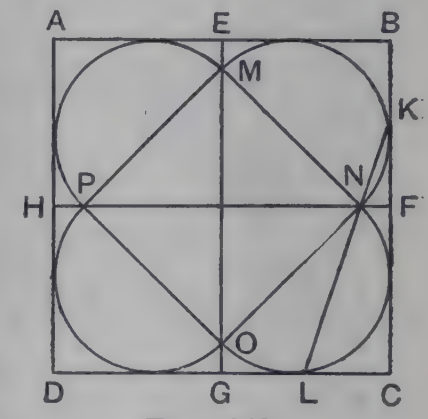


FIG. 249.

28. ABCD is a square of side 8 in. Join AC and BD intersecting at O. With A, B, C and D as centres and AO, BO, CO and DO as radii describe quadrants of circles as shown in the figure (250). Find the area of the hatchured portion.

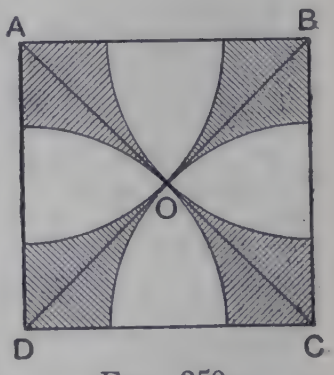


FIG. 250.

29. A cardboard in the form of a semi-circle of radius 4·2 in. is put in one scalepan of a balance. In the other scalepan, 4 squares of side 2·1 in. of the same cardboard are put. What fraction of a square of side 4·2 in. should be cut and inserted into the lighter scalepan so that the scales may counter-balance?

30. Mention two lines that are in the ratio of $\pi : 1$; mention two areas that are in the ratio of $\pi : 1$.

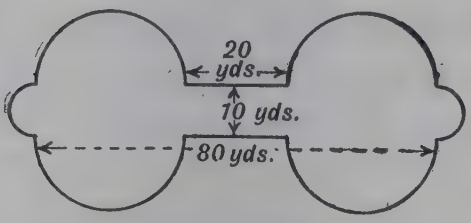


FIG. 251.

31. The figure (251), not drawn to scale, represents the ground plan of a bungalow consisting of a rectangular hall with 6 semi-circular rooms (as shown in the figure). Find the area of the ground.

32. A circle of diameter d inches, has a radius of r inches and a circumference of C inches and an area of A sq. inches.

(*a*) Express A in terms of C alone.
(*b*) ,, ,, ,, d ,,
(*c*) ,, ,, ,, r ,,
(*d*) ,, ,, ,, C and r.
(*e*) ,, ,, ,, d and r.
(*f*) ,, C ,, A and r.
(*g*) ,, ,, ,, A alone.
(*h*) ,, r ,, A alone.
(*i*) ,, d ,, A alone.
(*j*) ,, r ,, C alone.
(*k*) ,, r ,, A and C alone.

33. You know that, if a, b and c are the sides of a right-angled triangle, (c being the hypotenuse) $c^2 = a^2 + b^2$. Multiply every term in this relation by $\frac{\pi}{4}$. Then you get $\frac{\pi c^2}{4} = \frac{\pi a^2}{4} + \frac{\pi b^2}{4}$. You also know if d represents the diameter of a circle, $\pi\frac{d^2}{4}$ represents its area. Similarly what meaning can you give to $\frac{\pi c^2}{4}$, $\frac{\pi a^2}{4}$ and $\frac{\pi b^2}{4}$ and therefore what is the geometrical interpretation of the relation $\frac{\pi c^2}{4} = \frac{\pi a^2}{4} + \frac{\pi b^2}{4}$?

34. The circular ground of a circus has a circular arena in the centre. The area intended for spectators is $\frac{4}{11}$ of an acre. The distance between the entrance into the tent and the entrance into the arena is 22 yds. Find the area of the arena.

35. The minute and hour hands of a tower clock are 8 in. and 6 in. What is the difference in the areas traced out by the hands in 24 hrs. ?

36. Find the area of a cross section of the following (of the space occupied by iron) in each case :—

(*a*) An iron girder whose cross section is shown in figure 252.

(*b*) An iron pipe whose cross section is shown in figure 253.

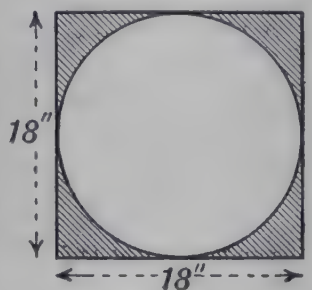


FIG. 252.

173. Areas of Sectors and Segments.

Let ABCDEF be a circle of which O is the centre. Take equal arcs AB, BC, CD, DE and EF. Then the angles AOB, BOC, COD, DOE and EOF are equal. These sectors cut out of the same circle and having equal angles may be made to coincide. Therefore they are equal in area. Thus it follows that when the angle of a sector is doubled, trebled, its area also gets doubled, trebled.

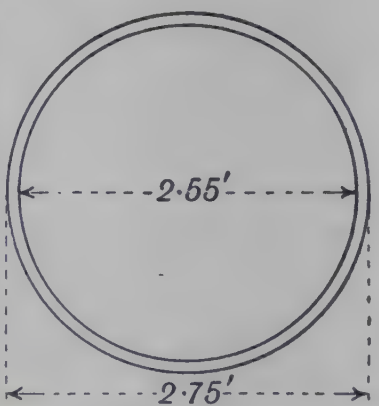


FIG. 253.

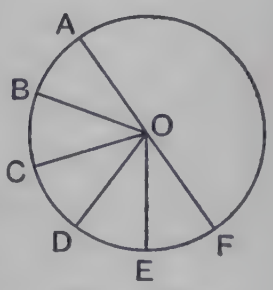


FIG. 253*a*.

$$\therefore \frac{\text{the area of sector AOB}}{\text{the area of sector AOC}} = \frac{\angle \text{AOB}}{\angle \text{AOC}}.$$

The whole circle may be regarded as a sector in which the two bounding radii coincide and the angle between the radii is 360°.

$$\therefore \frac{\text{area of sector AOB}}{\text{area of the circle ABCDA}} = \frac{\angle \text{AOB}}{360°} = \frac{n°}{360°} \text{ if } \angle \text{AOB contains } n°.$$

$\therefore$ area of sector $= \frac{n}{360^\circ} \times$ the area of the circle ABCD.

$= \frac{n}{360} \times \pi r^2$ ..... (1).

$= \frac{\text{length of the arc of the Sector}}{\text{the whole circumference}} \times \pi r^2$.

($\because$ arcs of sectors are proportional to the angles subtended by them at the centre).

$= \frac{l}{2\pi r} \times \pi r^2$.

$= \frac{lr}{2}$ ..... (2).

174. Area of a Segment.

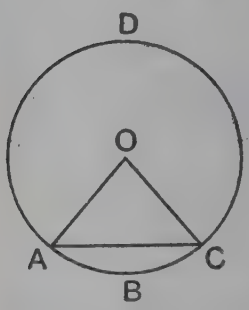


FIG. 254.

Area of the segment ABC = Area of the sector OABC — the area of the triangle OAC.

Area of the segment ADC = area of the sector OADC + triangle OAC.

Model No. (1) In a circle of radius 1 ft. 3 in., find the area of a sector whose angle is 84°. ($\pi = \frac{22}{7}$.)

Area of sector $= \frac{n}{360} \times \pi r^2$.

$= \frac{84}{360} \times \frac{22}{7} \times \left(\frac{5}{4}\right)^2$.

$= \frac{55}{48}$ sq. ft.

$= 1$ sq. ft. 21 sq. in.

ABC is **a minor segment** because it is less than a semicircle ADC is **a major segment** because it is greater than a semicircle.

Model No. (2) The radius of a circle is 9 ft. and the angle of the sector is 50°. Find the area of the minor segment ($\pi = \frac{22}{7}$).

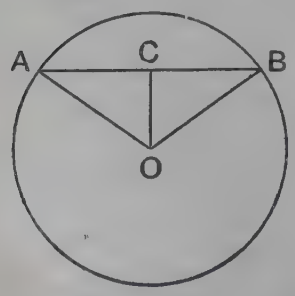


FIG. 255.

Area of the segment = sector OAB — triangle OAB.

$= \frac{50}{360} \times \frac{22}{7} \times 81 - \frac{1}{2} \times 7{\cdot}5 \times 8{\cdot}2$.

$= 35{\cdot}42 - 30{\cdot}75$.

$= 4{\cdot}67$ sq. ft.

N.B.—AB by measurement of a figure drawn to scale is 7·5 ft. and BC 8·2 ft. (this figure is not drawn to scale).

Exercise XXXII (C).

1. In a circle of radius 15 in., find the area of a sector whose angle measures 63°.

2. In a circle of radius 6 ft. 8 in., find the area of a sector whose angle measures $67\frac{1}{2}°$.

3. The area of a sector of radius 20 ft. is 60 sq. ft. Find the angle of the sector.

4. The area of a sector of radius 8 yds. is 25·6 sq. yds. Find the angle of the sector.

5. The area of a sector is 80 sq. in. The angle of the sector is 77°. Find the radius of the sector.

6. The area of a sector is 32 sq. yds. and its angle is 60° ; find the radius of the sector.

7. Find the area of a sector whose radius measures 4 yds. 2 ft. and whose arc 3 yds. 1 ft.

8. Find the length of the arc of a sector whose area is 27 sq. ft. and radius 9 ft.

9. Find the length of the arc of a sector whose area measures 5 sq. ft. 36 sq. in. and radius 3 yds. and 1 ft.

10. The area of a sector measures 6 sq. ft. 10 sq. in. and the length of its arc 6 ft. 9 in. Find its radius.

11. The chord of a sector is 8 in. ; the radius 6 in. ; find the area.

12. The area of a sector is 360 sq. ft. The area of the circle 1,440 sq. ft. Find the length of the arc ($\pi = 3{\cdot}14159$).

13. The area of a circle is a sq. mile. Find the area of a sector whose angle measures 81°.

14. The radius of a circle is 20 in. and the angle of the sector is 60°. Find the area of the segment.

15. Find the area of the segment of a circle of radius of 5 ft., if the chord subtends an angle of 135° at the centre.

16. Find the area of a segment of a circle whose chord measures 4 ft. and subtends an angle of 96°.

17. Find the area of the major segment cut off by a chord equal to the radius of the circle which is 5 ft.

18. The span of a circular arch of 80° is 160 ft. Find the area of the segment.

19. A bob is suspended by a thread. The distance of the centre of the bob from the point of suspension is 80 cm. The bob is drawn to one side making an angle of 15° with its original position. Find the distance traversed by the centre of the bob (*a*) in a vibration and (*b*) in an oscillation. What area does it sweep out (*a*) in a vibration and (*b*) in an oscillation?

20. The lengths of the hour and minute hands of a clock are 9 in. and $10\frac{1}{2}$ in. Find the difference of the areas of the sectors swept out by the hands between 10.45 a.m. and 11.12 a.m.

21. A tank bund is a circular arc subtending an angle of 36° at the centre. The radius of the curve is 960 yds. Find the length of the bund.

22. The cradle of a child is rocked to and fro being suspended from the ceiling by chains each 8 ft. long. Ten complete oscillations with a maximum extent of swing are enough to induce sleep. If it swings through an angle of 60° to the utmost extent what distance did it traverse before the child went to sleep?

23. A boy holds two rings in his hands suspended from a horizontal bar in a gymnasium field by ropes of equal length, viz., 8 ft. His maximum swing is through an angle of 60° on either side of the vertical. What more distance is traversed in a swing by his foot than the rings if the distance between the rings and his feet while in swing is 7 ft.? Find also the difference in the areas swept out by the rings and feet in a swing.

24. The following table gives the area in sq. miles of the various oceans :—

Name.	Area in sq. miles correct to the nearest million.
Atlantic.	35,000,000
Pacific.	65,000,000
Indian.	25,000,000
Arctic.	5,000,000
Antarctic.	5,000,000

Draw a circle dividing it into sectors to represent the areas of the several oceans according to scale.

25. The following table gives the area in sq. miles of the various continents :—

Name.	Sq. miles.
Europe.	4,000,000
Asia.	17,000,000
Africa.	12,000,000
N. America.	8,000,000
S. America.	7,000,000
Australia.	3,000,000

Draw a circle dividing it into sectors to represent the areas of the several continents according to scale.

26. Figure (256) shows the world's coal wealth in metric tons which is 475,000 and the several sectors indicate the portions produced by the several continents marked in it. Find the coal production of each continent.

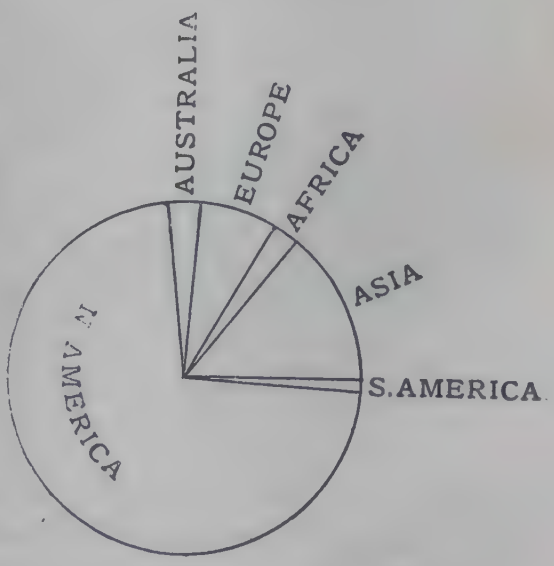


FIG. 256.

27. In a circular room, a stick 20 ft. long placed on the ground with its extremities against the wall subtends an angle of 120° at the centre. Find the area of the smaller-segment cut off by the stack.

28. The centres of two circles are 8 cm. apart and their radii are 6 and 4 cm. Find the area of the common portion. (Fig. 257).

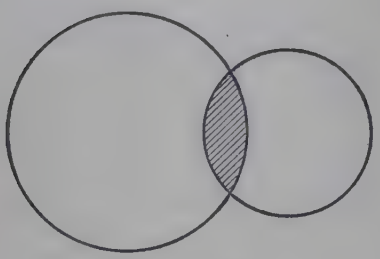


FIG. 257.

29. Two circles are described on the bounding radii, of a quadrant of a circle of radius 8 ft., as diameters. Find the area of the figure common to both circles $\pi = 3{\cdot}14159$. (Fig. 258).

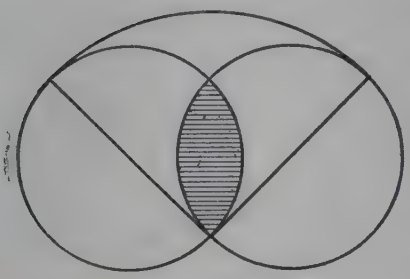


FIG. 258.

30. Three equal circles of diameter $1\frac{1}{2}$ ft. touch one another as shown in the figure. Find the area enclosed by the three circles. (Fig. 259).

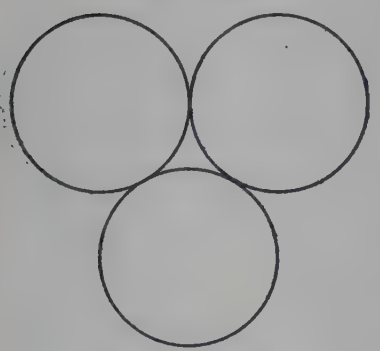


FIG. 259.

31. ACD is a circle whose radius is 10 ft. CDE is a circle passing through the centre of the circle ADC and having a radius of 20 ft. Find the area of the three portions divided by the curves. (Fig. 260).

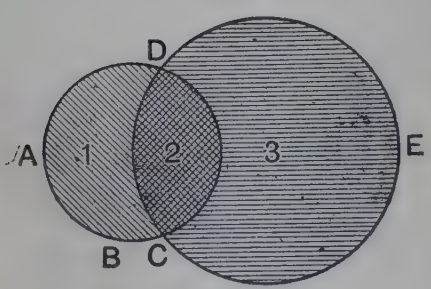


FIG. 260.

32. The longitudes of two places on the equator are W. 80° and E. 20°. What is the magnitude of the angle subtended by the arc joining these two places on the equator at the centre of the earth? Hence find the distance between the two places if the equatorial diameter is 7,900 miles.

33. In the previous question, the longitudes of the two places selected are both East 80° and 25°. How will your answer be affected?

34. The figure represents an actual semi-circular

protractors used for measuring angles. Find the area of the tin used for making it. (Fig. 261).

35. The radius of a circle is 20 ft. Two parallel chords are drawn on the same side of the centre, one subtending an angle of 60° and the other 100°. Find the area of the zone enclosed between the chords.

36. The diameter of a circle is 50 yds. Two parallel chords in it drawn on the same side of the centre are 45 yds. and 35 yds. Find the area of the zone formed.

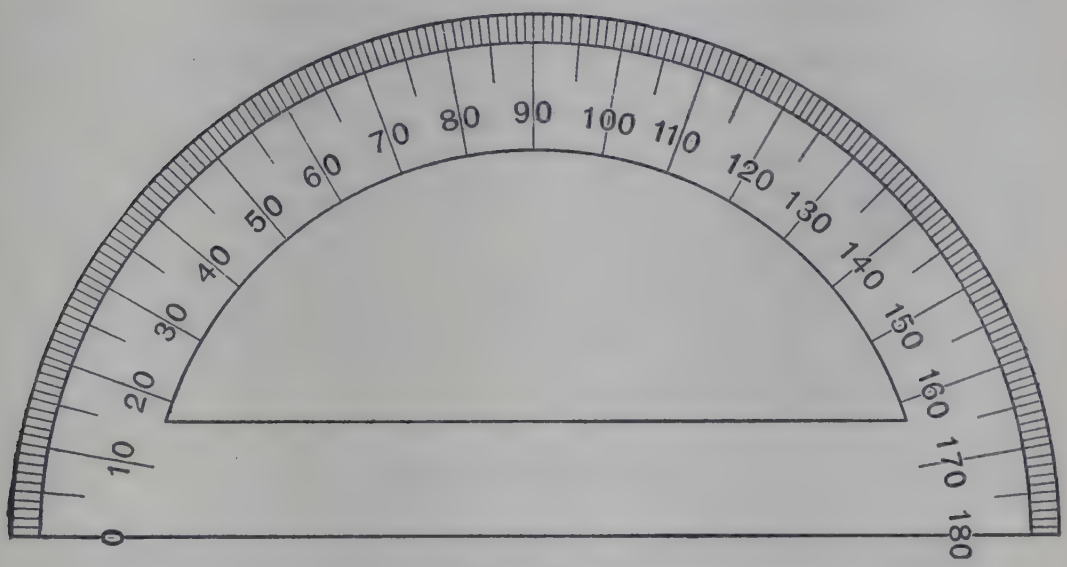


FIG. 261.

37. The height of an arc is 10 ft. The diameter of the circle of which it is an arc is 40 ft. Find the chord of the arc.

38. The chord of an arc is 8 ch. 40 links; the diameter of the circle 10 ch. 80 links. Find the height of the arc.

39. The span of an arch over a doorway is 4 ft. 6 in. The height of the arch is $7\frac{1}{2}$ in. Find the radius of the circle.

40. The span of a road bridge arch is 37 ft. and the height of the arch is 13 ft. Find the radius of the arch.

41. If the span of an arch is 40 ft. and if it subtend at the centre of the arch an angle of 90°, find the height of the arch.

42. The arch of a bridge is of 40 ft. span and 10 ft. rise. If the thickness of the arch is 2 ft. 3 in. and if the width of the bridge is 30 ft., find what volume of stone is required to complete the arch.

43. The Madras lighthouse is 150 ft. high. Taking the radius of the earth to be 4,000 miles, find to what distance all round it the light of the lighthouse is visible.

44. The height of a tower is 75 ft. What is the greatest distance from which it is visible, the radius of the earth being 4,000 miles ?

45. A person ascends in a balloon a height of 1 mile. What is the diameter of the horizon he will be able to see if the radius of the earth is 4,000 miles ?

46. A person up in a balloon sees a horizon which has a radius of 20 miles. Find at what height he is, the radius of the earth being 4,000 miles.

47. A person having his eye 15 ft. above the ground level is able to look round to a distance of 4·75 miles. Find the radius of the earth.

CHAPTER XXXIII.

Rectangular Solids. The Cube and Cuboid (Revision).

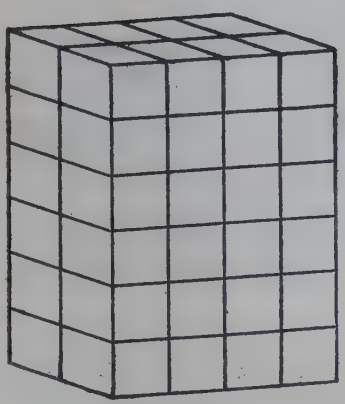


Fig. 262.

175. The figure contains 6 layers. Each layer contains 2 rows, each row contains 4 in. cubes. Then the whole rectangular solid contains $6 \times 2 \times 4$ or 48 cubic inches.

This figure is called a cuboid because the dimensions are unequal. If the length of a cuboid is l units, breadth b units and height h units, then volume $= lbh$ cub. units.

Hence if the volume is denoted by V cub in.

$$V = lbh \ldots\ldots (1).$$

or V = area of the base (lb) $\times$ height (h).

If the length, breadth and height are equal, then the figure is called a cube and its volume. $V = l \times l \times l = l^3$ **.....(2).**

In the case of a cube each face is a square, but in the case of a cuboid each face is a rectangle.

The area of the six faces of a cube or its surface is $6l^2$ **where** l **is the length of a side of the cube.**

The area of the six faces of a cuboid or its surface $= 2(lb + bh + lh)$ **where** l, b **and** h **are its sides.**

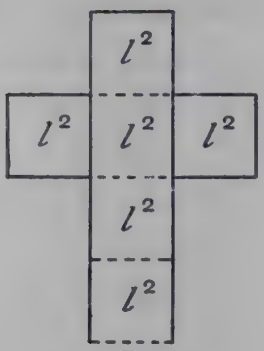


FIG. 263.

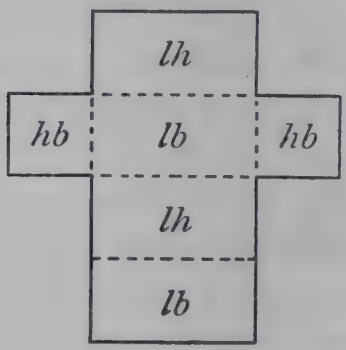


FIG. 264.

Exercise XXXIII.

1. Find the cubical contents of the following :—

(*a*) A wall forming the boundary of a rail road 66 yds. long, 10 ft. high and 2 ft. 3 in. thick.

(*b*) A log of wood 1 ft. 9 in. square and 25 ft. 6 in. long.

(*c*) A compound wall 80 ft. long, 1 ft. $1\frac{1}{2}$ in. thick and 7 ft. 6 in. high.

(*d*) A box whose internal dimensions are $2' 4'' \times 1' 6'' \times 12'$.

(*e*) A brick of the dimensions $9'' \times 4\frac{1}{2}'' \times 3''$.

(*f*) A kerosene tin-can of the dimensions $9'' \times 9'' \times 4''$.

2. To have 100 c. ft. of finished wall 1,400 bricks of the size $9'' \times 4\frac{1}{2}'' \times 3''$ are required. Find how much volume is taken up by mortar.

3. Find how many cubic feet of air there are in a room whose internal dimensions are 18′ 6″, 12′ 9″, 10′ 6″.

4. A water tank is 30′ × 30′ × 8′. If the water it contains is sufficient for a day's consumption for a hostel accommodating 400 students, find at how many gallons per head per day the supply works out. (1 cub. ft. of water equals $6\frac{1}{4}$ gallons.)

5. On a certain day there was a rainfall of $3\frac{1}{2}$ in. over a field 880 yds. by 660 yds. Find the quantity of rainfall on the field.

6. A beam 15″ × 12″ rests on two pillars 16 ft. apart. If each of the pillars bears the beam for a length of 9 in. find how much weight comes on each pillar. (1 cub. ft. of material weighs 50 lbs.)

7. A chemical laboratory 95 ft. $4\frac{1}{2}$ in. by 37 ft. 6 in. by 18 ft. (internal dimensions) is intended for 70 students. Find how many cub. ft. of air is allowed for one student.

8. A road 40 ft. wide is paved with asphalt 2 in. thick for a length of 100 yds. Find what volume of asphalt was required.

9. Find the volume of a rectangular solid the areas of whose adjacent faces are xy sq. in. yz sq. in. and xz sq. in. Find the volume if $xy = 12$, $yz = 15$ and $xz = 20$.

10. From the formula (*a*) $V = lbh$, find l, b and h in terms of the other quantities and (*b*) $V = A \times h$, find A and h in terms of the other quantities.

11. A tank is 20 ft. 6 in. long and 18 ft. 9 in. broad. It has a discharge pipe which discharges at the rate of 1 cub. ft. per minute. If the pipe is open for 20 min. 35·65 sec. find by how many inches the level of the water has fallen.

12. 562·5 lbs. of gold are beaten out into a sheet 56′ × 4″. Find the thickness of the sheet in decimals of an inch specific gravity of gold being 19·3. Also 1 cub. ft. of water weighs $62\frac{1}{2}$ lbs.

13. A brick tank is 10′ × 8′. Water stands to a height of 3 ft. 6 in. in it. If 125 counting bricks $8\frac{1}{2}$″ × $3\frac{3}{4}$″ × $1\frac{1}{8}$″

are soaked in it and the bricks absorb $\frac{1}{10}$ of their own volume of water after 24 hrs., find by how much the surface has risen ?

14. A teak box 2′ 6″ × 1′ 8″ × 9″ is made of wood ·38 of an inch thick. Find the inner volume of the box (*a*) when there is a lid and (*b*) when there is no lid.

15. Find the size of a cube of concrete block which weighs a ton. Concrete weighs 125 lbs. to the cubic foot.

16. The external dimensions of a hall are 25 ft. $4\frac{1}{2}$ in. and 18 ft. $7\frac{1}{2}$ in. If the thickness of the wall all round is 1 ft. 6 in., find the internal volume of the room. The height of the room is 16 ft.

17. A rectangular box $p'' \times q'' \times r''$ is made of cardboard m in. thick. If it has a lid, find the volume of the cardboard required.

18. A trench is 3 ft. 6 in. broad and 5 ft. deep. Find what length of trench has to be dug if 1,500 cub. ft. of earth are required for an embankment nearby.

19. Find the volume of brickwork and concrete in a compound wall 75 ft. long whose cross section is given in Fig. 265.

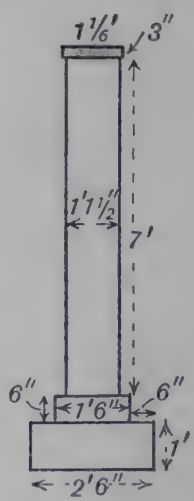


FIG. 265.

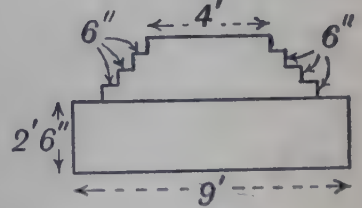


FIG. 266.

20. The foundation of a pedestal consists of a rectangular block of concrete 9′ × 9′ × $2\frac{1}{2}$′. The pedestal proper is 4′ × 4′ and 8′ high. At the bottom it has three footings each projecting 6 in. before the one immediately above it and each footing is 6 in. high (as in Fig. 266). Find the volume of the concrete and also the volume of brickwork.

21. The mean depth of a river at a particular cross-

section is 7 ft. $5\frac{1}{2}$ in. If the river is 3,360 ft. wide at that section and water flows at the rate of 3 miles an hour, find what volume of water passes that cross-section in an hour.

22. A cistern 4 ft. 8 in. by 3 ft. 6 in. by 2 ft. 3 in. has 28·15 cub. in. of water in it. Bricks which measure $9'' \times 4\frac{1}{2}'' \times 3''$ and absorb $\frac{1}{10}$ of their volume of water are put in the cistern. Determine how many of them can be put in so that not a drop of water may overflow.

23. Find the weight of air in a room $26' \times 18'\ 6'' \times 12\frac{1}{6}'$ if one cubic foot of water weighs 62·32 lbs. and if water is 100 times as heavy as air.

24. The amount of silt carried by the Krishna river annually amounts to 4,221 million cub. ft. Find over what area this can be spread if the average height of the silt is 1 ft. $5\frac{1}{2}$ in.

25. Find the cost of varnishing the outside of a deal-wood box $4'\ 6'' \times 3'\ 8'' \times 1'\ 9''$ made of material $\frac{1}{2}$ in. thick at 0-0-6 per sq. ft. If it is lined inside with lead of $\frac{1}{16}$ in. thickness, find what weight of lead is required. (Weight of lead being 632·5 lbs. per cubic foot.)

26. A rectangular tank is 220 yds. by 180 yds. Around the tank on all the four sides a road is made. Find how many cubic feet of broken stone would be required if before consolidation there should be a depth of 8 in. Assume the width of the road to be 30 ft.

27. Around a plot of ground which is $100' \times 120'$ outside a trench 6′ wide is dug. What should be its depth if the earth dug out is spread over the plot in order that the plot may be raised by $1\frac{1}{6}$ in. ?

28. A flower-bed is of the shape shown in Fig. 267. If all round it there is to be a path 2 ft. wide, find the cost of gravelling it to a depth of 6 in. at 3 as. a cub. ft.

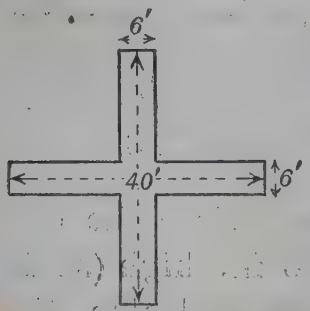


FIG. 267.

29. A tank has a catchment area of $3\frac{3}{4}$ sq. miles $2\frac{1}{2}$ in. of rainfall on the area raised the level of the water in the tank by $1\frac{1}{2}$ ft. Find the average area of the tank.

30. If the surface of a cube is 530·16 sq. in., find the length of an edge and that of the longest diagonal of the cube.

31. A tank 24 ft. long, 18 ft. broad and 12 ft. 6 in. deep is full of water. If 3254·82 cub. ft. are drawn off, find what depth of water remains in the tank.

32. A room $32' \times 26' \times 18'$ has a ventilator $3' \times 2'$ which empties out the hot air in the room. If $\frac{1}{4}$ of the whole amount of air is emptied in an hour find at what rate air is passing out through the ventilator.

33. A log of wood is 18 in. wide, 13 in. thick and 22 ft. long. How many sq. ft. of surface would there be altogether if the log were divided into planks 1 inch thick and if there were no loss of material in sawing.

34. Fig. 268 is the plan of trenching for the main walls of a building. If the trench is 3 ft. 6 in. wide and 4 ft. 6 in. deep, find the amount of the earth work to be excavated.

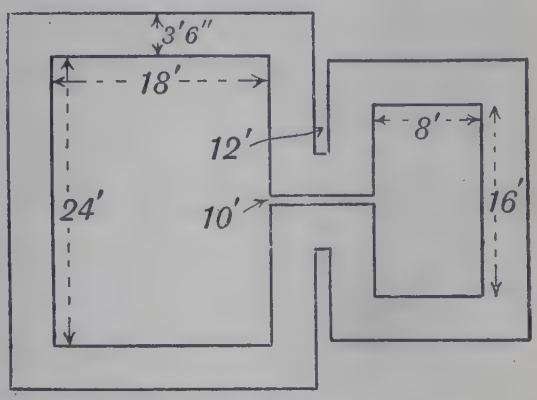


FIG. 268.

35. Find what amount of teak would be required to make the door frame in Fig. 269? Find what amount of wood there is in the frame.

36. Find the volume of material in the following steel box girder 20 ft. long. (Fig. 270). Neglect the curvature in the angle plates.

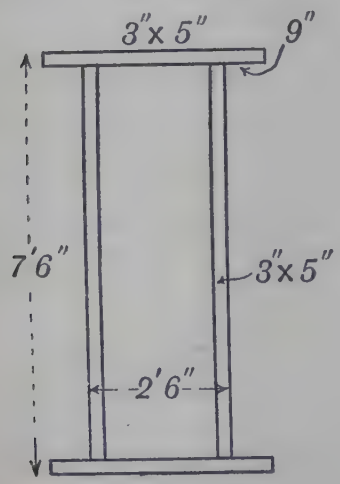


FIG. 269.

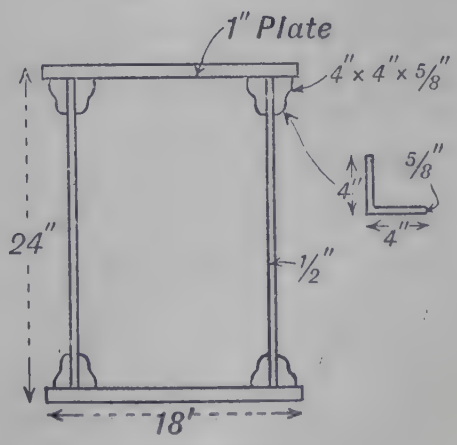


FIG. 270

37. Fig. 271 gives the section of a cast iron girder. Find what amount of material there is in a length of 18 ft. 6 in. Find also the weight of the girder if the specific gravity of iron is 7·4 and water weighs 62·5 lbs. per cubic foot.

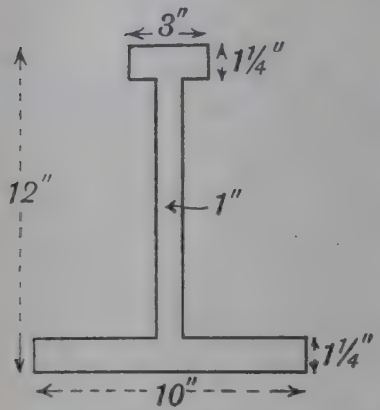


FIG. 271.

38. Find the amount of wood in the cross projecting of a wall in Figure 272.

39. Two rectangular vessels one of which has dimensions 18″ × 12″ × 3′ 6″ and the other 12″ × 9″ are connected by means of a small pipe which has a stop cock in the middle. If the first vessel is full of water to start with, the other empty and if the stop cock is opened, find what the height of the second vessel should be so that it may be full after water has found its level.

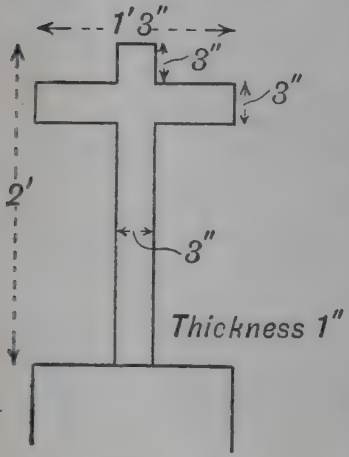


FIG. 272.

40. A class-room is 36′ × 28′ × 18′. If an area of 8 sq. ft. is allowed for each student, find how students can be seated in the room and how many cubic foot of air is allowed for each student.

41. A lake is 10 square miles in area and is on an average 25 ft. in depth. If water cannot be taken from a depth of 15 ft. from the bottom of the lake for a healthy supply of water, find for how long a time the supply of the lake would last for 50 lakhs of population at the rate of 30 gallons per head per day.

42. A cistern 2′ 6″ by 3′ 8″ by 1′ 9″ is partly filled with water. A rectangular block of iron 18″ × 20″ × 12″ is placed in it. If 28 cub. in. of water flows over, find to what depth there was water in the cistern.

43. A rectangular trough is 2 ft. 6 in. long, 1 ft. 8 in. broad and 7 in. deep. Find the cost of lining it internally with lead at Re. 1 per sq. ft.

44. A steel tank $8' \times 6' \times 3\frac{1}{6}'$ is tarred internally. Find the cost of tarring it at 5 as. per sq. ft.

45. Find the volume of a cuboid whose sides are

(*a*) $a^2 + b^2 + ab$, $a - b$ and $a^3 + b^3$ units.

(*b*) $x - y$, $x + y$ and $x^2 + y^2$.

(*c*) $x^2 + 4x + 3$, $x^2 + 6x + 8$ and $x^2 - 3x + 2$.

46. The sides of an iron cuboid are $x^2 + (p + q)x + pq$, $x^2 + 2px + p^2$ and $x^2 + 2qx + q^2$. If the cuboid were melted and made into a cube what would be a side of the cube?

47. A rectangular vessel is p in. long, q in. broad, and r in. deep. It is filled with water by pouring n potfuls. On a previous occasion, the same pot was used and x potfuls were poured into another vessel and the water was only, y ft. deep in it. Find the area of the bottom of the second vessel.

48. A copper rod is 18 in. long and has a cross-section of $2'' \times 3''$. Another rod is 2 ft. long and a cross-section of $5'' \times 2\frac{1}{2}''$. If both are melted and beaten into a rod, having a cross-section of $1\frac{1}{2}'' \times 3''$, find the length of the new rod.

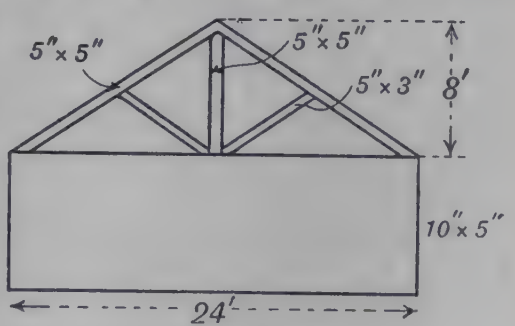


FIG. 273.

49. Fig. 273 is a representation of a king post truss. Calculate what amount of iron would be required to construct the truss.

50. A box is $6' \times 4' \times 1\frac{1}{2}'$. It is proposed to fill it with an exact number of copies of a book. The dimensions of a book are $9'' \times 6'' \times 4''$. How many books can be put into the box. Will there be any empty space? If so what is the volume of the empty space?

51. A box has a square bottom each side being 40 in.

inside measurement. How many books each 8 in. by $5\frac{1}{2}$ in by $2\frac{1}{2}$ in. can be laid on the bottom if they are laid flat and do not stand on edge. How many books can you put into the box if they are arranged in such layers and the box is 25 in. deep. How many books could be put into it if they stood on edge?

52. Out of a mass of iron 40 in. by 35 in. by 25 in. how many blocks each $7\frac{1}{2}$ in. by $3\frac{1}{2}$ in. by $1\frac{1}{4}$ in. can be cast?

53. Out of a block of assafoetida 25 in. by 20 in. by 16 in. how many small chips each 1·5 in. by $1\frac{1}{4}$ in. by $\frac{3}{8}$ in. can be cut?

54. The cost of a silver cube of side 3·56 in. is Rs. 108. Find the cost of a silver cube of side 4·45 in. If the cost of gilding the surface of the former is Rs. 21, find the cost of gilding the surface of the latter.

55. A sheet of paper is 13 in. long and 8 in. wide. Find the edge of the biggest cube you could make from the sheet. How much of the sheet will remain after the cuttings for the cube are made?

56. Water flows in a canal the cross section of which is shown in Fig. 274 at the rate of 4 miles an hour. Find the quantity of water flowing through it in a day.

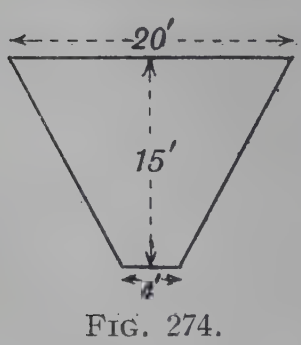


FIG. 274.

57. Find the number of bricks required for building a wall 40 ft. 3 in. long 10 ft. 2 in. high and $1\frac{1}{2}$ ft. thick each brick measuring $9'' \times 8'' \times 4''$.

58. A tank 12 ft. long 4 ft. wide and $1\frac{1}{2}$ ft. deep is filled with pulp for making paper. How many sheets each 4 ft. long 3 ft. wide, and $\frac{1}{2\,7\,5}$ in. thick can be formed from the pulp supposing that $\frac{1}{5}$ of the pulp is lost in drying it.

59. The external length, breadth and height of a wooden box are a, b and c in. respectively and the thickness of the wood is d inches. When the box is empty i

weighs e lbs. and when filled with sand f lbs. Compare the weights of equal bulks of sand and wood. Evaluate your answer when $a = 19{\cdot}5$, $b = 15{\cdot}5$, $c = 7{\cdot}5$, $d = 1$, $e = 40$ and $f = 150$.

60. The external dimensions of a closed cistern are 14 ft., 12 ft. and 10 ft. and the thickness of the material is an inch. If when the vessel is empty it weighs 205 oz., what does it weigh when full of water, a cubic foot of water weighing 1,000 oz. ?

61. A closed vessel formed of metal 1 in. thick, whose external dimensions are 7 ft. 3 in., 6 ft. 5 in. and 3 ft. 3 in. weighs 360 lbs. What would be the weight of a solid mass of metal of the same dimensions ?

62. The external length, breadth and depth of a rectangular closed tin vessel are 14, 10 and 9 in. respectively and the thickness of the tin half an inch. When the vessel is empty it weighs 1500 oz. and when filled with water 2041·6 oz. Find the weight of a cubic foot of water.

63. A room 21 ft. long by $13\frac{1}{2}$ ft. wide is surrounded by walls $1\frac{1}{2}$ ft. thick and 14 ft. high. There are two doors each $4\frac{1}{2}$ ft. by 6 ft. and one window 3 ft. by $4\frac{1}{2}$ ft. Find (*a*) the cost of building the wall at the rate of Rs. 10-2-0 per cubic yard and (*b*) the number of bricks each measuring $9' \times 4' \times 1\frac{1}{4}$ in. required for the work.

64. Take 5 inch cubes ; put them together in a row. Put 5 such rows together ; you get a square containing 25 cubes. Put 4 more layers on this bottom layer. Then the whole block represents 5^3. Separate this into 8 blocks — 1 block representing 3^3, another 2^3, 3 blocks each representing a cuboid $3^2 \times 2$ and another 3 blocks each representing a cuboid 3×2^2. Hence infer :—

that 5^3 or $(3+2)^3 = 3^3 + 2^3. + 3{\cdot}3^2.\ 2 + 3{\cdot}3{\cdot}2^2$.

What formula is this in Algebra ?

65. A teak wood drawer of an almirah is 6 ft. long 2 ft. 6 in. wide and $1\frac{1}{2}$ ft. deep inside. How many measures of rice will it hold ? One measure occupies 80 cub. in.

66. A petrol tin is a cuboid $12'' \times 7{\cdot}7'' \times 6''$. It contains 2 gallons. Find the number of cubic inches in a gallon. A U.S.A. Kerosene tin with the monkey brand is $15'' \times 10'' \times 40''$. Find how many gallons it holds.

67. In the formula $(x+y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$, put $x = a - b$ and $y = b$ and hence deduce the formula $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$.

68. From a cuboid $12' \times 10' \times 8'$ it is required to cut off the greatest cube. Draw a plan of the remaining portion. How will you find its volume from the shape it has taken. Check your result by finding the difference of the cuboid and the cube.

69. There are two cuboids whose dimensions are $12'' \times 12'' \times 13''$ and $5'' \times 5'' \times 3''$. Find a cuboid having a square base which is as big as the two cuboids put together.

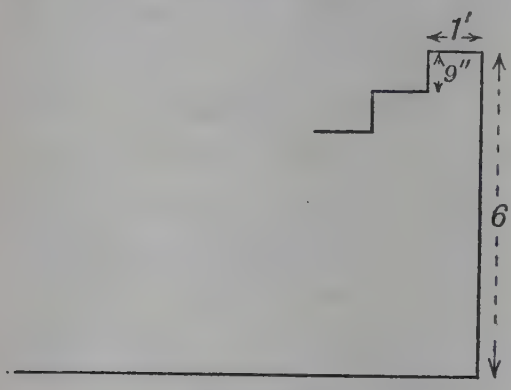


FIG. 275.

70. There is a room under the steps leading to the floor of a terraced building. The tread of each step is 1 ft. The rise of each step is 9 in. (Fig. 275). Find the cubic contents of the room if the width of the steps is 6 ft.

CHAPTER XXXIV.

RIGHT PRISM, CYLINDER AND CONE.

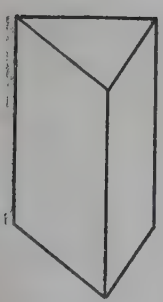


FIG. 276.

176. A prism is a solid body whose side faces are parallelograms and whose end faces are equal rectilineal figures lying in parallel planes. The end faces * are parallel as in Figs. 276, 277 and 278. These are called the ends and one of them on which the prism may be supposed to stand is called the base of

* In Fig. 276 one end face alone has been shown.

the prism. These ends may be triangles in shape as in Fig. 276 or quadrilaterals as in Fig. 277 or any polygons as in Fig. 278. **The perpendicular distance between the ends of a prism is called its height.**

FIG. 277.

A prism is called a right prism when its ends are perpendicular to the common edges of two adjacent faces. In all right prisms the lateral faces are rectangular. Prisms take their names from the shape of their ends; they are triangular, rectangular, pentagonal or hexagonal prisms according as their bases are triangles, rectangles, pentagons or hexagons.

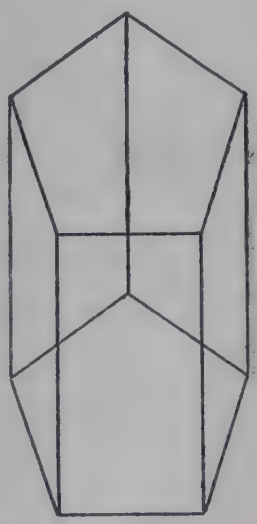


FIG. 278.

177. The volume of a Right Prism.

Method (1) The base triangle may be reduced to an equivalent rectangle; or the base polygon may be reduced to an equivalent triangle and then to an equivalent rectangle, but a rectangular base need not be changed. Similarly a right prism standing on a triangle or a parallelogram or any polygon can be reduced to an equivalent right prism standing on a rectangle (height remaining the same as before).

∴ the volume of a right prism = the area of the base × height.

Method (2) Let the area of each end be A square units and h the altitude of the prism. Take a plane D′ E′ F′ parallel to the end DEF at a distance DD′ equal to 1 unit (Fig. 279). The volume of the prism bounded by two ends DEF and D′ E′ F′ = A unit cubes, for on each unit area of DEF there is a cube bounded by the two planes DEF and D′ E′ F′. AD = h units. Therefore AD may be divided into h units and through these points of division $h-1$ planes parallel to the ends may be taken.

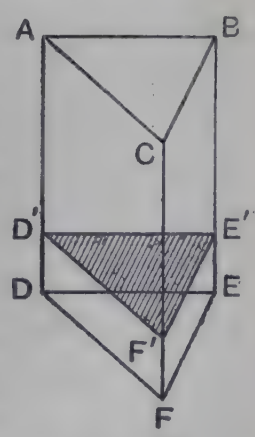


FIG. 279.

∴ **the volume of the whole prism** = the volume of h prisms each equal to A unit cubes.

= **Ah.**

The lateral area of the prism $= (a + b + c + d + \ldots)h$ where a, b, c, d . . . are the sides of the base polygo and h the height.

$=$ **perimeter of the prism** $\times$ **h.**

$\therefore$ total surface area = perimeter $\times$ height + twic the area of the base.

Model No. (1) The base of a prism is an equilatera triangle of side 4 inches. The height of the prism is inches. Find its total surface area and volume.

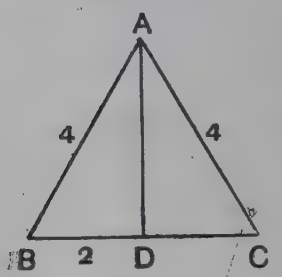


FIG. 280.

ABC is the base. From A draw AD perpendicular to BC.

Then BD $= 2$ in. $\therefore AD^2 = AB^2 - BD^2 = 4^2 - 2^2 = 12$.

$\therefore AD = \sqrt{12} = 2\sqrt{3}$.

$\therefore$ triangle $ABC = \frac{1}{2} \times 4 \times 2\sqrt{3} = 4\sqrt{3}$ sq. in.

The perimeter of the triangle $= 12$ in.

$\therefore$ lateral area $= 12 \times 3 = 36$ sq. inches.

The area of the ends $= 2 \times 4\sqrt{3} = 8\sqrt{3}$ sq. in.

$\therefore$ the total surface area $= (36 + 8\sqrt{3})$ sq. inches.

The volume = base $\times$ height,

$= 4\sqrt{3} \times 3 = 12\sqrt{3}$ cub. inches.

Exercise XXXIV (A).

Find the volumes of the following prisms :—

1. Base 5 sq. ft. 64 sq. in. and height 1 ft. 10 in.
2. Base 10 sq. ft. 30 sq. in. and height 3 yds.

Find the heights of the following prisms :—

3. Base 18 sq. ft. 130 sq. in. and volume 5 cub. ft 1230 cub. in.
4. Volume 6 cub. yds. 20 cub. ft. and base 15 sq. ft 120 sq. in.

Find the base areas of the following prisms :—

5. Volume 4 cub. ft. 1008 cub. in. and height 2 ft. 4 in
6. Volume 8 cub. ft. 208 cub. in. and height 5 ft. 7 in.

Find the volumes and total surface areas of the right prisms given below :—

7. Base an equilateral triangle of side 1·8 in. and height 7·2 in.

8. Base an isosceles triangle of sides 5 in., 5 in., 8 in. and height of 7 in.

9. Base a triangle of sides 15 in., 8 in., 17 in. and height 20 in.

10. Base a rhombus whose diagonals are 6 and 8 in. and height 15 in.

11. Base a square of side 2·8 in. and height 4·5 in.

12. Base an isosceles trapezium : parallel sides 8 in. and 10 in. and perpendicular distance between the parallel sides 7·5 in. and height 13 in.

13. Base a regular pentagon of side 2·3 in. and height 10 in.

14. Base a regular hexagon of side 1·5 in. and height 12 in.

15. Base a regular octagon of side 1·8 in. and height 2 ft. 5 in.

16. What weight of water will fill a tank in the form of a prism whose base is a regular hexagon of side 4 ft. and height $5\frac{1}{2}$ ft. ? (1 cub. ft. of water weighs $62\frac{1}{2}$ lbs.)

17. A vessel in the shape of a prism on a regular hexagonal base of side 4·5 in. is filled with water. Find to three places of decimals how much the liquid will sink if 1 gallon is taken away. (1 gallon $= 277\frac{1}{4}$ cub. in.)

18. What are the cubical contents of a shaft the mean section of which is a regular hexagon of side $3\frac{1}{2}$ ft. and whose height is 70 ft. ?

19. The base of a certain prism is a regular octagon. Every edge of the prism is 2 ft. Find the volume of the prism.

20. The section of a canal is 30 ft. wide at the top and 15 ft. wide at the bottom. It is 10 ft. deep. How many cubic yards were excavated in a mile of the canal ?

21. It is found that two vessels in the shape of a prism

one on an equilateral triangle of side 2 ft. and the othe on a sq. base of side 2·5 ft. have the same capacity Compare their heights.

22. The volumes of two prisms standing on a rectang 18 in. by 12 in. and on a right-angled triangle whose side containing the right angle are 24 in. and 10 in. are equa Compare their heights.

23. A hay-stack stands on a rectangular base ABCD wi gabled ends as in Fig. 281. Th sides of the ends are 35, 37 an 12 ft. and the length of the stack 40 ft. Find the quantity of hay the stack.

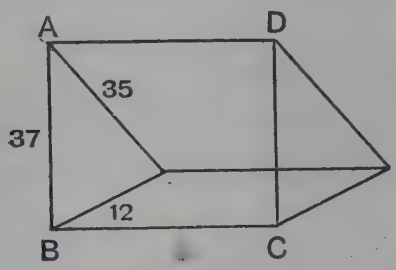


FIG. 281.

24. A rectangular hall with gabled roof is used as a hospita See Fig. 282. The length of the ha is 80 ft. The triangle is an isosceles right-angled o whose hypotenuse is 36 ft. Th greatest height of the buildi is 35 ft. If daily on an avera 60 patients and 4 doctors rema in the hospital for some tim how many cub. ft. of air a allowed to each?

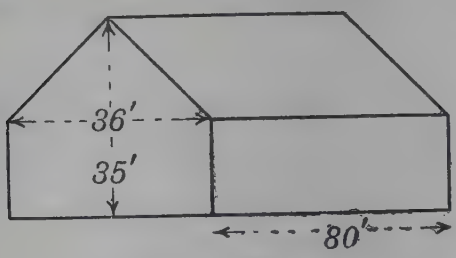


FIG. 282.

25. The vertical section of a canal is an isoscel trapezium. The parallel sides at the top and the bottom a 20 ft. and 12 ft. The non-parallel sides are each 10 ft. lon If water flows through it in full at 3 miles an hour on average, what volume of water flows through it in a day

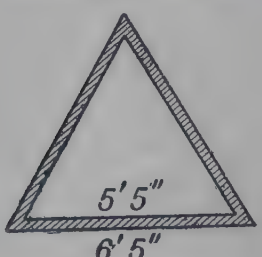


FIG. 283.

26. An iron stand placed on an oven in the form of a uniform hollow pri whose base is the difference of t equilateral triangles as shown in the figu The thickness of the material is $\frac{1}{2}$ Find the volume of iron in the stand.

27. The iron stand on which a kerosene-oil cart is made to rest when oil is sold in streets by a hawker, is in the form of a uniform hollow prism each of whose parallel ends is the difference of two trapeziums as shown in the Fig. 284. The thickness of the material is $\frac{3}{4}$ of an inch. Find the volume of iron in the cart stand.

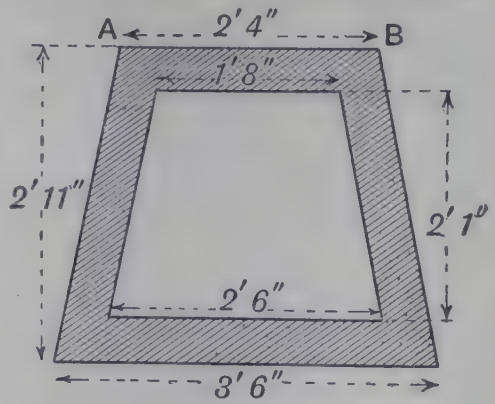


FIG. 284.

28. A cistern in front of an idol in a temple, consists of stone work which is a uniform hollow prism each of whose parallel ends is the difference of two regular hexagons as shown in Fig. 285. The height of the cistern is 3 in. Find the volume of the stone in the cistern.

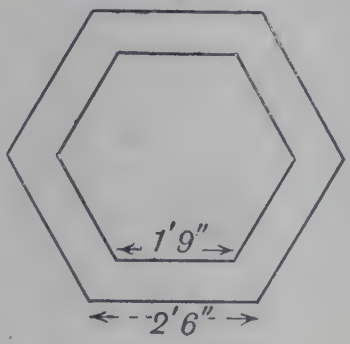


FIG. 285.

29. The portico of a bungalow is a right prism on a base as indicated in Fig. 285 (*a*). The height of the portico is 20 ft. Find the area of the floor and the volume of the portico.

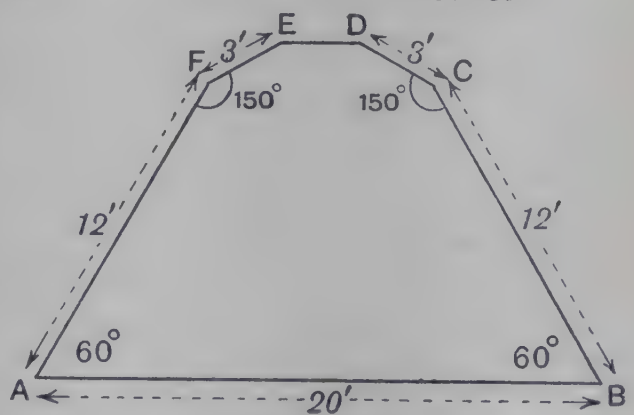


FIG. 285 (*a*).

30. The vertical section of a chimney constructed over a terraced kitchen is an isosceles trapezium as shown in Fig. 286. The chimney is a prism whose parallel ends are vertical. The inside distance between the two parallel sides is $1\frac{1}{2}$ ft. What volume of smoke can it hold at a time? (The slant side is $4\sqrt{17}$ and not 4′ as marked in the figure).

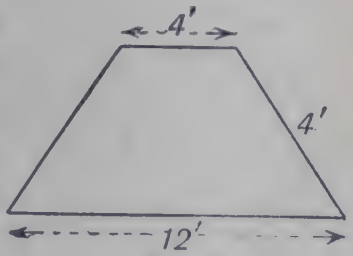


FIG. 286.

31. The vertical section of the frontage of a house consisting of one storey is shown in figure (287). It is 40 ft. long. Find the volume of the verandah below and above the terrace. Assume the thickness in the concrete of the terrace to be $1\frac{1}{2}$ ft. (The height of the verandah below the terrace is 12′.)

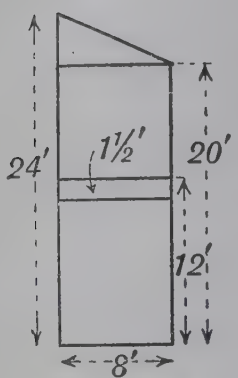
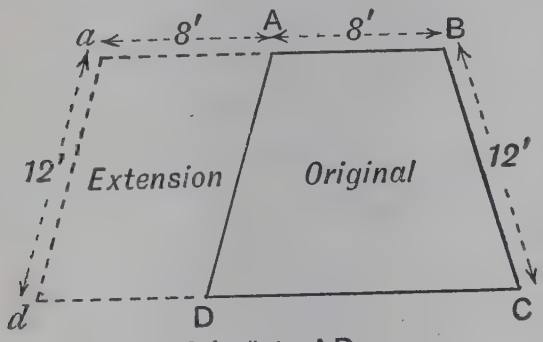


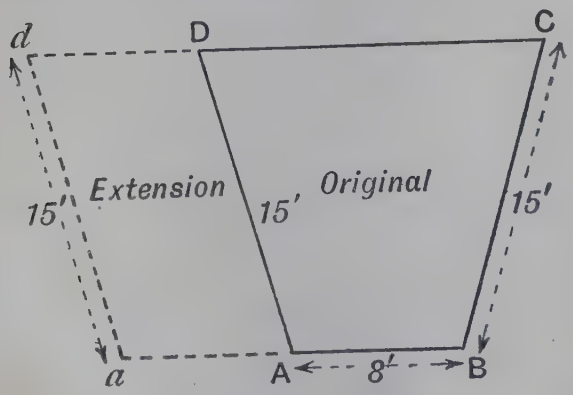
FIG. 287.

32. A rail-road consists of a level ground with descending slopes on both sides, 8 ft. wide, on which a single line of railway has been laid. The road descends 3 vertical to 4 along the slope. What amount of earth is needed to broaden the road to lay another parallel line of railway for a mile's length so that the new width of the whole road may be 16 ft. as shown in Fig. 288? (*ad* is parallel to AD.)



ad is ‖ to AD
FIG. 288.

33. In the previous question, at a certain place on the railway the ground beside the rail, road rises 4 vertical to every 5 measured along the slope. What amount of earth should be removed to open a double line for a mile's length of the line being 16 ft.? (Fig. 289) (*ad* is parallel to AD.)



ad is ‖ to AD
FIG. 289.

34. Find the quantity of earthwork in a straight portion of a bund 150 ft. long, top width 6 ft., water slope $2\frac{1}{2}$ vertical to 1 horizontal, outer slope $1\frac{1}{2}$ vertical to 1 horizontal. The height of the top of the bund is 12 ft. above ground level.

35. Each of the two parallel ends of a railway waggon in the form of a right prism is vertical and in the shape of a trapezium 20 ft. at the top and 12 ft. at the bottom. The height and the width of the waggon are 6 ft. and 8 ft. 6 in. It is filled with sand. Find the quantity of the sand in the waggon.

Cylinder.

178. A round pencil, a round pillar, and a round pipe are all examples of a right circular cylinder. It is also a right prism whose parallel ends are circles. Suppose the base of a prism is a regular polygon, and the number of sides is indefinitely increased, in which case it tends to become very nearly equal to a circle. Hence in the limit the polygon may be regarded to be equivalent to a circle. Therefore the volume of a cylinder = the area of the base circle $\times$ height = $\pi r^2 h$. The circle may be cut up into a very large number of small sectors which may be arranged in the form of a rectangle as we did when we found the area of the circle. Then the volume of the cylinder would be equivalent to that of a prism standing on a rectangle equal in area to the circle.

A cylinder is a solid generated by the revolution of a rectangle about one of its sides kept fixed. The fixed edge is the **axis** of the cylinder. Suppose a paper is wrapped round a wooden cylinder and marks are made with a pin where the paper overlaps itself and finishes one round. Now unfold the paper cutting off that portion that exactly covered the surface of the cylinder. Then a rectangle is got, one of whose sides is the circumference of the cylinder and the other is its height.

FIG. 290.

If r is the radius of a cylinder and h its height, the area of the curved surface of a cylinder = $2\pi rh$ or πdh where d is the diameter of the circle. The total surface of the cylinder

$=$ the curved surface $+$ the area of the two plane ends $=$ $2\pi rh + 2\pi r^2 = 2\pi r\ (h + r) = C\ (r + h)$ if C represents the circumference.

FIG. 291.

The volume of the material in a hollow cylinder of external diameter D and external radius R, of internal diameter d and internal radius r, and of height h = the difference of the two cylinders of diameters D and d = $\pi R^2 h - \pi r^2 h = \pi h\ (R^2 - r^2) = \pi h\ (R + r)\ (R - r)$ or $\frac{\pi}{4}D^2h - \frac{\pi}{4}d^2h = \frac{\pi h}{4}(D^2 - d^2) = \frac{\pi h}{4}(D + d)\ (D - d)$.

The total surface area of such a hollow cylinder = the area of inner and outer curved surfaces + that of flat rims at the ends.

$= 2\pi Rh + 2\pi rh + 2\pi\ (R^2 - r^2)$.

or $\pi\ Dh + \pi dh + \dfrac{\pi D^2 - \pi d^2}{2}$.

Model No. (2) Find the volume and surface area of a cylinder whose diameter is 6 in. and height 10 in. ($\pi = \frac{22}{7}$).

Volume $= \pi r^2 h = \frac{22}{7} \times 3^2 \times$ 10 cub. in. $= \frac{22}{7} \times 90 = \frac{1980}{7} = 282\frac{6}{7}$ cub. in.

Surface area $= 2\pi rh + 2\pi r^2 = 2\pi r\ (h + r) = 2 \times \frac{22}{7} \times 3\ (10 + 3) = \frac{44}{7} \times 3 \times 13 = \frac{1716}{7} = 245\frac{1}{7}$ sq. in.

Model No. (3) Find the volume and curved surface of a hollow cylinder whose external and internal diameters and height are 20 cm., 16 cm. and 5 cm. respectively. Find also the surface area.

Volume $= \pi R^2 h - \pi r^2 h = \pi h\ (R + r)\ (R - r)$.

$$= \frac{22}{7} \times 5\ (10 + 8)\ (10 - 8).$$

$$= \frac{22}{7} \times 5 \times 18 \times 2.$$

$$= \frac{3960}{7} = 565\frac{5}{7} \text{ c.cm.}$$

Curved surface of a hollow cylinder

$= 2\pi h\ (R + r) = 2 \times \frac{22}{7} \times 5 \times 18 = \frac{44}{7} \times 90$

$= \frac{3960}{7} = 565\frac{5}{7}$ sq. in.

Surface area $= 2\pi h\ (R + r) + 2\pi\ (R + r)\ (R - r)$.

$= 2 \times \frac{22}{7} \times 5\ (10 + 8) + 2 \times \frac{22}{7}\ (10 + 8)\ (10 - 8)$.

$= \frac{44}{7} \times 90 + \frac{44}{7} \times 36 = \frac{44}{7} \times 126 =$ 792 sq. in

Exercise XXXIV (B).

Find the volumes of the following circular cylinders :—

1. Radius 1·7 in., height 4 ft. 3 in.
2. Radius 2·8 in., height 6 ft. 5 in.
3. Radius 3·5 cm., height 8·9 cm.
4. Diameter 6·5 cm., height 10·8 cm.
5. Diameter 6 ft. 5 in., height 10 ft. 8 in.

Find the radii of the following cylinders :—

6. Volume 3·475 cub. in., height 3 ft. 4 in.
7. Volume 6750 cub. cm., height 2·5 dm.
8. Volume 1 cub. ft., height 1 ft. 2 in.
9. Volume 3·56 cub. ft., height 4 ft.

Find the heights of the following cylinders :—

10. Volume 6750 cub. in., diameter 1 ft. 2in.
11. Volume 235 cub. cm., diameter 13·5 cm.
12. Volume 1·2 cub. ft., radius 9 in.
13. Volume 1·8 cub. dm., radius 8·6 cm.

Find the value of π from the following data :—

14. Volume 2·75 cub. in., radius $=$ 5 in., height 3·5 in.
15. Volume 33·4769 cub. in., radius 1·2 in., height 7·4 in.
16. Volume 123·05 cub. in., radius $=$ 2·4 in., height 6·8 in.
17. A circular pillar is 1 ft. 6 in. in diameter and 7 ft. 6 in. high. Find how many cubic feet of masonry there are in the pillar.
18. A well 3 ft. 6 in. in diameter is 25 ft. deep and water stands at a height of 12 ft. from the bottom. Find the volume of the empty space.
19. A circular well of diameter 4 ft. 6 in. has a square top of 4 ft. 6 in. The square portion extends to a height of 3 ft. 6 in. Find the volume of space enclosed between the top of the well and the water surface which is 18 ft. below the top.
20. A circular bore has to be made through a post $4'' \times 4'' \times 6'$ so that an iron rod $\frac{3}{4}$ in. diameter may pass through it. If there is to be a clear space of $\frac{1}{8}$ in. all round

in the bore, find what volume of wood will be taken away from the wood and also the volume of the remaining portion of the wood.

21. A window has 6 iron bars of diameter $\frac{5}{8}$ in. The length of the bar between the insides of the frames is 4 ft. 6 in. The bar has to go $1\frac{1}{2}$ in. into the bottom frame and 1 in. into the top frame. If there are 6 such windows in a room, find what quantity of iron will be required to mould the bars.

22. A hollow cylinder has an external diameter of 12·3 in. and an internal diameter of 11·8 in. Find the volume of the solid if the height is 6·2 in.

23. A circular cylinder has a square hole passing right through it from edge to edge. If the diameter of the cylinder be 8 cm., its height 3·2 dm. and the side of the square be 4·5 cms., find the volume of hollow solid.

24. A square prism of side 6 in. and height 1·2 ft. has a circular hole in it passing right through it from edge to edge. If the diameter of the hole be 3·6″, find the volume of the hollow solid.

25. 1 cub. ft. of copper is drawn into a round wire of diameter $\frac{1}{20}$ of an inch and is just sufficient to pass round a circular garden. Find the area of the garden.

26. A rectangular solid is $7'' \times 7'' \times 10''$. A cylinder is 7 in. in diameter and 10 in. high. Compare their volumes.

27. A rectangular solid with a square base and a cylinder are of the same height and of the same volume. What is the ratio of a side of the square to the diameter of the cylinder?

28. A circular shaft 3 in. in diameter is sunk down to a depth of 30 ft. by a boring machine. Find what volume of earth has been taken out.

29. A well 10 ft. in diameter is dug in the centre of a square field $70' \times 60'$. The earth taken out from the well is sufficient to raise the level of the field by 5 in. Find to what depth the well was dug.

30. The velocity of water at a certain section of a pipe of diameter 3 ft. 6 in. is 4 miles per hour. If water is discharged for 25 minutes, find how many gallons would have passed through that section (a gallon = ·16 c. ft.)

31. A measure, $\frac{1}{4}$ measure and an ollock are cylindrical in form and 8·125 in., 4·25 in. and 3·5 in. high respectively. If the diameter of the $\frac{1}{4}$ measure is 3 in., find the diameter of the others.

32. If, in the previous question, it is given that a measure holds 100 cub. in., find the diameters of the three measures, measure, $\frac{1}{4}$ measure and an ollock.

33. A tin rectangular sheet $a'' \times b''$ is rotated about one of the larger edges measuring a in. What is the solid formed and what is its volume? If it is rotated about the line joining the middle points of the sides b in. long find the volume of the solid generated. Compare these volumes.

34. A cardboard 9 in. by 5 in. is to be rolled into a cylinder along (a) its length and (b) its breadth. Find the height and diameter in each case.

35. Two cylindrical tubs, the first of diameter 1 ft. 2 in. and 3 ft. high and the second 2 ft. 6 in. diameter and 5 ft. high, which are full of water, fill a third cylindrical tub of diameter 3 ft. 6 in. completely when emptied into it. Find the height of the third.

36. A drain is of the section shown in Fig. 292. If water flows full in it, what quantity of water, in cub. ft., will 100 yds. length of the drain hold?

37. The section of a silver box with semi-circular ends is as shown in Fig. 293. If it be 3 ft. 3 in. long find the cubical contents of the box.

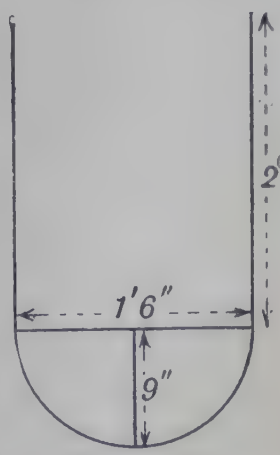


FIG. 292.

38. The roof of a bullock cart is as shown in figure 294. If the cart is 6 ft. long, find how many cub. ft. of air is enclosed by the roof.

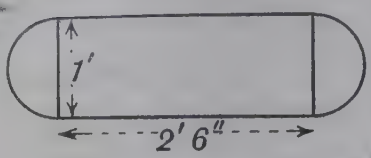


FIG. 293

39. A roller used for levelling the tennis ground is 3 ft. in diameter and $3\frac{1}{4}$ ft. long. Find its volume; also its weight, if one cub. ft. of the material weighs 180 lbs.

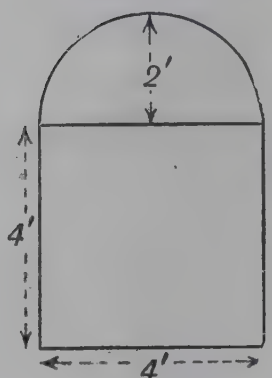


FIG. 294.

40. A road roller, in traversing a road 154 yds. long makes 42 revolutions and moves over an area of 205 sq. yds. 3 sq. ft. Find the volume of the road roller.

41. A circular kerosene-oil tank approaches a cylinder in form. Its diameter is 80 ft. and height 35 ft. How many gallons can it hold? assuming 1 gallon = $277\frac{1}{4}$ cub. in.

42. In the previous question, find the cost of painting the curved surface (C.S.) of the cylinder at Rs. 3-12-0 per 100 sq. ft.

43. A cylindrical vessel is without a lid. Its external height is 3 ft. 5 in. and external diameter 2 ft. 2 in. If it is made of metal $1\frac{1}{2}$ in. thick, what volume of water will it hold and what will be its weight if one cubic foot of the material weighs 350 lbs?

44. A silver box in a cylindrical form is made to hold spices. Its external height and diameter are 6 in. and 3 in. Its thickness is $\frac{1}{2}$ in. What is the capacity of the box?

45. The following experiment was made to find the rate of flow of water in a pipe:—The pipe of internal diameter $\frac{3}{4}$ in. was opened and water collected in a cylindrical tub of internal diameter 2 ft. and height 3 ft. The time taken for filling it was 20 minutes. Explain how the rate of flow was determined and give the result of the experiment.

46. A goldsmith has cylinders made of different materials. In one scalepan of a balance, 4 silver cylinders each of diameter 1 in. and height 2·5 in. are put. In the other scalepan he puts wooden cylinders each of diameter 2 in. and height 5 in. If the densities of silver and wood are $10\frac{1}{2} : \frac{3}{4}$, find the number of wooden cylinders put to counterpoise the silver cylinders.

47. An iron cast is in the form shown in Fig. 295. If the height of the cast is 2 ft. and a side of the square 1·2 in., find its weight if 1 cub. ft. of the iron weighs 450 lbs.

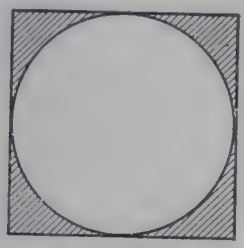


FIG. 295.

48. There is a cylindrical oil drum from which kerosene is discharged through a pipe fitted to its bottom. Find by how much the level of the oil in the drum will go down when a four gallon tin is filled from it. The diameter of the drum is $1\frac{1}{2}$ ft. [1 gallon = $277\frac{1}{4}$ c. in.]

49. A jar full of ink is cylindrical in form. From it 150 small bottles are filled for the supply of ink to students in an examination. If the internal diameter and height of the jar are 6 in. and 15 in. and the corresponding dimensions of the small bottles are $2\frac{1}{2}$ in. and 3 in., find how many jars will be required for the purpose.

50. Water is poured from a burette into a cylindrical flask. The initial reading when the stop-cork was opened was 50 c.c., the final reading when the tap was closed was 150 c.c. If the internal diameter of the flask is 45 cm., find to what height water stands in the flask.

51. In a candlestand there is a cylindrical tube into which a candle is thrust and screwed when the spring in it is compressed. The candle is 8 in. long and ·9 in. in diameter. The tube is 9·5 in. long and ·9 in. in diameter. Find the volume of the space occupied by the spring when a full candle is inserted given that the conical extremity of the candle which is $\frac{3}{4}$ in. long remains outside the tube.

52. Two cylindrical tubes full of water with internal diameters of 4 ft. and 3·5 ft. and heights of 2·8 ft. and 3·4 ft. respectively are emptied into a cylindrical vessel whose bottom has a diameter of 3·75 ft. If the latter vessel is full, find its height.

53. The area of a circular field is one acre. Around it outside there is a circular path 6 ft. wide. It is proposed to dig a semi-circular pit in it to a depth of 15 ft., its diameter being 30 yds. The earth thus taken out is spread over the whole field including the path. By how much has the level of the field been raised? What is the final depth of the semi-circular pit?

54. The vertical section of a small temple in a village is shown in Fig. 296 and is of the same shape at any part of its length, i.e., a rectangle surmounted by a semi-circle. BC = 30 ft. and AB = 15 ft. Length of the temple is 40 ft. If, on a festival day, 200 devotees enter the temple at the same time and occupy it how much air space is allowed to each?

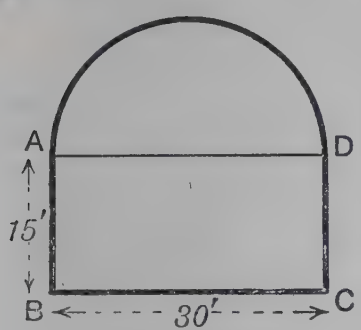


FIG. 296.

55. Suppose the figure in Exercise 54 represents the ground plan of a bungalow, BC = 100 ft. and AB = 40 ft. The horizontal section at any point of the height is of the same shape. Find the cubical contents of the bungalow if its height is 15 ft. Supposing this is used as a hospital for in-patients and that each in-patient requires 960 cub. ft. of air space, find the number of in-patients that can be accommodated. Suppose there is an extension of space by constructing a semi-circular room on the other side of AB, of the same height. How many more patients can be accommodated at the same rate?

56. A hall consists of a rectangular portion 45′ × 21′ with two semi-circular portions at the ends of the rectangle.

If the roof of the hall is flat and if the height of the hall is 18 ft. 6 in., find the cubical contents of the hall. What ambiguity is there in this question? Give all possible answers.

57. A copying pencil is 7 in. long and $\frac{5}{16}$ in. in diameter. If the lead is $\frac{3}{32}$ in. in diameter what quantity of wood is in the pencil?

58. A circular cylindrical vessel is inverted so that the open end rests on the ground. In the other end there is a hole into which another cylindrical vessel opens, the diameter of the hole and that of the upper vessel are the same. The area round the hole uncovered by the upper cylinder is $15\frac{5}{7}$ sq. in. Find the volume of both the cylinders (Fig. 297).

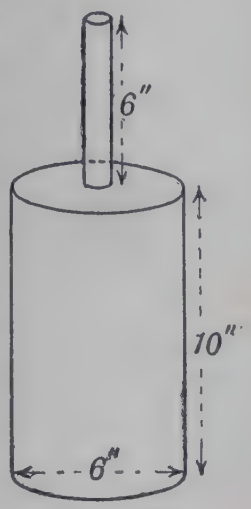


FIG. 297.

59. A cylindrical gold tube is bent in the form of a circular ring. The internal diameter of the ring is 2·4 in. The diameter of the cylinder which becomes the thickness of the ring is 0·18 in. Find the volume of the ring.

60. A verandah has 8 pillars each of diameter $1\frac{1}{2}$ ft. and $7\frac{1}{2}$ ft. high. Find the cost of plastering the pillars at Rs. 5-14-0 per 100 sq. ft.

61. There are two pillars of the same height, one circular, $1\frac{1}{2}$ in. in diameter and the other a square of side $1\frac{1}{2}$ in. Which is cheaper to whitewash? What fraction of the surface of the one is that of the other?

62. A piece of wood 6 ft. long having a cross-section of $9'' \times 9''$ is alternately cylindrical and square. The change occurs at the end of every foot and the beginning is made with a square. Find the surface area including the ends. The diameter of the cylindrical portions throughout is 9 in.

63. A gutter 100 yds. long has a cross-section as shown in the adjacent Fig. 298. Find the cost of cementing the gutter at Rs. 14-9-0 per 100 sq. ft.

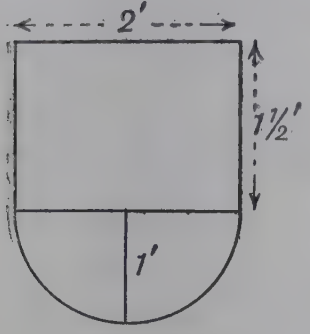


FIG. 298.

64. The area of the two ends of a cylinder is equal to the area of the curved surface. Find the ratio of the height of the cylinder to the radius of the base.

65. A silversmith is asked to prepare two cylindrical vessels having the same surface area, one with and the other without a lid. He makes the height of the cylinder of the first equal to the radius of the second and the height of the second equal to twice the radius of the base of the first. Compare their diameters.

66. Find the volume of a glass cylindrical tube of bore 4 inches bent in the form shown in Fig. 299. The curved portion is semi-circular in form.

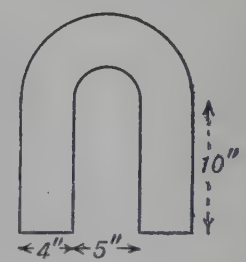


FIG. 299.

67. Find the cost of erecting a circular pillar 15ft. high, a horizontal section of which is shown in the figure. It consists of three portions, the outer portion is cement, the intermediate portion bricks and the central portion stone. The rates are Re. 1-4-0 per cub. ft. of stone, 6 as. per cub. ft. of brick and 3 as. per cub. ft. of cement.

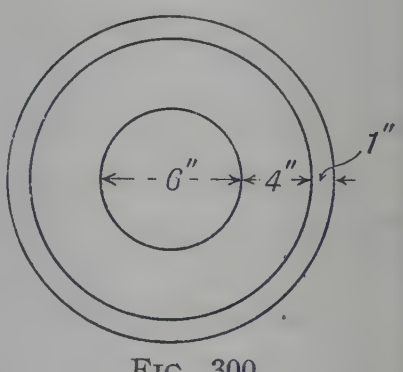


FIG. 300.

68. s c.c. of silver are melted and drawn into a cylindrical wire w cm. thick. If the length of the wire is l cm., express s in terms of l and w.

69. A goldsmith has two coils of gold wire. One is 20 ft. long and $\frac{1}{100}$ inch in diameter, the other 30 ft. long, $\frac{1}{50}$ in.

in diameter. He has to pass a wire through beads whose bore is $\frac{1}{75}$ of an inch in diameter. He melts the two wires and draws a wire of the required thickness. How many feet of wire does he get?

70. It is required to construct a case in the form of a square prism to hold just 4 pencils each of diameter 3 in. and 7 in. long. The length of the case is the same as that of the pencil. Find the volume of the space not occupied by the pencils in the case when they are put in it (Fig. 301).

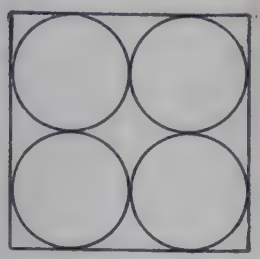


FIG. 301.

71. In the previous question, it is required to construct a cylindrical case so as to just hold 4 pencils and to be of the same height as the pencils. Find the volume of the space not occupied by the pencils in this case (Fig. 302).

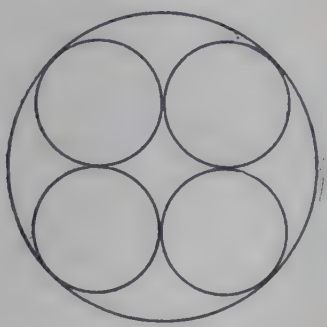


FIG. 302.

72. There is a cylindrical beaker which weighs 15·75 gm. Some mercury is poured into it. It then weighs 200·25 gm. The height of the column of mercury in the beaker is 4 cm. If the s.g. of mercury is 13·6, find the diameter of the beaker.

73. 25 gms. of a certain liquid, specific gravity ·9, are poured into a cylindrical vessel. The liquid is 8 cm. high. Find the diameter of the vessel.

74. There is a cube of side 4 in. A circular hole of diameter 1 in. is bored right through the tube on the upper face so that the centre of the face is the centre of the hole. On an adjacent face a hole in the form of a square of side 1 in. is bored right through the centre of the cube to the opposite face. Find the volume of the empty spaces in the cube.

75. How many cubes of edge 1 in. can be cast from a cylinder of lead of diameter 3·6 in. and of height 11 in.? ($\pi = 3{\cdot}1416$.)

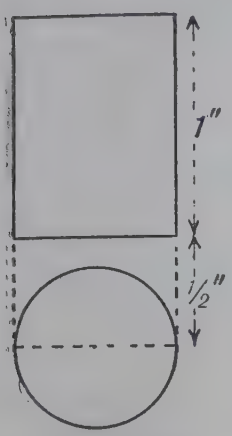


FIG. 303.

76. The figure shows the plan and elevation of a cylindrical tank to a scale of $\frac{1}{300}$ of the natural size. Find in gallons how much water it will hold and in inches to what depth it would be filled by 500 gallons of water. (1 gallon = ·16 c. ft.) Ignore the measurements given in the figure by mistake.

77. A coil of wire has 4 turns, the average diameter of the turns is 12 in. and the thickness of the wire is $\frac{1}{12}$ in. Find (*a*) the length of the wire and (*b*) the volume of wire in the coil.

78. Find the weight of a garden hose 65 metres long in the form of a hollow cylinder whose external diameter = 4·5 cm. and thickness ·5 cm.; s.g. of the material 1·68. What amount of water can it hold?

179. A right circular cone is the solid generated by revolving a right-angled triangle about one of the sides containing the right angle kept fixed. The fixed side is called the axis. The other side of the right angle describes the base of the cone which is a circle. The hypotenuse describes the curved surface of the cone. Hence it is called the generator of the cone. A cone is then bounded by two surfaces, viz., a curved surface and a plane surface forming the bottom of the cone. In the figure V is the apex. VO the line passing through the vertex and the centre of the base is perpendicular to the base and is called the **axis** for it is about this line the right-angled triangle should be revolved.

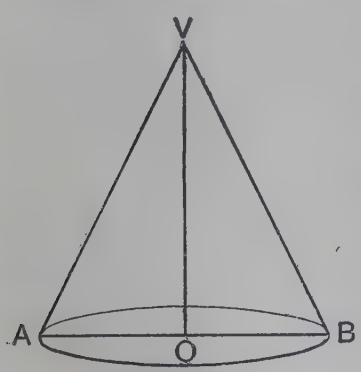


FIG. 304.

Sections of the cone. Any section parallel to the base circle is a circle.

Any section along the axis is an isosceles triangle. $VA = VB$ and VOA is a right-angled triangle. Therefore $VA^2 = AO^2 + OV^2$.

$\angle AVB$ is called the vertical angle of the cone.

$\angle AVO$ is called the semi-vertical angle.

180. Volume of the Cone. Take a hollow cylinder and a hollow cone having the same diameter and the same height. If the cone is filled with sand, and emptied into the cylinder and if this is done 3 times the cylinder is just filled.

Hence the volume of the cone on a base of radius r and height $h = \frac{1}{3}$ of the volume of the cylinder of radius r and height h. It is equal to $\frac{1}{3}\pi r^2 h$. While the right-angled triangle is half of the corresponding rectangle, the generated cone is $\frac{1}{3}$ of the generated cylinder.

Curved surface of the cone. If can be easily seen that **the sector of a circle can be folded into a cone, the arc of the sector becoming the circumference of the base of the cone and every radius becoming the slant height of the cone,** until the bounding radii of the sector overlap in the cone formation.

$\therefore$ the area of the curved surface of a cone of radius r and height h = the area of the sector.

= $\frac{1}{2}$ the arc of the sector $\times$ radius of the sector.

= $\frac{1}{2}$ the circumference of the base of the cone $\times$ slant height.

$= \frac{1}{2} \times 2\pi r \times \sqrt{r^2 + h^2}$ $\left[\text{for } \sqrt{A^2} = VO^2 + OA^2 = r^2 + h^2\right]$

$= \pi r \sqrt{r^2 + h^2}$.

$= \pi r \times l$ where l is the slant height of the cone.

$\therefore$ the total surface area of a cone $= \pi r \sqrt{r^2 + h^2} + \pi r^2$ (the area of the base) $= \pi r (r + \sqrt{r^2 + h^2})$.

Model No. (4) Find the total surface and volume of a cone whose diameter is 12 in. and whose vertical height is 2 in.

Total surface $= \pi r\,(r + \sqrt{r^2 + h^2})$.

$$= \frac{22}{7} \times 6\ (6 + \sqrt{36 + 4})\text{ sq. in.}$$

$$= \frac{22}{7} \times 6 \times (6 + 6{\cdot}3)\text{ sq. in.}$$

$$= \frac{22}{7} \times 6 \times 12{\cdot}3 = \frac{1623{\cdot}6}{7} = 231{\cdot}94\text{ sq. in.}$$

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 36 \times 2 = \frac{528}{7} = 75\frac{3}{7}\text{ c. in.}$$

Exercise XXXIV (C).

Find the volumes of the following cones :—

1. Radius 7 in., height 15 in.
2. Diameter 12 in., height 10 in.
3. Circumference 35 in., height 9 in.

Find the height of the following cones :—

4. Vol. = 275 cub. in., radius = 12 in.
5. Vol. = 178 cub. in., diameter = 18 in.
6. Vol. = 278 cub. in., circumference = 42 in.

Find the radius of the following cones :—

7. Vol. 375 cub. in., height 18 in.
8. Vol. = 570 cub. in., height 2 ft.
9. Vol. = 780 cub. in., height 3 ft. 6 in.
10. The generating line of a cone is 1 in. long. If the height of the cone is equal to the radius of its base, find the volume of the cone.
11. The generating line of a cone is 1 ft. long. If the angle of the cone is 60°, find the volume of the cone.
12. The height of a cone is 3 ft. and its slant height 3 ft. 6 in. Find the volume of the cone.
13. The diameter of the base of a cone is 1 ft. 4 in. If the slant height of the cone is $2\frac{1}{2}$ ft., find the volume of the cone.
14. An equilateral triangle of side 12 in. is rotated about a median of the triangle. Find the volume of the solid thus generated.

15. An isosceles triangle whose base is 8 in. and one of whose equal sides is 12 in. rotates about the perpendicular from the vertex on the base. Find the volume of the solid thus generated.

16. A tent is in the form of a cylinder surmounted by a cone. If the diameter of the tent be 40 ft. and the height 30 ft. of which the portion relating to the cone is 10 ft., find the amount of air enclosed in the tent.

17. A solid consists of a cylindrical portion in the middle and two equal cones attached to it at the extremities as shown in figure 305. Find its volume.

18. The two smaller sides of a right-angled triangle measure 12 in. and 5 in. Find the total surface area of the solid generated by the revolution of the triangle about one of these sides.

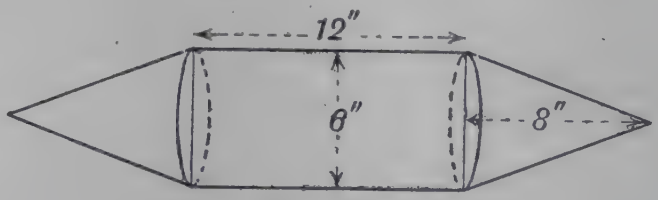


FIG. 305.

19. The 60° set square is rotated about the longer of the two sides containing the right angle. Find the volume of the solid generated (Hyp. 4 in.).

20. The rattling drum of a morning beggar is in the form of a double cone. If the diameter of each end be $1\frac{1}{2}$ in. and the height of the double cone be 3 in., find the volume of the drum.

FIG. 306.

21. A conical tent is to accommodate 15 men. Each must have a floor space of 20 sq. ft. and 200 cub. ft. of air to breathe. Find the height and the diameter of the tent.

22. A cone has a base of diameter 6 in. Its height is 12 in. If the cone be cut off by a plane at right angles to the axis of the cone, passing at a distance of 8 in. from the bottom, find the volumes of the two slices.

23. A funnel of diameter 6 in. and height 8 in. has a hole

at the bottom through which water flows at an average rate of $2\frac{1}{2}$ cub. in. per minute, find at what height it will stand after $5\frac{1}{2}$ minutes supposing the funnel is full before it flows.

24. A conical vessel 12 in. in height and 8 in. in diameter at the base is filled with water. The water in it is poured into another conical vessel whose height is 8 in. If, after filling it, there still remains 1 in. of water in the former held base upwards, find the radius of the base of the latter.

25. A quadrant of a circle whose diameter is 3 ft. is formed into a cone. Find the volume of the cone.

26. A cylindrical vessel of diameter 12 in. and an inverted conical vessel of height 8 in. and diameter 6 in. are connected by means of a tube. To begin with, water stands at a height of 2 in. in the conical vessel. If a piston rod is applied to the cylindrical vessel and pressed so as to make water flow into the conical vessel and raise its level to 4 inches, through what depth must the piston be pressed?

27. Find the area of the slant surface of a cone the diameter of whose base is 1·7 in. and whose height is 13 in.

28. Find the superficial area of a cone (including the base) whose height is 1 ft. 6 in. and whose slant height is 2 ft.

29. A cone is $\frac{a\sqrt{3}}{2}$ ft. high. If the vertical angle of the cone is 60°, find the area of the base and the slant surface developed. What angle does the arc formed by the circumference of the base subtend at the centre (the vertex of the cone) in the sector leading to the formation of the cone?

30. There is a funnel which is 8 in. high and whose diameter is 6 in. at the top. Find what area of filter paper is required to cover the inner surface of the funnel with only one layer of the filter paper.

31. What is the area of the canvas required for the construction of a tent which is in the shape of a cylinder surmounted by a cone? The diameter of the tent is 80 ft. Its height is 50 ft. The height of the cone is 25 ft.

32. Find the ratio of the height of a cone to the diameter of its base if the area of the slant surface is equal to twice that of the base.

33. The curved surface of a cone is 75 sq. in. The radius of the base is 3·7 in. Find the slant height of the cone.

34. The curved surface of a cone is 83 sq. in. The diameter of the base is 8 in. Find the height of the cone.

35. The curved surface of a cone is 125 sq. in. The height of the cone is 10 in. Find the radius of the base.

36. The curved surface of a cone is 220 sq. in. The height of the cone is 13 in. Find the slant height of the cone.

37. The 60° set square is rotated about the longest side. (5·5 in.) Find the surface of the solid formed.

38. The area of the whole surface of a cone is 45 sq. ft. If the radius of the base be $\frac{1}{3}$ of the height of the cone, find the volume of the cone.

39. Verify the following statement : Given the full area of the surface of a cone, the greatest volume of the cone is when its height is $2\sqrt{2}a$ where a is the radius of the cone.

40. An oil-can whose capacity is a gallon is conical in shape. Its height is found to be 1 ft. Find the diameter of its base. (1 gallon $= 277\frac{1}{4}$ cub. in.)

41. A packet contains 24 candles and weighs 3 lbs. Each candle consists of a conical and a cylindrical portion. The total length of the candle is 17·5 cm. and the greatest width 17 mm. The length of the conical portion is 2 cm. Find the s.g. of the candles. 1 kg $=$ 2·20 lbs.

42. The inside of a tower consists of a cylindrical portion

surmounted by a conical roof. If the inner base of the tower be 15 ft. in diameter, the height of the cylindrical portion 50 ft. and that of the conical portion 10 ft., find the cost of whitewashing the inside of the tower at 6 pies per sq. yd. and the volume of the tower.

43. A conical measure is made which will hold just as much as a measure containing 8 ollocks. The diameters of the cone and the 8 ollock measure are equal. Compare their heights.

44. A cylindrical piece of wood is cut into 4 cones (Fig. 307). If the height of each of the small cones is a fourth of that of the whole cylinder, find the quantity of wood left in the solid in terms of the diameter and the height of the cylinder.

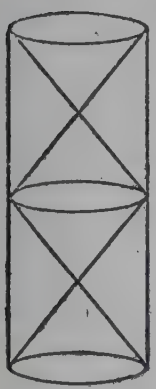


FIG. 307.

45. Water is poured into a cylindrical vessel of diameter 6 in. to a height of 7 in. (*a*) A solid glass cone whose base is 4 in. in diameter and whose height is 3 in. is immersed in it. How high will the water rise? (*b*) A solid glass cylinder of the same diameter and height as the cone is immersed while the cone is removed. How is the level of the water affected?

46. Find the volume of the largest cone that could be cut out from a cubical block of side 12 in. and also from a rectangular prism $12'' \times 10'' \times 8''$.

47. A conical heap of lime has a diameter of 15 ft. at the bottom and a vertical height of 7·5 ft. It is measured by means of a rectangular hollow trough whose internal dimensions are $3\frac{1}{2}' \times 3\frac{1}{2}' \times \frac{3}{4}'$ which is a called a parah. Find how many parahs there are in the cone.

48. A cone 4 ft. high and $2\frac{1}{2}$ ft. diameter at the bottom is placed on the ground and sand is poured over it until a conical heap is formed 5 ft. high and 25 ft. in circumference at the bottom. How many cubic feet of sand are there in the heap?

49. The sides of a right-angled triangle are 6 in. and 8 in. respectively. Find the volume of the double cone formed by the revolution of the triangle round the hypotenuse.

50. ACB is a triangle right-angled at C. D and E are the middle points of AB and AC. DF is perpendicular to BC. The triangle is rotated about AC. What solids are generated. Compare their volumes.

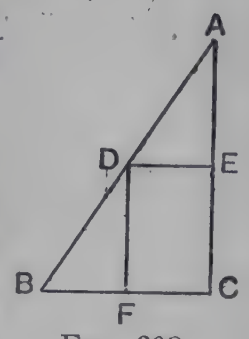


FIG. 308.

Miscellaneous Examples arranged according to Topic.

Section I.—Simplification of Fractions.

1. If $\frac{x}{y} = 3{\cdot}23$, find the value of :—

$$\frac{x+y}{y},\ \frac{x-y}{y},\ \frac{x+y}{x},\ \frac{x-y}{x},\ \frac{x}{x+y},\ \frac{x}{x-y},$$
$$\frac{y}{x+y},\ \frac{y}{x-y},\ \frac{x+y}{x-y},\ \frac{x-y}{x+y},\ \frac{x^2+xy}{x^2-xy},\ \frac{y^2-xy}{y^2+xy}.$$

2. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{1}{3}$, find the value of

$$\frac{2a+3c+4e}{2b+3d+4f} \text{ and } \frac{a-c+e}{b-d+f}.$$

Find the value of the following :—

3. $\dfrac{(\frac{1}{3}+\frac{1}{4})(\frac{1}{3}-\frac{1}{4})}{\frac{1}{2}+\frac{1}{16}+\frac{1}{6}}$

4. $\dfrac{(\frac{1}{5}+\frac{1}{6})(\frac{1}{5}-\frac{1}{6})}{\frac{1}{25}+\frac{1}{36}-\frac{1}{15}}$

5. $\dfrac{\frac{9}{8}\times\frac{9}{8}-\frac{17}{16}\times\frac{17}{16}}{\frac{9}{8}+\frac{17}{16}}$

6. $\dfrac{\cdot38\times\cdot38\times\cdot38+\cdot62\times\cdot62\times\cdot62}{\cdot38\times\cdot38-\cdot38\times\cdot62+\cdot62\times\cdot62}$

7. $\dfrac{11\frac{1}{5}-5\frac{7}{13}}{2\frac{4}{5}-1\frac{5}{13}}\times\dfrac{(4\frac{1}{2}+3\frac{3}{8})(6\frac{2}{3}+4\frac{2}{5})}{(4\frac{1}{2}-3\frac{3}{8})(1\frac{2}{3}+1{\cdot}1)}$

8. $$\frac{\left(1+\frac{a}{b+c}\right)\left(1+\frac{a}{b+c}\right)-\left(1-\frac{a}{b+c}\right)\left(1-\frac{a}{b+c}\right)}{\left(1+\frac{a}{b+c}\right)-\left(1-\frac{a}{b+c}\right)}.$$

9. $$4+\cfrac{5}{6+\cfrac{1}{7+\cfrac{1}{8+\cfrac{1}{9+\cfrac{1}{11}}}}}$$

10. $$a+\cfrac{b}{c+\cfrac{1}{d+\cfrac{1}{e-f}}}$$

Section II.—G. C. M. and L. C. M. and Motion Round Circles.

1. A $= ma$, B $= mb$. The H.C.F. of a and b is c. Find the H.C.F. of A and B.

2. The G.C.M. of two numbers is contained 40 times in one of them and 56 times in the other. Show that this is wrong. How many times the supposed G.C.M. is the real G.C.M?

3. The areas of two adjacent faces of a cuboid are 406 sq. in. and 464 sq. in. Find the dimensions of the cuboid. What is the area of the third adjacent face?

4. If x is the H.C.F. of A and B, it is also the H.C.F. of A—B. Use this principle to find the H.C.F. of 13716 and 13843.

5. $850 = ak + c$, $830 = bk + c$ and $870 = dk + c$. Find the value of k and c given that a, b and d are prime to one another.

6. Two men run round a circle 3 miles in circumference; one runs at the rate of 165 yds. and the other at 132 yds. per minute. If they both start from the same point at the same

time and run in the same direction, when will they be first together again at the starting-point?

7. If, in the previous question, they run in opposite directions, at what distances from their common starting-point will they first meet? If they continue, where and when will they next meet? When will they together reach their original starting-point and what distance will each have travelled altogether by then?

8. A and B start from the same point to run in opposite directions round a circular race course 2,310 ft. in circumference. A does not start till B has run 308 ft. They pass each other when A has run 1,028 ft. Which will come first to the starting-point and what distance will they then be apart?

9. Three horses A, B and C run at the rate of 436, 425, $419\frac{1}{2}$ yds. per minute respectively. If they start together in the same direction round a circular track $\frac{1}{8}$ of a mile in circumference, find how long after the start and at what part of the track they will be next all together.

10. Two men A and B begin at the same time to run opposite ways once round a circular course starting from the same point. From the moment at which they first meet A takes 40 sec. to finish the round, B takes 56 sec. A and B mended their paces in the ratio of 3 : 2 and 5 : 4 from the time when they first met. Find the time each takes to run the whole course.

Section III.—Decimals.

1. Divide 168·34576 by ·000346. Give only the integral part of the quotient and find the remainder.

2. Compute the value of $\frac{1}{\sqrt{5}+1} + \frac{1}{\sqrt{7}-1}$ correct to 2 places of decimals.

3. Calculate to 4 places of decimals the value of $\sqrt{\frac{1+y}{1-y}}$, where $y = \cdot 3875$.

4. Find the value of $1 + \frac{1}{1} + \frac{1}{1\cdot2} + \frac{1}{1\cdot2\cdot3} + \frac{1}{1\cdot2\cdot3\cdot4} +$ . . . to 2 places of decimals the series being unending.

5. Find the value of the series $\frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} - \frac{1}{\lfloor 5} +$ . . . . to 6 places of decimals if $\lfloor 5 = 5 \times 4 \times 3 \times 2 \times 1$, $\lfloor 4 = 4 \times 3 \times 2 \times 1$.

6. If $\pi = 3\frac{1}{7}$, prove that $\frac{1}{\pi} = \frac{1}{4} + \frac{1}{16} + \frac{1}{176}$ by the method of vulgar fractions and also decimally. Use this result to divide 642 by π.

7. Prove $\frac{1}{7\,3\,0\,0\,0} = (1 + \frac{1}{3} + \frac{1}{3\,0} + \frac{1}{3\,0\,0}) \times \frac{1}{1\,0\,0\,0\,0\,0}$ very nearly. Hence find the value of $\frac{8\,6\,7\,5\,4\,8\,9\,6}{7\,3\,0\,0\,0}$.

8. Show that $\frac{1}{1\,7\,6\,0} = \frac{1}{2\,0\,0\,0} (1 + \frac{1}{8} + \frac{1}{8\,8})$. Use this result to convert 6243897 yds. into miles.

9. Show that $\frac{1}{2\,2\,4\,0} = \frac{1}{2\,0\,0\,0} (1 - \frac{1}{1\,0} - \frac{1}{1\,4\,0})$. Use this result to convert 934768 lbs. into tons.

10. Show that $\frac{1}{6\,4\,0\,0} = \frac{1}{1\,0\,0\,0} (1 + \frac{1}{2} + \frac{1}{1\,6})$. Use this result to convert 184368 palams into candies.

11. Show that $\frac{1}{5\,7\,6\,0} = \frac{1}{1\,0\,0\,0} (1 + \frac{1}{2} + \frac{1}{4} - \frac{1}{7\,2})$. Use this result to convert 867843 grains into lbs. (troy).

12. Show that $\frac{1}{8\,6\,4\,0\,0} = \frac{1}{1\,0\,0\,0\,0} (1 + \frac{1}{6} - \frac{1}{1\,0\,8})$. Use this result to convert 1178643659 seconds into days.

Section IV.—Scale Representation.

1. The railway betwen two stations P and Q ascends at a gradient of 1 in 300 from P to a place R distant 15 miles and then descends at a gradient of 1 in 200 from R to Q. If the station at Q is 32 ft. lower than that at P, find the distance from P to Q.

2. A gentleman owns paddy fields which are in the form of a triangle ABC whose sides AB, BC and CA are 250 yds., 350 yds. and 450 yds. D,E,F are the middle points of AB, BC and CA respectively. Join DE, EF and FD. The four plots thus got are given to his four sons. Show

that the property has been equally divided among his four sons and find the extent of each son's share in acres.

3. A ladder 36 ft. long is placed against a wall at a distance of 18 ft. from the bottom. The ladder slips and then comes to rest making an angle of 45° with the wall. Draw a figure to scale 1″ = 10 ft. From the figure find the distance of the foot of the ladder from the wall and verify by calculation.

4. An acre meadow is in the form of an equilateral triangle. Draw its plan stating your scale.

5. A half acre meadow is in the form of a square. Draw its plan stating your scale.

6. A person at a place A cannot walk directly to another place B due north and he has to walk first due east 520 yds., then north-east 650 yds. and then north-west before reaching that place B. Find, by calculation, the distance direct from A to B and draw an accurate figure to any convenient scale.

7. Two sides of a field are parallel and the other two are not. The parallel sides AB and CD measure 175 and 55 ft. respectively and the two diagonals AC and BD are 165 and 75 ft. respectively. Draw a plan of the field to scale and locate the position of a well therein so as to be equidistant from the corners A, B and D.

8. A billiards table is 140 in. by 70 in. There are 6 pockets, 4 pockets A, B, C, D at the corners and 2 pockets P and Q at the centres of the lengthwise sides AB and CD respectively. The plain ball is $2\frac{1}{2}$ ft. from AD and 2 ft. from AB; the spotted ball 2 ft. from P in a direction making an angle of 60° with PB; and the red ball makes a right angle with the plain and spotted balls and is $3\frac{1}{2}$ ft. from Q. Draw a neat figure to any convenient scale and find the distance of the red ball from the two opposite corners B and D.

9. Construct a quadrilateral ABCD, AB = 3·5 cm., BC = 4·7 cm., AC = 5·3 cm., angle D = 72° and AD = 48 cms. Draw another quadrilateral of the same shape

having each of its sides $\frac{3}{5}$ of the corresponding side of the original figure. If the original figure represents a field drawn to scale 1 cm. = 1 furlong, find area of the second figure in acres.

10. ABCD is a quadrilateral field; AB = 176 yds., BC = 132 yds., CD = 121 yds. and DA = 154 yds., angle B = 90°. Draw a figure to scale 22 yds. = 1 cm. Find AC by calculation. Hence find the area of the field in acres and square yards.

11. Represent a length of 5 miles 660 yds. by a straight line drawn to scale 1 in. = $\frac{1}{2}$ a mile.

12. Represent a weight of 2 qrs. 15 lbs. by a straight line drawn to scale 2 in. = 1 qr.

Section V.—Partnership.

1. Three partners A, B and C invest Rs. 4,000, Rs. 5,000 and Rs. 7,500 respectively. A acts as manager for which he receives $\frac{5}{16}$ of the profits and the remaining profits are distributed in the ratio of their capitals. If A gets Rs. 260 more than B, find the total profits and the profits of each.

2. A, B and C enter into partnership with £1,100, £1,300 and £1,700 respectively. Some months later, £500 was needed and supplied by B. At the end of the year the profits amounted to £2,513-14-0 and A's share thereof was £623-14-0. When did B supply and what were B's and C's profits?

3. A and B enter into business together, A providing $\frac{3}{5}$ of the capital. At the end of the first year, they make a profit of $3\frac{1}{2}$ per cent. on the capital, at the end of the second year a profit of 5 per cent. on that year's capital. At the end of the third year they are bankrupts and can pay only 10 *s.* in the pound. If they are now worth £ 16,663-10-0, how much did each contribute?

4. A and B are partners in a book-selling business, their

capitals being as 5 : 7. In the first year they secure a profit of 20 per cent. on their capital and in the second 30 per cent. If 5 per cent. of the profits is set apart for expenses and 15 per cent. is paid to A for management, what average rate of interest per annum do the shareholders get on their money ?

5. A began to trade with a capital of a rupees. After m months B joined him with a capital of b rupees. At the end of the year the profits amounted to p rupees. Express A's and B's shares of the profits in terms of a, b, p and m. If $a = 4{,}500$, $b = 3{,}000$, $p = 1{,}300$, find m when B's share of the profits is Rs. 400.

6. A and B enter into partnership for a year. A contributes Rs. 4500 on January 3 and B contributes on May 3 as much as entitles him to half the gain at the end of the year ending January 2. What was B's contribution ?

7. A, B and C, start business contributing to the capital in the proportion of 6, 9 and 5 respectively. At the end of the first year A withdraws Rs. 500 of capital which is replaced by B. At the end of the second year, B's share of the profit for the two years is Rs. 1,610 and C's Rs. 700. Find the amount of capital invested by each supposing the profit was made in each year.

8. A and B engage in trade their capitals being as 5 : 7. At the end of 4 months A withdraws $\frac{5}{7}$ of his capital, B withdraws $\frac{1}{3}$ of his capital at the end of 3 months but at the end of $7\frac{1}{2}$ months added 6 times what A had withdrawn. How should a profit of Rs. 220-1-0 be divided at the end of the year. ?

9. Three graziers A, B and C agree to hold a pasture in common. A put in 10 oxen for 8 months ; B 12 oxen for 7 months ; C 5 oxen for 10 months at a rent of Rs. 428 per annum. How much should they each respectively contribute towards the rent ?

10. A and B enter into partnership, each contributing capital and A having salary as manager Rs. 3,700 per year ;

A supplies Rs. 40,000 and B Rs. 25,000. The gross receipts for a year are Rs. 20,000 of which $\frac{1}{8}$ is paid away for expenses, and 4 per cent. for insurance. Find the shares of A and B.

11. A and B are two partners. A's capital being Rs. 900, and B's Rs. 1,500. A is to have 10 per cent. of the profits for managing the business. The rest of the profits is to be divided between them in proportion to their capitals. If B's share of the profits is Rs. 630, find how much A receives.

12. A and B started a business with a capital of Rs. 3,000 of which Rs. 2,100 belonged to A. If at the end of the year the total profits amounted to Rs. 1,275 find how much A got more than B as his share of the profits.

13. A began to trade with Rs. 900. After 4 months B joined him with Rs. 600. At the end of the year the profits amounted to Rs. 273. How much will each get? How much more would each have got if B joined at the very commencement of the business.

14. A, B and C entered into a partnership with sums of Rs. 18,000, Rs. 15,000 and Rs. 14,000 respectively. A withdraws Rs. 6,000 at the end of three months and the deficit in the capital is made up equally by B and C. If the profits at the end of the year is Rs. 1,12,800, find A's share.

15. A, B and C enter into partnership with capitals of £44,000, £52,000 and £68,000 respectively. Some months later, an extra capital of £20,000 being required, it was supplied by B. At the end of twelve months the total profits were £10,340 and A's share was £2,640. When did B supply the extra capital?

16. A starts a business with a capital of £600. After 5 months he takes B with a capital of £400; one month later C with a capital of £900. At the end of the year the profits are divided and B finds that he had to pay an income tax of 10 guineas at the rate of 6*d*. in the £. Find the total profits.

17. A and B enter into partnership each contributing capital and B having a salary as manager. A supplies a lakh and a half of rupees. B supplies Rs. 90,000. The total profits for a year are 56,000 of which $\frac{1}{7}$ is paid away for expenses and 5 per cent. for insurance. A receives Rs. 25,000 as his share of the profits. What is the amount of B's salary as manager?

18. A and B enter into partnership. A contributes Rs. 6,000 for 8 months and B Rs. 4,000 for 6 months. They gain Rs. 1,260; find each man's share of the gain.

19. A and B are partners in a firm and contribute £1,500 and £2,500 respectively. A is to have 10 per cent. of the profits for managing the business and the remaining profits should be divided in proportion to the capital contributed by each. The profits at the end of the year amount to £900. How much of it must each receive?

20. A starts business with a capital of Rs. 2,000; a year later he takes B into partnership with a capital of Rs. 8,000 and in two years more they divide a profit of Rs. 1,650. What were their shares?

21. A and B rent a field for Rs. 250. A puts in 15 cows for 8 months and B puts in 20 cows for 6 months. How much should each contribute to the rent?

22. A and B engage in a trade their capitals being as 3 : 2. At the end of 3 months, A takes out a sum equivalent to $\frac{1}{3}$ of B's capital and at the end of another 3 months B puts in a sum equivalent to what A took out. (*a*) If the profits at the end of the year are Rs. 1,450 find the amount of the profits of each. (*b*) If A's profits at the end of the year are Rs. 110 more than B's, find the profits of each.

23. A starts business with a capital of £ 2,000. After 3 months he takes B with a capital of £3,500 and 4 months later C with a capital of £1,500. At the end of the year the profits are divided and A finds that he has to pay £3 as income-tax. Find the total profits conserved, the tax being at the rate of 6*d*. in the £.

24. Two partners invested Rs. 5,400 and Rs. 3,000 respectively in their business and arranged that 40 per cent. of the profits should be divided equally between them and the remaining profits treated as interest on the capital. If one partner's share of the profits is Rs. 90 more than the other's share, find what the whole amount of the profits was.

25. A and B hire a pasture for Rs. 123 for 30 days. A puts in 12 cows for a certain number of days and pays Rs. 63. B then puts 10 cows for the remaining days and pays the remaining sum. For how many days do A and B put in their cows?

26. A and B rent a pasture for Rs. 1,317 for 120 days. A puts in 180 sheep and B puts in 249, but at the end of 80 days A disposes of half his number and then C puts in 120. Find what each should give of the rent.

27. A, B and C rent a field for Rs. 6,296. A puts in 24 horses for 5 months and 90 sheep for 6 months; B puts in 30 oxen for 6 months and 108 sheep for 2 months; C puts in 12 horses and 96 oxen for 3 months. If 4 horses and 3 sheep together eat as much as 5 oxen and 1 horse, and 2 oxen eat as much as 7 sheep, how much of the rent should each person pay?

28. A starts a business at the beginning of a year with Rs. 3,000. On March 1, he takes a partner B with Rs. 4,000, and on June 1, he receives another partner C with Rs. 5,000. The profits at the end of the year amount to Rs. 1,480. What share of the profits should each partner receive? What is the rate per cent. per month of the profits on the capital invested?

29. A and B enter into partnership: A supplies the whole of the capital, amounting to Rs. 45,000 upon condition that the profits are to be equally divided and that B pays A interest on half the capital at 10 per cent. per annum, but receives Rs. 120 per mensem for carrying on the concern. Find their total yearly profits when B's share is equal to one half of A's share.

30. Two persons rented out some fields for £243 for 10 months. The one put in 27 oxen to graze for 3 months, and the other 270 sheep for 7 months. If 3 oxen eat as much as 11 sheep how much of the rent ought each to pay?

31. The capitals contributed by A and B to a joint concern are in the proportion of 19 : 21 and the profits received by them are in the ratio of 8 : 7. If A's capital is in the concern for 9 months, how long is B's?

32. A, B and C rent a field for Rs. 226. A puts in a number of cattle for 6 months; B put in 40 for some time; and C 50 for 7 months. If the shares of the rent paid by A, B and C are in the ratio of 42 : 36 : 35, find the number of cattle put in by A and the number of months for which B put in his cattle.

Section VI.—Proportion.

1. From the following sets of values of x and y, examine whether y varies inversely as x or y varies inversely as x^2. Find the variation constant.

x	3	4	5	6	7
y	1·33	·75	·48	·33	·24

2. The area of a circle is proportional to the square of its radius. If the area of a circle of radius 8 is 200 sq. in., what is the radius of a circle of 256 sq. in.? Find a formula giving the area of any circle when its circumference is known.

3. A copper plate 4 m. long, 3 m. 6 cm. broad and 1·5 cm. thick weighs 1,200 kg. How much more would it weigh if it had been made 6 cm. broader and 5 mm. thicker? Answer to the nearest gram.

4. The square of the time that a planet takes to revolve round the sun varies as the cube of its distance from the sun. If the earth whose distance is 93 millions of miles takes one year, how long will Saturn take whose distance is 870 millions of miles?

5. A map is on a scale of 1 inch to a mile. What fraction is a field on the map of its actual size?

6. On a map an area of 40,000,000 sq. m. is represented by 1·6 sq. m. What is the scale of the map?

7. The weight of a sphere of copper is 6 lb. What would be the weight of a sphere of gold of thrice the diameter, gold being $\frac{19}{9}$ times as heavy as copper bulk for bulk.

8. A solid expands so that every linear measurement is increased in the ratio of 1·0025 : 1. In what ratio is the volume increased?

9. A well is fed by a spring which flows continually and uniformly into it. When there are 10,000 c. ft. of water in the well, 7 men can empty in 20 days, but when there are 15,000 c. ft. of water 5 men can empty it in 50 days. How much flows into the well in a day?

10. On a map is drawn a triangular piece of land bounded by three roads measuring 3·5 in., 4·5 in. and 4 in. The whole map is 18 in. long, 12 in. wide and represents 216 sq. miles. What area does the triangle represent? Give the result in sq. miles and acres.

11. A recipe directs the mixture of a c. cm. of a substance A with b c. cm. of a substance B; but by mistake a grams and b grams respectively are taken. Find the proportion of A and B in the mixture estimated by volume it being given that p c. c. of A weigh x grams and q c.c. of B weigh y grams.

12. If the value of diamonds is proportional to the square of their weights and a diamond worth Rs. 1,28,750 is divided into three pieces whose weights are 11 : 13 : 15, find the value of the pieces.

13. The annual expenses of a convalescent house are partly constant and partly proportional to the number of inmates. The expenses are Rs. 7,680 for 24 patients and Rs. 8,640 for 32. Find the cost of maintaining 30 patients. Verify your answer graphically.

14. The grant to a school was partly fixed and partly dependent on the number of scholars at a fixed rate per scholar. When the number was 201, the total grant was Rs. 8,540 and when the number was 228, the total grant was Rs. 9,620. Find the payment per scholar and the total grant when the number of scholars was 285. Illustrate graphically.

15. The alloy of which the sovereign is made consists of gold and copper in the ratio of $g : c$. The alloy is worth £ a per lb. and fine gold £ p per lb. Express the value of copper in terms of a, g, c and p. What does copper cost per lb. to the nearest penny if $g = 22$, $c = 2$, $a = 44{\cdot}008$ and $p = 48$?

16. A vessel when lined with lead weighing 6 lb. per sq. ft. weighed $7\frac{1}{2}$ cwt. and weighed $4\frac{1}{2}$ cwt. when lined with copper weighing 3 lb. per sq. ft. How much did this vessel weigh when unlined?

17. The charge for luggage on a French Railway is proportional to the distance and to the excess of luggage above a certain weight. If the charge for 60 kg. for 100 km. is 2 fr. 50 cents, and for 120 kg. for 300 km. is 18·75 fr., find the charge for x kg. for y km.

18. A man's income-tax is proportional to the excess of his income above a certain amount. If he pays Rs. 15-10-0 on an income of Rs. 2,500 and Rs. 31-4-0 on an income of Rs. 3,000 find (i) the greatest income on which no tax is paid, (ii) the amount of tax on every rupee on which tax is paid, (iii) the tax on an income of Rs. 3,350 and (iv) the formula giving the tax on an income of R rupees.

19. For printing a circular a printer's estimate is Re. 1-5-0 for 50 copies or Rs. 2 per 100. Assuming each of these estimates to consist of (i) a charge for setting up the type, independent of the number of copies to be printed and (ii) a charge for printing and paper, proportional to the number of copies printed, find what his estimate for printing 2,000 copies would be.

20. An article imported from a foreign country pays a tax made up of a fixed sum and a sum which is a certain fraction of its value. If the tax on two articles worth Rs. 40 and Rs. 100 is Rs. 15 and Rs. 32 respectively, find how the tax is calculated.

21. A contractor undertook to build a railway station in 1 year 6 months. He employed 100 men, 50 women and 40 boys. At the end of 6 months he found only $\frac{1}{4}$ of the work completed. How many more men should be employed so that he could finish the work within the stipulated time? Three men were equivalent to 5 women and 5 boys were equivalent to 2 women.

22. A contractor undertook to finish a piece of work in 150 days. He employed 20 men, 30 women and 75 children; but at the end of 60 days finding that only $\frac{1}{4}$ of the work was done he dismissed all the women and 50 of the children and employed more men. The work was then finished 5 days before the stipulated time. Assuming that 3 men could do as much as 5 women and 2 women as much as 3 children, find how many additional men were employed.

23. When iron was at £2-14-2 per ton and the rupee at 1s. $7\frac{1}{2}d.$, the cost of laying a railway with iron rails weighing 50 lbs. per yard was Rs. 2,78,250. Find what will be the cost of relaying it with steel rails weighing 75 lbs. per yard when the price of steel is £3-17-6 per ton and the rupee is at 1$s.$ $1\frac{1}{4}d.$

24. The cost of rice for a family of 2 adults and 3 children from January 1, 1889 to December 11, 1889 both days inclusive, during which time rice was selling at 15·4 seers per rupee, was Rs. 70-7-0. What was the cost of rice for a family of 3 adults and 3 children from December 19, 1889 to May 11, 1890 both days inclusive assuming the price of rice was 14·7 seers per rupee, that the quantity required per day by each adult was the same in both cases, and the quantity required by a child was $\frac{2}{5}$ of the quantity required by an adult?

Section VII.—Percentages.

1. p per cent. of a certain ore is lost in heating, s per cent. of the remainder is lost in smelting. If the rest be pure metal, find how much of pure metal could be got out of t tons of ore. Also find what quantity of ore should be used to yield y tons of pure metal.

2. When 10 per cent. is lost in grinding wheat a country has to import 9 million qrs. of wheat; it can export 2 million qrs. of wheat if only 4 per cent. is lost. Find the quantity of wheat grown in the country.

3. In a certain year 2·5 per cent. of the articles given out for delivery from post offices in the Presidency of Madras were returned undelivered. Next year there was an increase of 7·5 per cent. in the number of articles given out for delivery and an increase of 10·5 per cent. in the number of articles returned undelivered. If in this year the number of such articles was 1,957,176, find how many articles were given out for delivery in each year.

4. In a certain year the total value of the exports from the Presidency of Madras showed an increase of 12·5 per cent. as compared with the total value of the exports for the previous year. Of the various items of export, coffee which in the first of these two years represented 13·59 per cent. of the total value of the exports showed an increase of 7·5 per cent. What percentage did coffee represent of the total value of the exports in the second of the two years?

5. In a certain year $\frac{3}{8}$ of the quantity of wheat raised in a country was exported at Rs. 2-6-0 per bushel. In the following year the acreage under wheat showed an increase of $12\frac{1}{2}$ per cent. but the yield per acre was only $\frac{5}{6}$ of what it was in the former year, and of the total amount of wheat raised only one-third was exported. If the value of this at Rs. 2-8-0 per bushel was 375 lakhs of rupees, find the value of the wheat exported in the first of the two years.

6. In an S.S.L.C. Public Examination x candidates

appeared. Out of these y candidates brought up subjects that would not qualify them for University courses of study. z per cent. of the remainder were students belonging to Group I and II, the rest belonged to other groups. l per cent. of the former (Groups I and II) and m per cent. of the latter (other groups) were declared eligible for University courses of study. Find the total number of eligibles in the list published by the University.

7. A man bought 2,000 apples. He sold 500 at a gain of 40 per cent., 500 at a gain of 30 per cent., 500 at a gain of 20 per cent. and the remainder at a loss of 10 per cent. and realized a profit of $100d$. At what price per apple did he buy? Also find at what price per apple the remainder should have been sold to give him a profit of 25 per cent. on the whole.

8. From a vessel filled with spirits and containing a gallons b per cent. is drawn off and the vessel is filled with water. This operation is done three times. Find the amount of spirit left after the third operation. What is the result when $a = 20$, $b = 5$?

9. The maximum number of marks in an examination was 550; and it was distributed in the proportion of 2 : 1 : 1 : 0·5 : 0·5 among the subjects English, Mathematics, Vernacular Composition, History and Elementary Science. A boy secured 248 marks in all. In English he secured as many marks as he did in Mathemetics and Composition taken together, and 80 per cent. in each of the other two subjects. If he obtained 30 per cent. in Elementary Mathematics, find how much per cent. he got in Vernacular Composition and also in English.

10. A merchant buys two kinds of wine, the price of one being 50 per cent. cheaper than the other. By selling the dearer one at 39s. per bottle he made a profit of 17 per cent. At what price per bottle must he sell the other so as to make on it a profit of 20 per cent?

11. Of the total expenditure in schools and colleges for the year 1914–15, provincial funds contributed 82·64 lakhs, local and municipal funds 21·77 lakhs and fees and other sources about 89·11 lakhs. If the total expenditure for the year 1913–14 was 180 lakhs find (*a*) the increase per cent. in the total expenditure for the year 1914–15 and (*b*) express also the contribution from each source as a percentage of the total expenditure to one place of decimals.

12. The following table appears in a report of the Government of india :—

	1911–12	1916–17	Increase	Percentage of Increase
Provincial Revenues	Rs. 26,95,88,080	Rs. 39,16,28,530	Rs. ...	...
Local funds.	10,58,01,140	...	6,79,84,210	...
Municipal funds ...	2,98,41,500	4,93,90,830	...	...

Supply the missing figures (the per centage of increase correct to an integer).

13. One vessel contains milk, m per cent. of which is milk and the rest water. Another vessel contains milk, n per cent. of which is milk and the rest water. How much of liquid must be taken from each vessel in order to obtain a third mixture containing p oz. of milk and q oz. of water ?

14. When the export duty on an article was decreased by x per cent. the export increased by y per cent. Find the percentage of increase in the revenue from the export duty on the article.

15. The side of an equilateral triangle is increased by 25 per cent. By what per cent. has the area been increased ?

16. The side of a cube is increased by p per cent. By what per cent. has the volume been increased ?

17. Of a company's capital 16 per cent. was invested in a farm, 28 per cent. in ships, 36 per cent. in a railway and the rest in mining. The ships rose $22\frac{1}{2}$ per cent. in value, railway rose 30 per cent., farm fell in value $12\frac{1}{2}$ per cent. and the company was just as rich as before. How much had the mineral fallen in value?

18. A man sells a house worth Rs. p to an agent at a loss of l per cent. If the agent disposes of it at a gain of p per cent. what does the purchaser pay? What would he have gained if he had sold it directly?

Section VIII.—Averages.

1. If from a case containing a gross of oranges costing on an average x shillings per dozen, b scores are selected costing y d. each on an average, find the average value per dozen of the remainder. Verify your answer by putting $a = 10$, $b = 12$, $x = 1\frac{1}{4}$ and $y = 2$.

2. The following are the returns from 4 mining companies :—

Companies	Tons of ore	Yield of gold in ounces	Value in £
A ...	18,700	6,260	26,338
B ...	4,034	2,868	11,539
C ...	5,035	1,456	4,560
D ...	1,325	949	3,479
Total ...	...	...	...

Find (i) to the nearest penny the average value of an oz. of gold in A and C and also for the group and (ii) the average number of oz. of gold per 100 tons of ore to the nearest integer for B and D and for the group.

3. The following table gives the population of Travancore in 1911 according to religion :—

Name	Literate	Illiterate	Total	Average number of illiterates per head of literate population
Hindus ...	3,24,843	...	2,282,617	...
Christians ...	1,66,762	...	903,868	...
Mohammedans	21,342	...	226,617	...
All sects ...	...	...	...	...

In the above table fill up the columns that are left blank.

4. The average of $a + b + c$ numbers is $a + b - c$; the average of $b + c$ numbers is $b - c$. Find the average of the rest.

5. The area of England and Wales is 58,324 sq. miles and the average population in 1901 was 558 per sq. mile. The area of London is 117 sq. miles and the average population was 38,774 per sq. mile. Find the average population per sq. mile for the rest of England and Wales.

6. From a return as to the wages of railway men it appears that

9,432 earn on an average weekly wages of Rs. 8-12-0.
7,677 ,, ,, ,, 10-8-0.
3,841 ,, ,, ,, 11-4-0.
504 ,, ,, ,, 12-4-0.
989 ,, ,, ,, 17-8-0.

What are the average weekly wages of the whole lot ?

7. In a workshop there are 35 coolies earning an average daily wage of Rs. 2-8-0; 45 coolies earning a daily wage of Re. 1-12-0; 120 coolies Re. 1-4-0; 230 coolies 14 as. Find the average daily wage of a cooly in the workshop. How will the average be affected if the

coolies get a uniform increase of 25 per cent. in their wages.

8. 320 men dine together four days a week; but on the remaining three days some are absent; the consumption of food is thus reduced for the whole week in the ratio 112 : 109. Find the number of absentees.

9. A cyclist can ride s miles in still weather, x miles more per hour with a certain wind, y miles less per hour against a wind. In what time could he go d miles with the wind and return against it? What is his average speed?

10. The average population of Madras is q inhabitants per sq. mile. Georgetown has an average population of g inhabitants per square mile and its area is a sq. miles. The remaining portion of Madras has s inhabitants per square mile and its area is r sq. miles. Find the area of Madras.

Section IX.—Interest.

1. What advantage does a man gain who, owing Rs. 2,000 which ought to be paid at once is allowed to cancel his debt by four equal quarterly payments of Rs. 500 extending over the next twelve months, money being worth 5 per cent. per annum?

2. The sum of £2,100 is due in 4 years, but it is paid by instalments as follows:—£275 at the end of 2 years, £460 at the end of the third year, £500 at the end of the fourth year and £600 at the end of the fifth year. What amount should be paid at the end of the sixth year in order to clear off the balance? Simple interest is reckoned at the rate of 5 per cent. per annum.

3. What sum will amount to Rs. 1,115-13-9 in 2 years at $3\frac{1}{2}$ per cent. compound interest?

4. A debt of Rs. 2,725 receivable at the end of 2 years is paid at once at a certain rate of discount and the sum is invested in the bank at 10 per cent. interest so as to gain Rs. 25. At what rate was discount allowed?

5. A pays B a debt a year before it is due, discount being allowed at 5 per cent. If B had waited for payment till the end of the year, he would then, money being supposed to produce 5 per cent. interest, have been £5 richer than by the actual arrangement. Determine the amount of A's debt.

6. An estate is brought at 20 years' purchase for Rs. 30,000, $\frac{1}{3}$ of the purchase money remaining on mortgage at 5 per cent. The cost of repairs amounts to Rs. 200 per annum. What interest does the purchaser make on his investment ?

7. Determine the present value of an annuity of £200, at 4 per cent. simple interest, payable for 3 years the first payment being due in a year's time.

8. A sum of money was invested for 4 years, interest payable annually. The rate of interest was 5 per cent. per annum for the first two years and 4 per cent. per annum for the last two; and the amount at the end of four years was £1,164-10-3$\frac{3}{4}$. What was the sum invested ?

9. If I borrow money at 3 per cent. per annum, interest payable yearly and lend it immediately at 5 per cent. per annum interest payable half-yearly (receiving compound interest for the second half-year) and gain thereby at the end of the year Rs. 660, what was the sum of money which I borrowed ?

10. A person puts out £1,197 at 30 per cent. per annum interest and spends at the end of each year £300 more than the annual interest on £1,197 and thus at the end of a certain time has nothing left. If he had spent £300 less than the same annual interest, how much would he have had at the end of the same time ?

11. A person obtains from his banker a loan of Rs. 1,250 on June 2nd. He pays Rs. 450 on July 2nd; Rs. 600 on October 1st and finally pays Rs. 800 on November 1st. What is the credit of the person with the bank on the

31st December, assuming the bank charges 5 per cent. interest per annum on all sums due to it but allows interest at $2\frac{1}{2}$ per cent. per annum on all sums due from it?

12. The population of a country increases at the rate of 7 per cent. every 10 years. What was the population 20 years ago of a country whose present population is 4,007,150?

Section X.—Formulae and Substitution.

1. $V = \frac{d + 34}{v}$. If $v = \frac{4}{3} \times \frac{22}{7} \times (\cdot 05)^3 \times \frac{1}{1728}$, and $d = 150$, find V.

2. $\frac{A - s_2 (A - a)}{1 - \frac{s_2}{s_1}} = k$. Find k when $A = 150$; $a = 140$; $s_1 = 19$, $s_2 = 10$.

3. $s = \frac{n(n+1)(2n+1)}{6}$. Find s, when $n = 50$.

4. $d = 1 \cdot 42 \sqrt{H}$. Find d, if $H = 10$, and H if $d = 15$.

5. $A = \frac{1950 B}{E}\left(1 + \frac{40}{E^2}\right)$. Find B, when $A = 15{,}000$, $E = 60$.

6. Evaluate (i) $\sqrt{s(s-a)(s-b)(s-c)}$.

(ii) $\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ when $a = 2 \cdot 5$, $b = 3 \cdot 5$, $c = 4 \cdot 5$ and $2s = a + b + c$.

7. $R = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$. Find R with the same values as in Question 6.

8. In Question (1) express v in terms of V and d.

9. In ,, (2) ,, a, ,, of A, k, s_1 and s_2.

10. Express B in terms of other quantities in Question 5.

Section XI.—Short Methods.

1. Multiply 186248 by 11998 in the shortest method possible.

2. Divide 98706245 by 15625 in the shortest method possible.

3. Find the value of 987 articles at Rs. 5-15-11 in the shortest method possible.

State with reasons why the following answers are wrong :—

4. $\frac{86{\cdot}78493 \times 800{\cdot}946}{{\cdot}0123} = 100.$

5. $\sqrt{0{\cdot}8136} = {\cdot}4024.$

6. The area of a right-angled triangle whose sides are 160 and 80 units is 12,800 sq. units.

7. The area of an equilateral triangle whose side is 10, is 20 sq. inches.

8. $x^2 + 12x - 35 = (x - 7)(x + 5).$

9. $4x^2 + 14x + 49 = (2x + 7)^2.$

10. Assuming that $\frac{1}{993} = \frac{1}{1000} + \frac{7}{1000^2}$, find the value of $\frac{4}{993} \times$ £87,654 to the nearest penny.

11. Find the value of 99 tons 19 cwt. 3 qrs. at Rs. 20-8-0 per ton.

12. Why is £3-11-11 = £3·786 obviously wrong ?

Section XII.—Square Root.

1. A ladder 52 ft. long will reach to a window 48 ft. above the ground on one side of a street ; and if the foot of the ladder be retained in the same position, it will reach to a window 42 ft. high on the other side of the street. Find the breadth of the street and the distance between the windows.

2. A person observes two light-houses lying in a line due west of him. He then goes towards north-west. After

walking 24 miles, the first is seen due south and after walking 16 miles more the second is seen due south. Find the distance between them.

3. The length of a field containing 21 ac. 3 ro. 25 sq. po. $3\frac{7}{8}$ sq. yds. is twice its breadth. Find the length of the field.

4. A certain number of men and women subscribe a sum of money, the number of women being four times the number of men. Each man subscribes as many annas as there are men altogether and each woman as many pies as there are women altogether. If the total amount subscribed is Rs. 756, find the number of men and women.

5. In a certain lake the tip of a bud of lotus was seen half a foot above the surface of the water. Forced by the wind it gradually advanced and was submerged at a distance of $2\frac{1}{2}$ ft. Compute the depth of the water.

6. ABC is a circle of which AD is a diameter 6″ long. P is a point which divides it in the ratio 5 : 1. MPN is drawn at right angles to AD. Calculate the length of MN.

7. In the previous question DA is divided externally in the ratio of 5 : 1 at Q. From Q a tangent QR is drawn to the circle. Calculate the length of the tangent QR.

8. The volume of a wooden cone of height 6 in. is 924 cubic inches. Find the volume of a cone cut off from it by a plane parallel to the base at a vertical distance of 2 in. from the vertex.

9. Draw two straight lines whose lengths are $\frac{\sqrt{5}-1}{2}$ in. and $\frac{\sqrt{5}+1}{2}$ in. respectively.

10. A room is 40 ft. long, 30 ft. broad and 20 ft. high. What is the length of the longest pole that may be placed in the room ?

Section XIII.—Equations and Problems.

1. Solve graphically $5x + 6y = 11$, $8x - 2y = 6$. Verify by calculation.

2. The values of x and y which satisfy both the equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ are given by

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}.$$

Verify the roots of x and y by solving the equations.

3. Solve (1) $2x - 3y - 4 = 0$ (2) $4x + 3y = 11$.
$3x + 4y + 9 = 0$ $6x + 2y = 15$.

4. The number of sides of a regular polygon is two more than that of another. The exterior angle of the latter is 6° more than that of the former. Find the number of sides of the polygon.

5. The cost of y articles is £x. The selling price of x articles is £y. The profits amount to 44 per cent. Find the ratio of $x : y$.

6. A company buys the copyright of a book for Rs. 10,000 and gets 20,000 copies of it printed. The printing charges and paper amount to Rs. 12 per 1,000 copies of each forme of 16 pages. The book contains 624 pages. Binding charges amount to Rs. 4 per 100 books. The company sends 500 presentation copies incurring postal charges of 1 anna per copy and sells the rest at Re. 1-12-0 per copy. Find its gain or loss. Frame an equation.

7. The manager of a daily newspaper gained Rs. 2,000 in a certain month when the number of copies sold was 5,000. The monthly subscription for the paper was Rs. 2-4-0. Another month he found that he got thrice as much from advertisements as he did before but the number of copies sold was only 4,500 and the expenses increased by 20 per cent. Find the income derived from advertisements and the expenditure incurred in the first of these two months if his gain in the second month was Rs. 1,425.

8. The head mistress of a private boarding school has 90 and 100 pupils respectively in two consecutive years. She finds that this addition of ten pupils increases her

yearly expenditure by Rs. 262-8-0 but diminishes the average cost by Rs. 3. What was the total expenditure in each of the two years? Frame an equation and solve.

9. A farmer hired 8 men and 6 boys one day for Rs. 6 and the next day at the same rates he hired 6 men and 11 boys for Rs. 6-2-0. Frame the equations and find how much per day he gave to each. Draw also the graphs of the equations.

10. A clerk A gets Rs. 132 after the fourth year of service and Rs. 220 after the fifteenth year; while another clerk B gets Rs. 130 in his fifth year of service and Rs. 190 in his eleventh year. Supposing they both get uniform annual increments find (*a*) the initial salary of each, (*b*) the rate of increment and (*c*) when they get the same salary. Illustrate your answers graphically.

11. Divide unity into two parts such that $\frac{3}{2}$ times one part may exceed $\frac{9}{4}$ times the other part by $\frac{5}{9}$.

12. For the ordinary parts of the Intermediate examination held a second time in the course of an academical year 1,144 candidates appeared; $\frac{8}{11}$ of the number entered for Part I, $\frac{9}{13}$ for Part II. How many men entered for both parts?

Section XIV.—Practical Geometry.

1. Construct a right-angled isosceles triangle on a base 5 cm. long. What is its area?

2. Construct a rhombus of side 3 in. having an acute angle of 60°. Measure the diagonals and hence find its area.

3. ABCD is a grass plot of the shape of a rhombus; AB = 100 yds. and $\angle A = 60°$. If two cows are tethered by ropes 100 yds. long one at A and the other at C represent in a figure the portion of the grass plot which the two cows can graze in common.

4. Construct a trapezium ABCD in which AB is parallel to DC. AB = 100 yds. CD = 60 yds., AD = 40 yds., and $\angle A = 50°$. Bisect BC at E. Through E draw FEG

parallel to AD meeting AB in F and DC produced in G. Compare the areas of the triangles BFE and ECG. Hence compare the areas of trapezium ABCD and the parallelogram AFGD.

5. Describe two concentric circles of radii 3 and 5 cms. Circumscribe a square ABCD about the inner circle. If AB and DC are produced both ways to meet the outer circumference in P and Q, and R and S respectively and PR and QS are joined, find PQ by calculation and calculate the area of the figure PRSQ.

6. A circular race course is 660 yds. in circumference. If there are three winning posts at equal intervals, find by drawing, the distance between any two posts $\left(\pi = \frac{22}{7}\right)$.

7. A section of railway tube is a segment of a circle greater than a semi-circle. The breadth of the tube at the level of the rails is 12 ft. and its greatest height above the rails is 15 ft. Draw the section on any convenient scale and write down the greatest breadth of the tube.

8. Construct a quadrilateral ABCD given AB = 5 cm., AC = 6 cm. BC = CD = DA = 4 cm. Measure BD, $\angle$ BAC and $\angle$ CAD. Join the middle points of the sides of the quadrilateral in order. What figure is formed? Compare the areas of the quadrilateral and the parallelogram.

9. A goat is placed in an acre meadow, that is in the shape of a square. It is tethered to a post at one corner of the field. What should be the length of the tether (to the nearest inch) in order that the goat shall be able to eat just half the grass in the field. (It is assumed that the goat can feed to the end of the tether.) Draw a plan of the meadow and the portion grazed by the goat.

10. Construct a regular octagon of side $\sqrt{2}$ and find its area by any method.

11. A circle passes through three points (12, 17), (12, 17), (10, 25). Find the points at which the circle cuts the axis of y and the axis of x.

Section XV.—Graphs.

1.

Time	2 a.m.	4 a.m.	6 a.m.	8 a.m.	10 a.m.	12 noon	2 p.m.	4 p.m.	6 p.m.	8 p.m.	10 p.m.	Mid-night.
Temperature in degrees ...	36	41	43	46	51	54	58	56	53	45	37	37

Draw a graph showing the changes in temperature during the day.

2. A and B ride to meet each other from two towns **x** and **y** 60 miles apart. A starts at 1 p.m. and B 36 minutes later. If they meet at 4 p.m. and A gets to **y** at 6 p.m., find by means of a graph the time when B gets to **x** and the time when they are 22 miles apart. When A is half way between **x** and **y** where is B?

3. The distance between Jalarpet and Madras is 132 miles. The following trains run :—

(*a*) Madras departure 7 a.m. (to-day).
Jalarpet arrival 3 p.m. (to-day).

(*b*) Madras departure 6 p.m. (to-day).
Jalarpet arrival 7 a.m. (to-morrow).

(*c*) Jalarpet departure 6 a.m. (to-day).
Madras arrival 3 p.m. (to-day).

(*d*) Jalarpet departure 11 a.m. (to-day).
Madras arrival 8 p.m. (to-day).

Represent these journeys graphically and in one diagram. From the diagram say (1) when and where *a* passes *c* and (2) *b* passes *d* omitting stoppages and assuming each train to travel uniformly.

4. The following table gives the average annual expenditure in rupees on a boys' primary school in the Madras Presidency. Plot a graph to represent the variations in the expenditure and find from your graph what the expenditure in 1898–99 was :—

Year	1886–87	1891–92	1896–97	1901–2	1906–7
Expenditure ...	85	89	91	102	119

5. The distance between Triplicane and Adayar is 6 miles. A man leaves Triplicane for Adayar at 6 a.m. walking at 4 miles an hour, stops at Adayar 15 minutes and returns to Triplicane at the same rate. Another person leaves Adayar at 6 a.m. for Triplicane cycling at 6 miles an hour, stops at Triplicane for half an hour and returns at the same rate. Find, graphically, the distance between the two places of meeting and the time that elapses between the first and second meetings.

6. Below is given the temperature record of a patient from midnight to midnight for a day : —

12 midnight	103·6°	12 noon	102·2°
3 a.m.	103·4°	3 p.m.	102·8°
6 a.m.	102·4°	6 p.m.	103·6°
9 a.m.	101·8°	12 midnight	103·8°

Draw a smooth curve showing the variations in temperature that day and find when the temperature was 102° and the temperature at 4.30 p.m.

7. A farmer has two farms for each of which he pays a rent of Rs. 15 an acre and his total rent is Rs. 675. If the rent of one farm were reduced by Rs. 2-8-0 per acre, and that of the other by Rs. 5 an acre, his rent would be Rs. 500. What is the acreage of each of the farms ? Solve graphically.

8. The distance between Madras and Nellore on the North-East line is 108 miles. The following trains run :—

(1) Nellore departure 5 a.m.
Madras arrival 9 a.m.
(2) Madras departure 7 a.m.
Nellore arrival 1·30 p.m.

Supposing the trains travel uniformly, No. (1) train without stopping and No. (2) train stopping at the 36th mile for ½ hour, represent their motions graphically and hence find when and where they meet.

9. Work out, graphically, the following quantities as accurately as you can :—

(i) $\frac{91\cdot3}{64\cdot8}$, (ii) $\frac{91\cdot3}{64\cdot8} \times 4\cdot32$, (iii) $\frac{91\cdot3}{64\cdot8} \times 6\cdot48$.

10. The following table shows the premia payable for five years insured for Rs. 1,000 according to the age at which payments begin :—

Age	20		22		24		26		28		30	
	RS	A	RS	A	RS	A	RS	A	RS	A	RS	A
Premium ...	85	6	88	0	90	13	93	12	96	15	100	4
Age	32		34		36		38		40		42	
	RS	A	RS	A	RS	A	RS	A	RS	A	RS	A
Premuim ...	103	13	107	10	111	10	115	15	120	6	125	2

Draw a graph to give premia for intermediate ages and from it write down the probable premia for a man beginning to pay at the age of (*a*) 27 and (*b*) 39.

Section XVI.—Mensuration.

1. In the centre of a compound 88 yds. by 110 yds. there is a bungalow in the form of a regular octagon having its side equal to 36 ft. Find the cost of raising the level of the compound by 2 ft. at the rate of 2 annas per cubic yard.

2. A kerosene tin with a square base is 13·5 inches high; a kerosene bottle, cylindrical at the bottom and conical at the top, is 3 inches in diameter and 10·5 inches high, the height of the cylindrical portion being 7 inches. If the tin holds 24 such bottles, find the side of the base of the tin ($\pi = \frac{22}{7}$).

3. A stream of water, 8 yards broad at the surface of the water and 6 yards at the bottom, and 2 yards deep flows at the rate of $1\frac{1}{2}$ miles an hour into a tank, 220 yards long and 56 yards broad, which can hold 74,250 tons of water. Find the depth of the tank and the time in which it will be filled (a c. ft. of water weighs $62\frac{1}{2}$ lbs.).

4. A right-angled triangle whose other angles are 60° and 30° revolves about the hypotenuse 6 inches in length. Find, correct to 2 places of decimals, the surface and volume of the solid formed ($\pi = 3{\cdot}1416$).

5. A circular plate of metal of uniform thickness, 1 ft. 6 in. in diameter, is pierced by 15 circular rivet holes, all of the same diameter. If, by the piercing of the holes, the weight of the plate is reduced by $1\frac{2}{3}$ per cent. find the diameter of each hole.

6. If the owner of a square plot of ground of diagonal 220 yds. gets, in exchange, from his neighbour a hexagonal field, having all its sides equal, and if the value of 1 acre of the first plot is $\sqrt{3}$ times that of an acre of the second plot, find the side of the second plot correct to a foot.

7. The diameter of the collecting funnel of a rain-gauge is 4·5 inches at the top and the diameter of the measuring glass (a cylinder) is 1·6 inches. Find the distance between the graduations, each representing one-hundredth of an inch in rainfall, on the measuring glass.

8. A rectangular stone trough is 4 ft. 2 in. long, 2 ft. 6 in. broad and 1 ft. 4 in. high externally and weighs 1,450 lbs. when empty. The interior surface is irregular but it is found that the trough holds 40 gallons. Find, to one place of decimals, how many times the stone is as heavy as water'

bulk for bulk (1 gallon = 277 c. in.; 1 c. ft. of water = 1,000 oz.).

9. The reservoir from which a certain city draws its water-supply has a surface area of $2\frac{1}{2}$ sq. miles. If the city has a population of 450,000 and if the average daily supply is at the rate of 20 gallons to each inhabitant, find, to the nearest inch, what must be average depth of the reservoir so that, when full, it may contain a year's supply (1 gallon = 277·274 c. inches; a year = 365 days).

10. The dimensions of a rectangular room are 25 ft. 4 in. by 18 ft. by 13 ft. 6 in. Find how many cubic feet of air must be supplied by the ventilating pipes in order that the whole atmosphere of the room may be removed in an hour. Also find (in square inches) what must be the least total sectional area of the pipes in order that the current may not exceed $1\frac{1}{2}$ ft. per second.

11. A rectangular column of stone of square section is to be worked down to a cylindrical shape. If the final diameter be less than the side of the square by $\frac{1}{50}$ part, find by what fraction the weight of the column will be reduced. (The area of a circle is ·7854 of that of a circumscribed square.) Also calculate the loss of weight to the nearest ounce, if the original weight be $2\frac{1}{2}$ tons.

12. In a hollow rectangular case of cast iron, the external dimensions of which are length 14 ft. 4 in., breadth 2 ft. and height 2 ft. and the thickness of which is 2 in. are placed 4 equal hollow closed cylinders of steel each 14 ft. long and 1 in. thick and having an internal radius of 4 in. These cylinders are full of mercury, and the remaining spaces inside the case are full of water. If the area of a circle be $\frac{22}{7}$ × square of its radius, and if cast iron be 7 times, steel 8 times and mercury $13\frac{1}{2}$ times as heavy as water and if a cubic foot of water be taken to weigh $62\frac{1}{2}$ lbs., find the weights of the water, the mercury, the steel and the iron respectively and their sum.

13. The height of a solid right cone is 14 in., and its

vertical angle is two-thirds of a right angle. If this cone is cut into two parts by a plane bisecting the axis at right angles, find to 3 places of decimals the volume of each part.

14. A towel 4 cubits long hangs over a roller 3·5 inches in a diameter. One end of the towel touches the floor and the other end is 1 ft. 9 in. above the floor. Find the height above the floor of the centre of the roller (1 yard = 2 cubits).

15. A cylindrical tin of coffee measuring internally $4\frac{1}{2}$ inches in height and 3 inches in diameter is sold at 10 annas; and one $5\frac{1}{2}$ inches in height and $3\frac{1}{2}$ in. diameter is sold at 14 annas. Compare the prices of the coffee in the larger and smaller tins.

16. The surface areas of two reservoirs are 2,500 sq. ft. and 4,000 sq. ft. respectively. If the first discharges into the second at the rate of 600 gallons a minute, how many minutes will it be before the difference in the two levels is diminished by 1 foot? (1 c. ft. = 6·23 gallons.)

17. What is the weight of a canvas tent consisting of a cylindrical lower part 20 ft. high and 48 ft. in diameter surmounted by a conical portion the highest point of which is 27 ft. from the ground, the weight of a square yard of the canvas being $1\frac{1}{5}$ lb.? ($\pi = 3{\cdot}1416$).

18. Find the total cost of making a circular well. First, it is to be dug 28 ft. deep at 1 anna 6 pies per cubic foot and it should be 6 ft. in diameter. It is then to be lined with brick and chunam one foot thick at 3 as. a cubic foot. And finally the sides are to be plastered at the rate of Rs. 4-11-0 per 100 square feet. ($\pi = \frac{22}{7}$.)

19. In the centre of a rectangular plot 110 yards by 70 yards are dug a circular well of radius 21 ft. and a square prismal well of side 20 ft. both having a depth of 40 ft. and the earth so dug out is spread evenly over the plot, find by how much the level of the plot is raised.

20. The average width and depth of a river at its mouth are 360 yards and 6 ft. respectively; the average rate of flow

is 2 miles per hour, and the amount of sediment per cubic foot of water discharged is 1·5 cubic inches. Find the amount of sediment deposited annually; and the depth of the deposit, supposing it spread uniformly over an area of 146 square miles (1 year = 365 days).

Section XVII.—Profit and Loss (Percentage).

1. From the price in a manufacturer's list a trade discount of 20 per cent. is taken off and then from the reduced price a further discount of 5 per cent. is allowed for cash. This leaves the manufacturer a profit of 16 per cent. on the list price. What per cent. of profit does he get on the cost of manufacture? If the cost price of an article is Rs. 8-7-3, what profit does he make?

2. A merchant in Madras consents to supply 100 tons of tobacco to a company in London at $4\frac{3}{4}d.$ per lb. and pays an advance of Rs. 5,000 to a merchant in Rajamundry who promises to supply the required quantity in six months at Rs. 3-12-0 per maund. If the freight and other charges (which the Madras merchant pays) amount to £3-10-0 per ton of 50 cubic feet and if a bale of 250 lbs. occupies a space of 10 cubic feet, find the probable gain or loss of the merchant. (Interest on the advance is calculated at 8 per cent.; 1 maund = 25 lbs.; Re. 1 = 1*s.* 4*d.*)

3. A grocer makes a mixture of chicory and coffee in the proportion of 1 to 6 and sold it at Re. 1-2-0 per lb., thereby making a profit of 20 per cent. If he had paid 1 anna per lb. more for the chicory and 2 annas per lb. less for the coffee, then mixed them in the proportion of 1 to 5 and sold the mixture at Re. 1-4-0 per lb., his profit would have been 50 per cent. Find the price per lb. paid for the chicory and the coffee.

4. A merchant in Madras owes 12,270 marks to a merchant in Hamburg. If exchange on Hamburg is at the rate of 1·32 marks per rupee, while exchange on London is at the rate of 1*s.* $3\frac{3}{4}d.$ per rupee and the exchange between

London and Hamburg is 20·45 marks per £, find, to the nearest pie, how much the merchant will gain by remitting through London instead of direct.

5. A merchant who sold his goods at a profit of $12\frac{1}{2}$ per cent. found that when he allowed 2 per cent. discount off his selling price his business increased by $\frac{1}{4}$. Find whether his total profits were increased or diminished by adopting this plan and in what proportion ?

6. A merchant pays a lakh of rupees for a season's goods. He marks his goods 25 per cent. over the prime cost and from what he sells at this rate realizes Rs. 1,12,500. At the end of the season he sells the remaining goods at reduced rates—one half at a reduction of 25 per cent. on the former prices, one-quarter at a reduction of 50 per cent. and the remainder at a quarter of the former prices. If the expenses amount to 12 per cent. of the sale receipts, what is his rate of profit on the transaction of the season ?

7. A person bought unslaked lime at the rate of 7 parahs per pagoda (= Rs. 3-8-0). On slaking it, every parah gave 3 cubic feet of lime, of which 5 per cent. was unserviceable. The remainder had to be carried to a place 8 miles distant, the cost of carriage being 4 annas per 100 cubic feet per mile. At what rate per cubic foot must he sell it then in order to gain 25 per cent ? Find also the total gain if he buys 350 parahs.

8. The price of flour is Rs. 22-8-0 per sack of 300 lb. In making bread 2 parts by weight of flour are mixed with 1 part of water to make dough, and during the baking some of the water evaporates, so that 120 lbs. of dough make only 100 lb., of bread. If labour, etc., come to 30 per cent. on the cost of the flour and a 2 lb. loaf is sold for 3 annas what percentage of 3 as. is the profit ?

9. Ice is manufactured at 5 pies a lb. and sold at 9 pies per lb. Of the total quantity made $\frac{2}{5}$ is kept for sale at the factory and the remainder sent to branch shops. The loss from melting is 10 per cent. in the case of the former and

20 per cent. in the case of the latter and the agents at the branches receive commission at the rate of $12\frac{1}{2}$ per cent. on the price of every pound sold by them. Find the profit or loss on every ton of ice manufactured.

10. The expense attending the production of a book, the retail price of which is Re. 1-4-0 is 8 as. 6 ps. per copy. The publisher allows the bookseller 20 per cent. on the retail price and gives 13 copies to the dozen ; 6,500 copies are printed and sold. The author is to have 40 per cent. of the profits. How much will he receive ?

11. A merchant invested Rs. 50,000 in purchasing and storing castor seeds at Rs. 12-8-0 per bag of 176 lbs. He sold them all in 3 different markets ; at Rs. 16-8-0 per bag in one market, at Rs. 16 per bag on another and at Rs. 420 per ton in the third market. If the quantities sold in the three markets were as 3 : 4 : 9 and if the expenditure in the transaction amounted to 30 per cent. of the sale receipts, find the gain or loss per cent. in the whole transaction.

12. To make an article takes 24 hours of a workman's time paid at Re. 1-4-0 per hour. The material 5 per cent. of which is wasted in the working costs Rs. 6-8-0 a pagoda weight. The final weight of the article is $14\frac{1}{4}$ pagodas' weight. At what price must it be sold to gain 10 per cent. on the total cost ?

13. A ship which moves at 30 miles in 4 hours requires $3\frac{1}{2}$ tons of coal for 3 hours. If coal costs Rs. 7-8-0 per ton and if the wages of the sailors, etc., amount to Rs. 150 a day, find the cargo it should carry, supposing it to earn one anna per ton freight per mile.

14. A ghee merchant mixes two kinds of ghee and makes a profit of $24\frac{4}{9}$ per cent. by selling the mixture at a certain price per viss ; had he sold each sort at the same price per viss as he sold the mixture, he would have gained $16\frac{2}{3}$ per cent. and 40 per cent. respectively on the cost price of each. Determine the ratio of the mixture.

15. A rectangular box is increased a per cent. in length

and b per cent. in breadth but diminished c per cent. in height. Find the increase per cent. in volume. If $a = 20$, $b = 24$ and if the increase in volume is 25 per cent, find c.

Section XVIII.—Time and Distance.

1. A man rode a certain distance at a uniform rate in 4 hours. If the distance had been 1 mile less and his rate per hour 3 miles more he would have taken 3 hours. Find his rate and the distance.

2. A man rode a certain distance at a uniform rate in 7 hours. If the distance had been 2 miles less and his rate per hour 6 miles more, he would have taken 4 hours and a quarter. Find his rate and the distance.

3. A man took $5\frac{1}{2}$ hours to travel 21 miles driving for 9 miles and walking the remaining distance. If he had driven only 6 miles the journey would have taken $\frac{1}{2}$ hour longer. Find his rates of driving and walking.

4. The distance between two places P and Q is 56 miles. A and B start from P and Q, just at the same time and walk towards one another at the rate of $2\frac{1}{4}$ and $3\frac{3}{4}$ miles per hour respectively. At what distance from P will they meet?

5. If in the above question A and B reach Q and P and return to P and Q when and where will they meet?

6. A can travel a miles in x hours. While B can travel b miles in y hours. How many miles more than B can A travel in z hours? Evaluate the result when $a = 16$, $x = 4$, $b = 6$, $y = 2$, and $z = 2\frac{1}{2}$.

7. A and B are x miles apart. They walk towards each other at a and b miles an hour respectively and meet in c hours after their starting. Express x in terms of a, b and c.

8. A is m miles ahead of B and travels at a miles an hour. At what rate should B travel so as to overtake him in x hours. Find the rate when $m = 10$, $a = 3\frac{1}{2}$ and $x = 2$.

9. A man had to travel a miles in x hours. He ran for y hours at m miles an hour and rode on his cycle for z hours at n miles per hour. If he walked the remaining distance, find an expression for the rate of his walking. Evaluate the expression when $a = 40$, $x = 6$, $y = 2\frac{1}{2}$, $m = 6$, $n = 10$ and $z = 1\frac{1}{2}$.

10. A prisoner of war in attempting to escape to a friendly ship moored within sight of land, swims at the rate of 3 miles per hour and is assisted by the tide which ebbs at the rate of 200 yds. per hour; after he swims for $3\frac{1}{4}$ hours the tide turns, and impedes him as much as it had previously assisted him. He also slackens his speed from 3 to $2\frac{1}{2}$ miles per hour and succeeded in reaching the vessel in another hour. How far was the vessel from the shore?

11. A detective sets out in pursuit of a thief who has a start of 68 miles. At the end of 8 days, he finds that the thief travels 16 miles a day faster than he does so trebles his speed and overtakes him in 6 days. Find their rates of travelling.

12. A and B start at the same time from X to Y riding at 7 and 4 miles an hour respectively. A reaches Y, returns immediately and meets B at Z, 5 miles from Y. Find the distance from X to Y.

13. A passenger train moving at the rate of 45 miles per hour overtakes a mineral train $1\frac{1}{2}$ times as long moving at the rate of 27 miles per hour and passes it completely in 25 seconds. How long would the passenger train take to pass completely a station 165 yds. in length.

14. A and B start at the same time from P and Q walking at the rate of 3 miles and 5 miles respectively. Both of them meet for the first time at Z, 25 miles from Q. Find the distance between P and Q. After reaching Q and P they immediately return and meet for the second time. Find the distance between the first point of meeting and the second point of meeting.

15. A train going at the rate of 60 miles an hour overtakes another train 160 yds. long going at 45 miles an hour in the same direction on a parallel line and passed it in 30 seconds. In what time would the first train pass another of its own length going in the opposite direction at the rate of 30 miles an hour ?

16. A train 176 yds. long overtook a person walking along the line at the rate of 4 miles an hour and passed him completely in 20 seconds. It afterwards overtook another person and passed him in $19\frac{1}{5}$ seconds. At what rate per hour was the second person walking ?

17. A man sitting in a train travelling at the rate of 50 miles an hour observes that it takes $7\frac{1}{2}$ seconds for a luggage train travelling in an opposite direction to pass him. If the luggage train is 330 yds. long find the rate at which it is travelling.

18. A person standing on a railway platform 198 yds. long noticed that a train passed the platform in 15 seconds and himself in 6 seconds. Find the length of the train and its rate per hour.

19. A train travelling at the rate of 40 miles an hour, while inside a tunnel, meets another train of its length travelling at 60 miles an hour and passes it completely in $4\frac{1}{2}$ seconds. Find the length of the tunnel if the first train passes completely through it in 4 minutes and $37\frac{1}{2}$ seconds.

20. A ship goes 12 miles down a river in $2\frac{5}{8}$ hours. If the crew had worked twice as hard, they would have performed the distance in $1\frac{7}{8}$ hours. Find the rate of the river.

21. A boat's crew rowed down from A to B and up again in 100 minutes. If the stream flows at 2 miles an hour and the crew can row in still water at 3 miles an hour, find the distance from A to B.

22. Two rivers P and Q flow at $1\frac{1}{2}$ and $2\frac{1}{2}$ miles per hour respectively from A to B which are 40 miles distant from one another. Two boats R and S start from A to B at the

same time R going in P and S in Q. They reach B and return to A. If the men in R and S can row them in still water at 3 and 4 miles an hour, find which boat will reach A before the other. Find what time is taken by each.

23. A starts from P and walks at the rate of 3 miles per hour. B leaves Q an hour later and walking at the rate of 4 miles an hour takes rest for 30 minutes at the end of 2 hours and continues to walk at the same rate. If the distance between P and Q be 35 miles, find when and where they will meet and verify your answer graphically.

24. A and B row on a river starting simultaneously from the same point, A down stream, B up stream. In $3\frac{3}{4}$ minutes they are 5 furlongs apart. A then turns to follow B and at the end of $12\frac{1}{2}$ minutes from that time the boats have together rowed 1 mile 7 furlongs in all. If the speed of A, B and the stream are constant, how many miles per hour does the river flow?

25. I row against a stream flowing $1\frac{1}{2}$ miles an hour to a certain place and then turn back stopping 2 miles short of the place whence I originally started. If the whole time occupied in rowing be 2 hours 10 minutes and my uniform speed in still water be $4\frac{1}{2}$ miles per hour, find how far up stream I went.

26. Three trains 200, 528, 130 yds. long are travelling on parallel rails at the rates of 15, 24, 30 miles per hour respectively, the first two in the same direction the third meeting them. The second reaches the rear of the first train at the same moment as the third reaches its front. Find the interval in time between the moments when each of the two trains clears the third.

27. Two persons A and B set out on a journey at the same time. They walked at 3 miles an hour and after they had proceeded for $\frac{3}{4}$ of a mile, B returned walking at the same rate to the place of starting. Here he was detained for $\frac{3}{4}$ of an hour and set out again overtaking A who had

all the time been walking, at the end of $2\frac{1}{2}$ hours from the second time of starting. At what rate did he walk ?

28. A train 130 ft. long passes a man travelling in opposite direction in 3 seconds. If he were travelling in the same direction, it would have passed him in $3\frac{3}{4}$ seconds. Find the rate of the man and the train.

29. Two persons A and B leave Tiruvattyur for Mylapore (8 miles distant) : A leaves at 6 a.m. and walks at 4 miles an hour : and B at 6.30 a.m. and bykes at 10 miles an hour. Another person C leaves Mylapore at 6.30 a.m. and goes to Tiruvattyur in a carriage at 8 miles an hour. Find, graphically and by calculation (*a*) when B overtakes A and (*b*) when C meets both A and B.

30. A thief steals a motor-car at 7 p.m. and drives off at 30 miles an hour followed 10 minutes later by the owner in another car at 24 miles an hour. The thief drives 20 miles and then has a break down. After spending 10 minutes in repairing the machine he finds that he can drive only at 16 miles an hour. At what time will he be overtaken ? Illustrate also graphically.

31. Three cyclists leave Madras at 6 a.m., 7 a.m. and 8 a.m. travelling along the same road at uniform rates. The second travels $1\frac{1}{2}$ miles an hour faster than the first, and the third $2\frac{1}{2}$ miles an hour faster than the second. If all the three meet at a particular instant, find the distance from Madras of the place of meeting and the rates at which they have travelled.

32. Three places P, Q and R on a railway are passed by a train at 8, 8.50, and 10.15 ; another train passes P and Q at 9 and 10.15 both trains travelling uniformly. At what time should the second train pass R ? If the first is travelling 25 miles an hour what is the rate of the second ? Illustrate also graphically.

33. A, who cycles to business every morning at $12\frac{1}{2}$ miles an hour regularly passes at the same spot a man B, who is walking at 5 miles an hour. One morning A passes

B a quarter of a mile further along the road than usual. Supposing B is punctual, how many minutes late is A ?

34. An up train, travelling at 30 miles an hour, always passes a down train at the same spot when both are punctual, the down train travelling at 20 miles an hour. One day the up train is 3 minutes late and the down train 4 minutes late. Where will they pass each other ?

35. I have to go to a railway station, after covering a distance of 20 miles and $4\frac{1}{2}$ hours to get there in. On my bicycle I go at the rate of 10 miles an hour, but my front tyre is likely to burst, in which case I shall have to walk with the machine at the rate of 3 miles an hour. How many miles must I ride before I am free from anxiety as to whether I shall catch the train ?

36. A man driving to a railway station at the rate of 10 miles an hour discovers when 15 miles from home that he has left one of his articles behind. He sends word back by a motor-bicycle that happens to be passing, which travels at the rate of 24 miles an hour ; and 10 minutes after the news is received the article is despatched by motor car at the rate of 35 miles an hour. At what distance from home does the article overtake the owner ?

37. A steamer P is in distress and stationary and fires a gun which is heard on another steamer Q coming direct towards her. P fires another gun 5 minutes after the first, and this is heard on Q 4 minutes 45 seconds after the first. At this moment Q is 3 miles from P. How soon will the two steamers be side by side (sound travels at the rate of 1,100 ft. per second).

38. A and B run a mile race and A wins by 66 yards ; when B and C run a mile race B wins by 55 yards. Supposing each runs at the same rate as before and that A and C run a mile race, by how much would A win ?

39. A person standing on a railway platform 275 yards long noticed that a train passed the platform in $22\frac{1}{2}$ seconds

and himself in 10 seconds. Find the length of the train and its rate per hour.

40. Two motorists A and B set out for a place 48 miles distant, A travelling at the rate of 16 miles an hour starting at 6 a.m. and stopping for $\frac{1}{2}$ hour at the end of each hour and B at the rate of 30 miles an hour starting at 8 a.m. without stopping at any place. Represent the motions graphically and from the graph state when and where B passes A and the times when they arrive at their destination. Also calculate the results.

41. A train passed a man standing at the entrance of a tunnel in $4\frac{1}{2}$ seconds and passed completely through the tunnel in 40 seconds: if the tunnel is 781 yards, find the length of the train and its velocity per hour.

42. Two trains A and B running on parallel lines enter a tunnel 2 miles long at the opposite ends at the same time. They meet in $1\frac{1}{2}$ minutes and A has then travelled in the tunnel 440 yards more than B. In $4\frac{1}{2}$ seconds they cross each other and 1 minute 9 seconds later, A has completely passed through the tunnel. Find the lengths and the rates of the trains.

43. A and B start at the same time from Madras and Tiruttani to Tiruttani and Madras (distant 48 miles) at 6 and 8 miles an hour respectively meet at a village P, reach Tiruttani and Madras, return immediately and meet at another village Q. Find the distance from P to Q.

44. The period of revolution of the Earth round the Sun is 365 days and that of Mars (in the same direction) is 687 days. Show that, if the motions are uniform the apparent year of Mars with regard to the Earth is 770 days (i.e. the time in which the Earth gains one revolution with respect to Mars). Show also that the actual year of Venus is about 225 days when the apparent year of Venus as seen from the Earth is 584 days.

Section XIX.—Approximations, Errors, etc.

1. Show that $\frac{1}{997} = \frac{1}{1000} + \frac{3}{1000^2}$ approximately; and use this result to find the value of $\frac{1}{997}$ of Rs. 54,321 correct to a pie.

2. A merchant's monthly average income amounts to Rs. 3,500 and his monthly average expenditure amounts to Rs. 1,700. If these figures are correct to 2 significant figures, find the limits of his annual savings.

3. The sides of a rectangular field are measured as 1,200 links and 750 links. If there is an error of 2 per cent. in the measurement of the sides, find the greatest possible error in the calculated area of the field.

4. In a long account the error of writing Rs. 3-3-0 instead of Rs. 0-3-3 occurred 960 times. But the account added up to Rs. 82-8-0, too little, because another error of writing Rs. 0-4-6 for Rs. 4-6-0 occurred a sufficient number of times. How many times did this error occur?

Do the same when a is substituted for 960, Rs. b, Rs. c, Rs. d and Rs. p substituted for Rs. 3-3-0, Rs. 0-3-3 Rs. 0-4-6, and Rs. 4-6-0.

5. The following are the returns from two Indian Gold Mines for November 1920:—

MYSORE.

18,169 tons of quartz produced	...	8,059·24 oz. of fine gold.
45,575 tons of sand and slime produced	...	4,189·71 ,,

OOREGUM.

12,800 tons of quartz produced	...	7,597·46 oz. of fine gold.
13,320 tons of tailings, etc., produced.		851·65 ,,

Find (a) the average number of ounces of gold (to 3 decimal places) per ton of ore from each mine, (b) the

average number of ounces of gold (to 3 decimal places) per ton of ore from the two mines and (*c*) the value of gold got from the mines at Rs. 72-8-0 per ounce.

6. The following table gives the population and birth-rate per thousand for three great cities :—

	Population	Birth-rate
A	4,675,000	28
B	786,000	32
C	672,000	34

If the population is correct to a thousand and the birth-rate to a unit, find the limits within which the birth-rate for the three cities taken together lies.

7. A boy having to find the value of ·679 of Rs. 127-6-3 to the nearest rupee, finds the value of ·68 of Rs. 127 to the nearest rupee. Show, by clear reasoning, that this gives the correct result.

8. A star is $1{\cdot}375 \times 10^{12} \times 924{\cdot}67$ miles from the earth. If light travels at the rate of 186,000 miles per second, find, to the nearest year, the time taken by light to reach us from the star.

9. With as little working as possible, write down

(*a*) the cost of 190 articles at 1 anna 3 ps. each article ;

(*b*) given $271352 = 13 \times 20873\frac{3}{13}$, the value of ·0271352 ÷ ·013 to five significant figures ;

(*c*) the value of 2511×2489 ; and

(*d*) the value of $\sqrt{112} \times \sqrt{175}$.

10. The following table gives the number of Educational Institutions and Scholars in the Madras Presidency :—

	No. of Institutions		Scholars	
	1911	1921	1911	1921
Public Institutions ...	25,344	35,804	1,087,562	1,688,736
Private Institutions ...	5,291	3,927	128,163	111,177

Find correct to three significant figures (*a*) the increase or decrease per cent. in the total number of institutions and scholars in 1921 ; and (*b*) the increase per cent. in the number of scholars in Public Institutions in 1921.

11. The traffic receipts of a railway company for the week ending June 30, 1920, were Rs. 36,01,866 and the number of miles of railway open was 2,659. For the corresponding week in 1919 the figures were Rs. 36,66,265 and 2,597. Find the decrease or increase in the receipts per mile of railway correct to the nearest anna.

12. Calculate, in kilograms, the weight of rainfall of 2 inches over 1·5 acres of ground, given a cubic foot of water = 1,000 oz. and 1 kg. = 2·2 lb. avoirdupois.

Section XX.—Time and Work.

1. A certain field could be reaped by 9 men in a certain time and 5 boys could do as much as 2 men. Find how many boys would be required, in addition to 35 men, for the reaping of a field of twice the size in a third part of the time.

2. A well is fed by a spring which flows continuously and uniformly into it. When there are 10,000 cub. ft. of water in the well, 17 men can empty it in 7 days and when there are 12,000 cub. ft. of water in the well 14 men can empty it in 12 days. Find the quantity of water which flows into the well every day.

3. A can do a piece of work in 22 days, B in 40 days and C in 110 days : how soon can the work be done if A is assisted by B and C on alternate days ?

4. A certain field can be planted by 2 men in 12 days or 10 women in 4 days or 6 boys in 10 days. One man, three women and three boys are employed : what is the total cost of planting if the daily wages of a man, a woman and a boy be 8 as., 5 as. and 4 as. respectively ?

5. The capacities of A and B are as 4 : 3. If A's wages be Rs. 6-14-0 a week and B's Rs. 5-6-3 a week what would A have charged for doing a piece of work for which B received Rs. 43-2-0 ?

6. A cistern has a tap P, by which water runs in, and

two others Q and R by which water runs out. When P and Q are open the cistern is filled in $12\frac{1}{2}$ minutes; when P and R are open in 10 minutes; and when P, Q and R open in 15 minutes. Find how long it will take to empty the cistern when Q and R are opened and P is kept closed.

7. A garrison has a supply of water for 50 days. Owing to a leak in the bottom of the reservoir, 10 gallons waste away every day and then the supply suffices for 10 days less; find for how many days the supply would suffice if 15 gallons leak off every day.

8. If 38 men working $7\frac{1}{2}$ hrs. a day can do a piece of work in 20 days, find in what time 57 men working 10 hrs. a day can do a work thrice as great, supposing 2 men of the first set to do as much work in 1 hr. as 3 men of the second set can do in $1\frac{1}{2}$ hrs.

9. A certain number of men, half as many women and twice as many boys, were employed to make a tank bund. The men worked 10 hrs. a day, the women 8 hrs. and the boys 5 hrs.; and all working together finished the bund in 28 days. If each man did in 3 hrs. as much work as each woman did in 4 hrs. and each woman did in 1 hr. as much as each boy did in $1\frac{1}{2}$ hrs. and if the bund could have been made in 39 days by 210 men working 8 hrs. a day, find how many men, women and children were actually employed on it.

10. A and B can do a piece of work in 30 and 45 days respectively; A and B together work for 6 days, when B goes away and C works with A for 5 days; then B rejoins them and the work is finished in $5\frac{10}{13}$ days more. How long would it have taken A, B and C to do the piece of work if they had all worked together?

Section XXI.—Construction of Quadrilaterals.

1. Construct a quadrilateral ABCD given $\angle ABD = 35°$, $AB = 5$ cm., $\angle BAD = 120°$, $BC = 6{\cdot}2$ cm., $\angle DBC = 55°$.

2. Construct a quadrilateral ABCD, given AB = 5 cm., BC = 7·4 cm., CD = DA = 6·2 cm., $\angle$ ABC = 90°.

3. Construct a quadrilateral ABCD given AC = 6·4 cm., $\angle$ CAB = 32°, $\angle$ ACB = 25°, $\angle$ ACD = 32°, CD = 4·2 cm.

4. Construct a quadrilateral ABCD in which $\angle$ ABD = 42°, $\angle$ AD = 4 cm., $\angle$ ADB = 25°, CD = CB = 4 cm.

5. Construct a quadrilateral ABCD in which $\angle$ A = 90°, BD = 5 in., AB = 3 in., AC = 6 in., BC = 4 in.

6. Construct a quadrilateral ABCD in which $\angle$ A = $\angle$ C = 90°, BD = 7·3 cm., AD = 5 cm. and CD = 5·3 cm.

7. Construct a quadrilateral ABCD in which AB = 4·2 cm., $\angle$ BAC = 25°, $\angle$ ABC = 101°, $\angle$ CAD = 20°, and AD = 5 cm.

8. A man starting from his house walks in the eastern direction for 4 miles, then he turns to his right through an angle of 120°. He walks in that direction 2 miles, then again turns through an angle of 45° to the right. If he continues to walk for 1 mile more, find how far he will be from his house in his final position.

9. Construct a parallelogram ABCD in which AB, BC are respectively equal to 3·2 cm. and 5·3 cm. and $\angle$ ABC = 60°.

10. Construct a parallelogram ABCD in which AB = 5 cm., $\angle$ BAD = 110°, AD = 4·8 cm. Measure the opposite sides and remaining angles.

11. Construct a parallelogram in which one side = 5 cm. and two diagonals are 6·2 cm. and 7 cm.

12. Construct a rhombus given its diagonals to be equal to 8 cm. and 7 cm.

13. Construct a rhombus given the two diagonals to be 5·6 cm., and 6·2 cm. Find also the area of the rhombus.

14. Construct a parallelogram ABCD in which AB = 2·5 in., BD = 2·9 in., AC = 3·2 in. Measure the remaining sides and angles.

15. Construct a parallelogram given that AB = 2·6 in., $\angle$ BAC = 36°, AD = 2·8 in.

16. The diagonals of a parallelogram are 5 in. and 7 in. long and cross at an angle of 48°. Construct it and measure its angles.

17. Construct a rhombus ABCD given AB = 2·2 in., and $\angle A = 48°$.

18. Construct a rhombus ABCD in which BC = 6·4 cm., and AC = 8 cm.

19. Construct a parallelogram given two adjacent sides to be equal to 5 cm. and 3 cm. and one diagonal to be 6 cm.

20. Construct a trapezium ABCD given that AB and CD are parallel and that AB = 5·3 cm., CD = 2·3 cm., and $\angle B = 72°$ and BC = 3·4 cm.

21. Construct a trapezium ABCD in which CD = 3 cm., BC = 5·2 cm., DB = 5·6 cm. and DA = 6 cm. (AB being parallel to CD.)

22. Construct a trapezium to the scale 100 ft. = 1 inch, given that its parallel sides are 110 ft. and 154 ft. the remaining two sides are 55 ft. and 66 ft. respectively. Find its area in sq. yds.

23. The oblique sides of a trapezium are both inclined to the base at an angle of 35°, and the parallel sides are 20 and 28 cm. Draw a figure and find its area.

24. The cross section of a railway road is a trapezium 14 ft. wide at the top and 20 ft. at the bottom and 6 ft. in height. If the oblique sides are equally inclined to the base, draw a section of the road and find its area.

25. Construct a trapezium ABCD so that the two parallel sides are 40 yds. and 30 yds., respectively and the two oblique sides are equally inclined at an angle of 40°.

26. Construct an isosceles trapezium whose parallel sides are 20 cm. and 15 cm. and whose oblique sides are each equal to 12 cm. If this represents the plan of a field drawn to scale 1 cm. = 120 yds., find, in acres, the area of the field. Mark in the field the position of a point 3 cm. distant from one parallel side and 5 cm. distant from one extremity of that parallel side.

27. Construct a quadrilateral ABCD in which $\angle A = 90°$, AB = 5 cm., BD = 10 cm., AC = 8 cm., and DC = 5·2 cm. Find its area in sq. cm.

28. Construct a rhombus of diagonals 10 cms. and 12 cms. What is its area if it is a field drawn to scale 1 cm. = 200 yds. ?

Section XXII.—Bankruptcy.

1. A bankrupt pays 10 as. 8 ps. in the rupee and his assets amount to Rs. 1,050. What is the amount of his liabilities ?

2. A person owes Rs. 30,000 to his creditors. But his assets amount to Rs. 12,750. How much in the rupee can he pay ?

3. A ghee merchant sells 1,000 tins of ghee each containing $1\frac{1}{2}$ mds. at Rs. 25 a maund to A who becomes insolvent. If the latter pays 14 as. in the rupee, how much does the ghee merchant lose ?

4. A bankrupt has book-debts equal in amount to his liabilities. All his book-debts amounting to Rs. 6,000 were recovered in full excepting Rs. 3,000 on which he recovered only 10 as. 8 ps. in the rupee. The bankruptcy charges amounted $3\frac{1}{8}$ per cent. of the total book-debts. At what rate does he pay his creditors ? (Answer to the nearest pie.)

5. A bankrupt has book-debts equal in amount to half his liabilities. His liabilities amount to Rs. 10,000; on Rs. 1,200 of the debts he recovers only 10 as. in the rupee and on Rs. 800 he recovers only 4 as. in the rupee. The expenses of the bankruptcy amount to Rs. 100. How much does he pay in the rupee ?

6. A bankrupt has book-debts equal in amount to his liabilities. All his book-debts were recovered in full excepting Rs. 2,000 on which he recovered only 12 as. in the rupee. The bankruptcy charges amounted to $12\frac{1}{2}$ per

cent. of the book-debts and the creditors get 12 as. in the rupee. Find the amount of his liabilities.

7. A bankrupt has debts equal in amount to half his liabilities; but on Rs. 3,000 of them he recovers only 8 as. in the rupee. The expenses of bankruptcy amount to Rs. 500. He finds he can pay only 5 as. 4 ps. in the rupee. Find the amount of his debts.

8. A bankrupt has book-debts equal in amount to his liabilities, but on Rs. 8,640 of such debts he can recover only $8\frac{1}{2}$ as. in the rupee, and on Rs. 6,300 only $5\frac{1}{4}$ as. in the rupee. After allowing Rs. 1,054-11-0 for the expenses of bankruptcy, he finds he can pay his creditors 12 as. in the rupee. Find the total amount of his debts.

9. A bankrupt's assets amount to Rs. 7,500; his known liabilities are such that he can pay only a dividend of 8 as. in the rupee. He has three creditors A, B and C; the sums he owes them are in the ratio of 6 : 5 : 4. In the mean time another creditor D puts forward his claims, and it is found that the assets can only pay a dividend of $7\frac{1}{2}$ as. in the rupee. What amount did the bankrupt owe A, B, C and D respectively?

10. The only three creditors of an insolvent whose assets amounting to £ 200 can only pay 5*d*. in the £ agree among themselves to take dividends in the proportion of the number of pounds, shillings and pence respectively contained in the amounts due to them. The dividends thus taken are in the porportion of 12 : 7 : 6. What are the amounts of their debts?

11. The only three creditors of an insolvent whose assets amounting to Rs. 200 can only pay $8\frac{5}{8}\frac{6}{9}$ pies in the rupee agree among themselves to take dividends in the proportion of the number of rupees, annas and pies respectively contained in the amounts due to them. The dividends thus taken are in the ratio of 5 : 8 : 12. What are the amounts of their debts?

12. Two dividends are paid under a bankruptcy, the

first of 12*s.* 6*d.* in the £, the second of 7*s.* 6*d.* in the £. On the deficiency after the payment of the first dividend a creditor receives as his second dividend £81. What is the debt?

Revision Papers—Second Series.

I.

1. Do not work the following questions, but give a reason for thinking that the suggested answers are obviously wrong:—

(*a*) Which is the greater 26 cub. ft. or a cubic metre? (1 m. = 39·3701 in.). Answer 26 cub. ft.

(*b*) 573×903. Answer 550,799.

(*c*) $\frac{764 \times 2356}{5864 \times 3650}$. Answer $\frac{1}{3}$.

(*d*) $\sqrt{\cdot 784} = \cdot 28$.

(*e*) The area of a triangle of sides 10 cms., 14 cms. and 16 cms. Answer 98 sq. cms.

2. A man borrowed Rs. 20,000 from a money-lender at 7 per cent. per annum simple interest. He lent Rs. 8,000 at 12 per cent. per annum and invested the balance in purchasing a house yielding a rent of Rs. 600 at the beginning of every year. This was invested in a bank paying 8 per cent. compound interest. At the end of two years he sold the house for Rs. 13,000 and collected all his dues and paid his creditor to the full extent. Find his gain correct to the nearest rupee?

3. A farm consists of two kinds of lands wet and dry. In a certain year the income from one acre of the former was Rs. 40 and that from the latter Rs. 25 and the total amounted to Rs. 600. In the next year the income from the wet lands decreased by Rs. 8 per acre and that from the dry increased by Rs. 5 per acre and the total fell short by Rs. 40. Find the number of acres under each head? Give a graphical solution mentioning the scale you choose.

4. A motor car travelling at the rate of 40 miles an hour starts from place P to Q a distance of 150 miles. Fifteen minutes later another car travelling at the rate of 60 miles an hour leaves P for Q stopping at a place 35 miles from P for 5 minutes ; find graphically

(*a*) at what distance from P the second car overtakes the first and

(*b*) the difference between the times of arrival of the two cars at Q.

5. Construct a quadrilateral ABCD given BD = 4 in., angles A and C are right angles, $\angle ABC = 75°$, $\angle ABD = 30°$, measure the sides BC, CD, DA and AB. Verify, by calculation, and hence, find the area. If an inch represents 100 yds., express the area in acres and sq. yds.

6. Express $\frac{1}{108{\cdot}566}$ as a decimal, correct to the nearest millionth. Use this value to find the diameter of the earth from that of the sun which is 842,765 miles assuming the diameters of the earth and the sun are in the ratio of 1 : 108·566.

7. Find the square root of ·00099856.

8. Find roughly the value of (*a*) $\frac{2088}{808}$, (*b*) $\frac{1748 \times 8756}{24356 \times 693}$.

II.

1. Resolve into factors :—

(*a*) $25x^2 + 55x - 42$.

(*b*) $(x^2 - 2x + 5)^2 - (x^2 + 4x + 1)^2$.

2. Construct on squared paper a quadrilateral ABCD of which two sides AB and BC of lengths 1·1 and 2·4 inches contain an angle of 130°, and the other two sides CD and DA are of lengths 2·5 and 3 inches. Find the area of the figure by counting the squares.

3. A man measures the length and breadth of a room by means of a walking stick and estimates it to be 15 ft. by $12\frac{1}{2}$ ft. In making the calculation he assumes that the

stick is 3 ft. long, the actual length being 2 ft. 9 inches. Find the real measurements of the room and the error in the area to the nearest square foot and also the relative and percentage errors to two significant figures.

4. An Express train starts from Egmore at 3 p.m. and the Boat Mail at 7 p.m. and both go south, the former at 15 miles per hour stopping at every tenth mile for 10 minutes, while the Boat Mail goes at 30 miles an hour and stops at 36th, 72nd, 108th miles for 20 minutes. Find graphically when and where the Boat Mail overtakes the Express.

5. A and B are 2 points on the circumference of a circle whose diameter is 3 inches. AB $= 1\frac{1}{2}$ in. Draw tangents at A and B. Measure them and also the angle between them. Verify by calculation. Also find the area of the figure formed by the tangents and the radii at their points of contact.

6. A man performs his journey partly by rail and partly by river, the former at 3 pies per mile and the latter at 6 pies per mile ; the fare amounts to Rs. 3-12-0. But if the rate of the former be reduced by $\frac{1}{2}$ a pie per mile, and the latter increased by 1 pie the fare is increased by 5 as. Find the distance performed by rail and river. Verify graphically (scale $1'' = 100$ units for both axes).

7. Given that a gallon of water weighs 10 lbs. avoirdupois, that 1 lb. avoirdupois contains 7,000 grains and that a cubic inch of water weighs 252·458 grains, calculate the number of cubic inches in a gallon, correct to three places of decimals.

8. Construct a rectangle whose diagonal is 1·3 inches and one side is 0·5 inches. What is the length of the other side ? Find the area of the rectangle.

III.

1. Find to within 10,000 sq. miles the area of the earth's surface given that its radius is 3,959 miles and that

the surface area of a sphere is 3·1416 times the square on the diameter.

2. Find the value correct to 5 places of decimals :—

$$1 + \frac{1}{9} + \frac{1}{9^2} + \frac{1}{9^3} + \ldots\ldots$$

3. Construct a quadrilateral ABCD having AC equal to 2·5″, ∠ B equal to 90° and AB equal to 1·5″, D being equidistant from B and C. ∠ CAD = 45°. Measure the remaining sides and angles and find the area. Verify by calculation.

4. In a circle two parallel chords 6 cms. and 8 cms. are placed at a distance of 1 cm. Find the radius of the circle.

5. A father's age is four times that of his eldest son and 5 times that of his youngest son; when the eldest son has lived to 3 times his present age the father will exceed twice that of his youngest son by 3 years. Find their present ages.

6. A man has two plots of ground one a square whose side is 3,806 yds. long and another a rectangle 587 yds. long and 374 yds. broad. He exchanges them for a single square plot of the same area. Find the length of a side of the new plot and the cost of fencing it at 2 as. 4 ps. per foot.

7. Solve graphically :—

$2x + 3y = 9.$

$3x + 8y - 10 = 0.$

8. A man hires a bicycle by paying Re. 1 for the use of the bicycle and 8 as. for every mile he rides. Express the cost of hiring in terms of the number of miles ridden. Draw a graph.

IV.

1. Without adding or writing down unnecessary figures find the sum correct to the nearest hundred thousand 4682531 + 806908 + 17492714 + 93602.

2. The area of the southern districts of the Madras Presidency is 53,578 sq. miles while the population is 19,366,486.

Find very roughly, with as little work as possible, the population per square mile.

3. Show that when a fraction of a £ is expressed correct to three decimal places, the error is less than $\frac{1}{8}$ of a penny.

4. The diagonals of a rhombus are 8 cms. and 6 cms. Construct the rhombus. What is the length of the side of the rhombus and what is its area ?

5. (*a*) For what value of x is x^2 equal to 389·6679 ?

(*b*) The number of feet (s) through which a body falling from rest descends in (t) seconds under the action of gravity is given by the formula $s = \frac{1}{2} gt^2$ where $g = 32{\cdot}2$ correct to the third significant digit.

A stone dropped from the edge of an overhanging cliff reaches the water in $4\frac{1}{2}$ sec. Find the height of the cliff correct to the units' place.

6. A square field is bordered by a path of uniform width outside, the field and the path together occupying 6 acres 1,936 sq. yds. Find the width of the path if the cost of making it at 6 annas a sq. yd. comes to Rs. 650-10-0.

7. Construct a rhombus of side 3 in. one of the diagonals being 2·4 inches. Find the length of the other diagonal without measurement and calculate its area in acres and sq. yds., when one inch represents 200 yds.

8. A man bought a horse and sold it at a loss of 6 per cent. If he had received Rs. 67-8-0 more he would have gained $12\frac{1}{2}$ per cent. What should the horse be sold for so as to fetch a profit of $22\frac{1}{2}$ per cent ?

V.

1. Prove, with as little calculation as possible, that $39{\cdot}4771 \times 2{\cdot}499531 \div 1249835$ is greater than ·00007 and less than ·00009.

2. Find the value of :—

$$\left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)\left(\frac{\sqrt{7}+1}{\sqrt{7}-1} + \frac{\sqrt{7}-1}{\sqrt{7}+1}\right)$$

correct to two decimal places.

3. A merchant mixes chicory at 3 as. a lb. with coffee at Re. 1-8-0 per lb. and sells the mixture at Rs. 1-9-0 per lb. thereby gaining 25 per cent. on his outlay. Find the ratio of the mixture.

4. A person bought $(a + b)$ tons of hay at Rs. $(a^2 - ab + b^2)$ per ton and $(a - b)$ tons at Rs. $(a^2 + ab + b^2)$ per ton. If he mixes the two and sells the mixture at Rs. $(a^2 + b^2 + c^2)$ per ton, what is his percentage of profit?

5. From a point without a circle whose radius is 1·2 inches draw 2 tangents to it so that the angle between them may be 60°.

6. Solve the equations graphically :—

$$1\tfrac{1}{2}x + y = 5.$$
$$2x - 3y + 2 = 0.$$

7. I wish to buy a certain number of cows and sheep for Rs. 760 at Rs. 20 per cow and Rs. 12 per sheep; had I bought as many sheep as I bought cows and *vice versa*, I should have paid Rs. 80 more. How many animals of each kind did I buy?

8. Two trains start from two places P and Q, 194 miles apart, and travel towards each other at 12 and 16 miles per hour respectively. The train from P starts at 8.10 a.m. and that from Q at 6 a.m. Find, graphically, when and where they will meet.

VI.

1. Given that 1 metre = 39·371 inches, 1 kilogram = 2·2046 lbs. and a cubic metre of water weighs 1,000 kg. find the weight of a cubic foot of water in lbs. correct to the nearest tenth of an ounce using contracted methods of multiplication and division.

2. From the equation $t = 2\pi\sqrt{\frac{l}{g}}$, find l in terms of the other quantities and calculate its value correct to 3 significant figures when $t = 1{\cdot}1$, $g = 32{\cdot}18$, and $\pi = 3{\cdot}1416$.

3. A circular plot of ground is crossed by a foot-track which measures 32 feet from where it meets the ground to

where it leaves it. The distance of this from the middle point of the smaller arc it cuts off is found to measure 8 ft. What is the diameter of the circular plot? Show how a figure of the plot with the track intercepted can be drawn without finding the diameter first.

4. Show that the triangle whose sides are $n^2 + 1$, $n^2 - 1$, and $2n$ units long is right-angled and hence find, geometrically correct to one decimal place, the square root of 12.

5. A lawn tennis ground is half as long again as it is wide. The cost of levelling it at 2*d.* a sq. yd. is £88-4-0. Find the cost of enclosing it with iron railing at £3-4-0 a yard.

6. By what amount does the compound interest on £195-6-8 for 2 years at 4 per cent. per annum exceed the simple interest on the same sum for 2 years at $6\frac{1}{5}$ per cent. per annum?

7. Twenty-five men are engaged in a work which is to be finished in 12 days. If 5 more men were employed after 10 days the work would fall 2 days behind. How many men more should be engaged after 6 days, if the work is to be finished a day sooner?

8. Solve:—

$$\frac{x}{8} - \frac{x-1}{2\frac{1}{2}} = \frac{3x-4}{15} + \frac{2-3x}{12}.$$

VII.

1. Find, correct to two decimal places, the value of $\frac{\sqrt{3}-1}{\sqrt{3}+1}$.

2. The length of a lawn tennis ground is one and a half times its breadth. The cost of levelling it at 9*d.* per sq. yard is £176-8-0. Find the cost of enclosing it with iron railing at 7*s.* 6*d.* per yard.

3. The breadth of a room is 10 ft. less than the length

but if the length be diminished by 15 ft. and the breadth increased by 5 ft., the area of the room is diminished by $21\frac{2}{3}$ sq. yds. Find the dimensions of the room.

4. Construct a quadrilateral ABCD given diagonal AC = 10 cm., $\angle B = 90°$, sides AB and CD are respectively 6 and 5·4 cms. and $ADC = 90°$. Find, by calculation, the lengths of the remaining sides and verify your answers.

5. The diagonals of a parallelogram are 3·6″ and 2·6″ respectively and one side is 2·4″. Measure the other side of the parallelogram and its smaller angle.

6. A train 138 yds. long moving at the rate of 42 miles per hour overtakes another train 192 yds. long moving in the same direction on parallel lines and passes it in 45 seconds. How long will it take to pass if they move in opposite directions?

7. Water has to be diverted from a river through an eight inch diameter pipe running full bore with a velocity of $2\frac{1}{2}$ ft. per second to irrigate a field of 4 acres. How long will it take to deliver an inch of water over the whole area?

8. At noon A starts to cycle from P to Q a distance of 40 miles at 6 miles an hour resting for an hour after riding 12 miles. At 3 p.m. B starts at 10 miles per hour. Find graphically (*a*) when and where B overtakes A? (*b*) If a third cyclist C starts at noon riding from Q to P and meets B at 4 p.m., at what speed does he ride? When will he meet A?

VIII.

1. A square lawn is bordered by a path 4 ft. 6 in. wide the path and lawn together occupying $\frac{1}{10}$ of an acre. Find the cost of paving the path with bricks at 7*s.* 6*d.* per square yard.

2. The diameter of a copper wire is 2 mm. and a cubic decimetre of the copper wire weighs 9·0 kgs. Find to the nearest metre the length of the wire which weighs 7·5 kg.

3. The wages of a man for a day's work are 5*s.* 6*d.* and those of a woman 2*s.* 9*d.* A man can do $1\frac{1}{2}$ times as much

work as a woman can do in the same time. A certain work has to be done which would occupy 50 men for 80 days. Find the difference in the cost if 20 men and 30 women were employed to do the work instead of 50 men.

4. Draw a quadrilateral ABCD having given AB = 3·2″, BC = 2·1″, CD = 3″, DA = 4·1″ and ∠ ABC = 30°. Find its area in square inches.

5. Solve :—

$$\frac{3x+4y+3}{10}-\frac{2x+7-y}{15}=5+\frac{y-8}{5}.$$

$$\frac{9y+5x-8}{12}-\frac{x+y}{4}=\frac{7x+6}{11}.$$

6. In a quadrilateral ABCD, AB = 5 cm., BC = 6 cm., AC = 6·5 cm., ∠ ADC = 150° and AD = 3·4 cm., construct the quadrilateral. Measure the remaining side and diagonal.

7. Construct a graph showing the relation between a rate in so many miles an hour and its equivalent in feet per second; from it deduce how many miles per hour are equivalent to 66 feet per second and also how many feet per second are equivalent to 25 miles per hour. Verify by calculation.

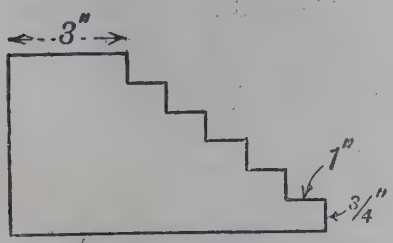


FIG. 309.

8. The figure shows the section of a stair case and landing. Each step is 1 foot wide and ¾ ft. high and 3½ ft. long. There is a cupboard under stair case and landing. How many cubic feet are there in the cupboard?

IX.

1. Find, to the nearest penny, the price per lb. in English money of tea which costs in France 3·50 francs per kilogram, assuming that 1 kg. = 2·204 lbs., £1 = 25·34 francs.

2. From the following data find (to the nearest whole number) how many gallons of beer were drunk per head and

to the nearest shilling, how much per head was contributed to the revenue in this way. (One barrel = 36 gallons.)

Year	Population	Barrels of beer drunk	Revenue in £
1904–5	43,331,000	35,862,068	11,323,679

3. I know the average weight of 5 metal bars to be 2 kg. 75 gm. and the weights of 4 of them to be 3 kg. 8 gm., 2 kg. 92 gm., 2 kg. 14 gm., 1 kg. 9 gm. All these weights are correct to the nearest gram. Find the weight of the fifth bar.

4. Construct a quadrilateral in which AB = 4·5 cms. ∠ B = 82°, ∠ A = 95°, BD = 6·5 cms. and AC = 5·4 cms. Measure CD and the remaining angles.

5. Solve $8x - 5y = 5$.
$3x - 2y = 1$. Verify graphically.

6. A cyclist starts at 4 p.m. and rides from the Iron bridge to San Thome in Mylapore at 4 miles an hour. 15 minutes later a pedestrian walks from San Thome towards the cyclist at 3 miles an hour. When will they meet? When will they be 2 miles apart? The distance between their starting points is 4·25 miles.

7. Find the weight of a heavy steel joint whose cross section is shown in figure 310 and whose length is 16 feet. (1 cubic foot weighs 490 lb.)

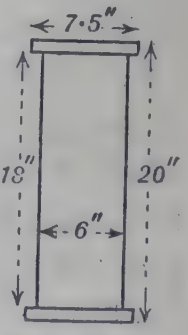


FIG. 310.

8. Describe a circle of 2·8 in. diameter and inscribe in it a square. Measure its sides. Inscribe in it a regular octagon. Find the length of its side and compare their areas.

X.

1 Find the value of $\frac{35{\cdot}705682 \times 581{\cdot}35823}{82{\cdot}05972}$ to two places of decimals, using contracted method.

2. Find the value correct to five significant figures of :—

$$\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4} + \ldots$$

3. What is the error per cent. if $a = \cdot0002$, $b = \cdot0005$, in the assertion that $(1 + a)(1 + b) = 1 + a + b$. Using the value $(1 - a)^n = 1 - na$, find the value of $(\cdot99)^3$ and find the error per cent.

4. Construct a quadrilateral ABCD in which AB = 3·2 cms., BC = 2·9 cm. CD = 4·1 cms., DA = 3·4 cms. and AC = 5·9 cm.

5. The price per bag of rice is Rs. 10-8-0 in Mylapore and Rs. 10 in town ; and the rate for the cooly for carrying the bag is 3 as. per mile. Supposing the distance between the town and Mylapore is 6 miles, find a place on the road between them, where the price of the bag is the same, whether bought in town or in Mylapore.

6. The linear scale of a map is 1 in 400,000. Find, correct to one place of decimals, the number of square miles of a country represented by an area of 6 sq. in. on the map.

7. In a sugar-refining factory the cost of production is partly constant and party dependent on the quantity of sugar produced. When 2,600 tons of sugar are produced the total cost is £7,870, and when 3,000 tons are produced the total cost is £8,350, what will be the expense when 10,000 tons are produced ? (Illustrate graphically.)

8. There is a rectangular pandal 14 ft. long 10 ft. 6 in. broad whose corners rest on four vertical pillars 25 ft. high above the level ground. A rope suspended from the centre of the pandal will just reach the ground, but is tied to one of the pillars at a point 4 ft. above the ground. What length of it will have to be coiled round the pillar, if the rest of it is tight ?

Madras Secondary School Leaving Certificate Public Examination.

1911.

Only *ten* questions to be attempted.

1. Find, graphically, what the remainder is when 189 is divided by factors.

2. A rectangle measures 10·05 cm. by 9·93 cm. ; find the area of the rectangle, without directly multiplying these two numbers together.

3. Find the cost, to the nearest anna, of a school library as follows :—

No. of books	Published Price of each	
	s.	*d.*
128	7	6
200	5	0
20	3	6
236	3	0
256	2	6
104	2	6 net
350	2	0
202	1	0 net

(*a*) 12 annas to the shilling.

(*b*) $6\frac{1}{4}$ per cent. discount allowed off the published prices except for the net books.

(*c*) Packing and carriage 2 per cent. of the published prices of the books.

4. The following are the results of a recent examination:

Centre	No. of candidates sent up	No. of passes
A	24	10
B	119	60
C	327	106
D	218	49
E	196	37
F	623	214
G	257	98
H	69	44

Find, correct to ·1, the percentage of passes for the whole examination; and give the percentage of passes at the most and at the least successful centres.

5. I bought a house for Rs. 6,500 ten years ago. I charge Rs. 40 a month for rent, and pay the house tax (2*s.* 8*d.* a month) and the repairs (averaging Rs. 60 a year) myself. It has been let for an average of $9\frac{1}{2}$ months in every year; and the while it stood empty I had to pay a man Rs. 5 a month to look after it. At the same time I had a friend of mine who deposited Rs. 6,500 in a bank and had drawn annually interest at the rate of $4\frac{1}{2}$ per cent. per annum. Which of us has benefited the more from his investment and by how much?

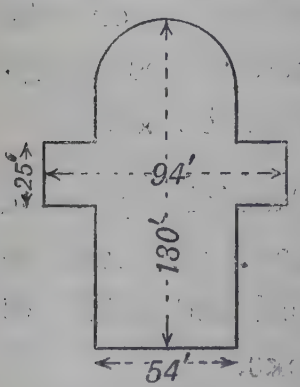


FIG. 311.

6. This is the plan of a lecture hall (not drawn accurately to scale) one end terminating in a semicircle. Find the area of the floor. The average height is 30 ft.; each person requires for health 200 cub. ft. of fresh air every hour: if a lecture lasts $1\frac{1}{2}$ hours and the air in the hall is not renewed, how many people can be accommodated without injury to health.

7. At some school sports the mile race is run on a perfectly circular course, the track being 8 yds. wide.

Four times round the course on the inner edge of the track is an exact mile. What is the length of the outer track?

8. A boy uses a badly divided ruler supposed to be 6 in. long and to be divided into inches. As a matter of fact it is so divided that the first, second, etc., so-called inches are ·93, 1·03, 1·01, ·96, ·94, and 1·01 in. (each 'inch' being divided into ten equal parts). He measures a line and finds it according to his ruler, 3 ft. 4·6 in. What is the real length of the line?

9. A boy hires a bicycle by paying one rupee for the use of the bicycle plus eight annas for every mile that he rides :—

(*a*) Express the cost of hiring in terms of the number of miles ridden.

(*b*) Draw a graph representing the relation between the cost and the distance.

(*c*) From the graph find how many miles he can ride for Rs. 5.

10. The distance from Madras to Tuticorin is 440 miles. The following trains run :—

(*a*)	Monday, Madras	dep.	6 p.m.	Represent these three journeys graphically in one diagram. With no help but the diagram say (*a*) how far each train is from Madras at 11 o'clock on Monday night (*b*) at what time the train from Madras (No. 1) passes the trains (No. 2) and (No. 3) from Tuticorin, omitting stoppages and assuming each train to travel uniformly.
	Tuesday, Tuticorin	arr.	4 ,,	
(*b*)	Monday, Tuticorin	dep.	8 ,,	
	Tuesday, Madras	arr.	6 a.m.	
(*c*)	Monday, Tuticorin	dep.	11 ,,	
	Tuesday, Madras	arr.	7 p.m.	

11. (*a*) Draw a line exactly equal in length to AB.
(*b*) Draw another line $\sqrt{3}$ times the length of AB.

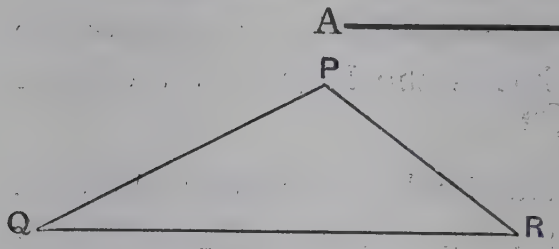


FIG. 312.

12. Measure as accurately as you can, each of the angles and each of the sides of PQR and find its area.

13. On a black-board measuring 4 ft. by 3 ft. is drawn a triangle so that :—(1) one vertex is 18 in. from the top left corner of the black-board, and 24 in. from the bottom left corner. (2) Another vertex is 24 in. from the top right corner and 18 in. from the bottom right corner. (3) The third vertex is 24 in. from the bottom right corner and the angle at this vertex is a right angle. Draw a plan showing the position of the triangle relative to the sides of the black-board. State your scale below the plan. (What is the length of the hypotenuse?)

14. A goat is tied by a rope to a post 15 ft. from a long straight fence. The rope is such that the goat can reach with its mouth 23 ft. from the post. Along what length of fence can it nibble ? Draw to scale, choosing and stating your scale.

15. The full pension granted to an officer retiring from Government service is half his average salary for the last three years of his service ; but in the case of an officer who enters service after an age of 30, the full pension is reduced by $\frac{1}{40}$ for every year by which his age at admission into service exceeds 30. Express the pension of such an officer in terms of a full pension and of his age at admission into service.

Determine what pension a man who enters service at the age of 35 receives upon retiring, if his salary per mensem during his last 3 years of service is Rs. 375 for 10 mos. Rs. 350 for 3 mos., Rs. 370 for 8 mos., Rs. 380 for 9 mos., and Rs. 400 for 6 months.

1912.

Only *ten* questions to be attempted.

	Elementary Mathematics										Total	per cent	Rank
Max ...	20	10	6	16	20	22	14	10	14	18	150		
A ...	14	8	3	13	13	16	12	4	10	12			
B ...	6	1	1	9	5	7	8	2	6	5			
C ...	2	2	0	6	1	4	5	1	3	2			
D ...	20	10	6	16	19	22	14	7	13	18			
E ...	13	3	2	12	12	14	11	3	9	11			
F ...	4	4	0	7	3	6	6	1	4	3			
G ...	1	5	0	5	0	2	4	1	2	1			
H ...	18	9	5	15	16	19	14	6	12	17			
L ...	*a*	4	1	11	*a*	11	10	*a*	8	8			
M ...	5	3	0	8	4	7	7	1	5	5			
N ...	7	2	1	10	6	8	9	2	7	6			
O ...	16	7	4	14	14	17	13	5	11	13			

In a table of 4 columns give—(1) the names (A, B and C) of these boys in alphabetical order; (2) the total of each boy's marks; (3) his percentage (in whole numbers) of the maximum; (4) his rank (determined by his percentage).

In case of absence the mark in column (3) should be the boy's percentage of those maxima only for which he was present.

2. To celebrate the procession of Edward VII a certain town subscribed a sum of money which was spent as follows:—a procession Rs. 57; feeding the poor Rs. 214; fireworks etc. Rs. 136; sports Rs. 143; entertaining children Rs. 184; local hospital Rs. 250; office

expenses Rs. 36. If Rs. 145 had been subscribed last year and spent upon the same objects in the same proportion, how much (to the nearest anna) would have been spent upon each ?

3. (1) Omit (stocks and shares).

(2) Find the simple interest for 9 months and 20 days on Rs. 1,264 at 10 as. per Rs. 100 per mensem.

4. Allowing an average of 23 in. for each boy to stand in, if 198 boys are arranged in a circle what would be its radius ? If they are arranged in a rectangle of which each long side is double each short side what would be the length of its diagonal, in feet, correct to five significant figures ?

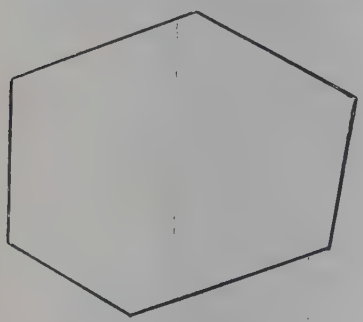


FIG. 313.

5. If this figure (313) is an accurate plan of the boundary of an estate find the area of the estate in sq. yds. (Scale $\frac{1}{10}$ cm. to 50 yds.)

6. This figure (314) is intended to represent a crane, the arm of which can be completely revolved around the central post the bottom of the weight ('w') can be made to touch the ground and can also be raised 20 ft. above the ground. The distance of the vertical rope from the central post is 12 ft. and the radius of the central post is 6 inches. If the weight is moved to every possible position by the crane, find the area of the surface over which a point on the bottom of the weight and vertically under the rope moves.

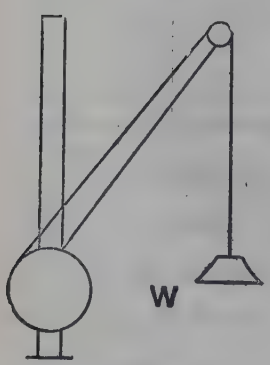


FIG. 314.

7. Draw a line 4·6 cm. in length and upon it describe an isosceles triangle having the angle at the vertex equal to 30°. Draw another triangle of the same form the sides of which are to the corresponding sides of the first triangle as 7 : 3.

8. The temperature of a patient taken every 3 hours was found to vary as follows :—

Monday	4 p.m.	100°·2	Tuesday	7 a.m.	100°·5
,,	7 ,,	101°·4	,,	10 ,,	100°·0
,,	10 ,,	102°·8	,,	1 ,,	100°·1
Tuesday	1 ,,	103°·6	,,	4 p.m.	100°·3
,,	4 a.m.	102°·7			

Draw on squared paper a curve indicating these changes. From it deduce what was probably his temperature at midnight on Monday and at 8.30 on Tuesday morning.

9. If a certain substance weighs 4 tolas for every cubic inch, find the weight of a solid right circular cone of the substance, of which the height is 20 inches and the diameter of the base 14·6 inches.

10. Draw a circle of 3·3 cm. radius ; insert in it a chord of 4·1 cm. length, and bisect the smaller arc of the circle.

11. Draw a graph showing the relation between so many rupees a month and its equivalent pounds a year (£1 = Rs. 15). From it deduce how many pounds a year are equivalent to Rs. 40 a month and also how many rupees a month are equivalent to £60 a year.

12. If a metre = 39·37 in. and a kilogramme = 2·2 pounds, express a pressure of so many pounds per sq. foot in terms of its equivalent in grammes per sq. cm.

13. The estimated cost of an irrigation project is 2 lakhs and it is expected to yield, after deducting for maintenance charges, a net return of 5·2 per cent. ; without such deduction, however, the return would be 6·4 per cent. Assuming the maintenance charges to be 12 as. per acre, find (1) the extent of land proposed to be irrigated and (2) the water rate per acre proposed to be levied.

14. The time t in seconds of an oscillation of a pendulum is given by the formula $t = \pi\sqrt{\frac{l}{g}}$ where l = the length of the pendulum. Find the length in centimetres of a

pendulum which oscillates 89 times in a minute. $g = 981$; $\pi = 3{\cdot}142$.

15. The income and expenditure of a certain school are as follows :—School fees, Rs. 12,672-12-0 ; fines, Rs. 9-7-0 ; stationery fees, Rs. 226-8-0 ; entrance fees, Rs. 56 ; library fees, Rs. 318-6-0 ; laboratory fees, Rs. 116-8-0 ; miscellaneous receipt, Rs. 27-7-0 ; salaries, Rs. 12,028-8-0 ; contingencies, Rs. 125-4-0 ; stationery, etc., Rs. 314-8-0 ; library, Rs. 367-3-0 ; class books, Rs. 63-8-0 ; furniture, Rs. 281-5-0 ; magazines etc., Rs. 213-6-0 ; laboratory, Rs. 187-8-0 ; prizes, Rs. 129-3-0 ; annual repairs, Rs. 253-7-0.

If the government pays a grant of half the deficit, find the cost to the proprietors per boy, there being 540 boys.

1913.

Only *nine* questions are to be attempted.

1. (*a*)

Cities		Population	No. of females to 1,000 males
Madras	...	518,660	946
Madura	...	134,130	999
Trichinopoly	...	123,512	1,006
Calicut	...	78,417	928
Kumbakonam	...	64,647	1,064

From the above table find (1) the total population of these 5 towns and (2) the number of males in each town.

(*b*) Find, correct to three significant figures, $0{\cdot}0923 \times 13{\cdot}215 \times 2{\cdot}654364$.

2. (*a*) What is Rs. 698-12-0 in English money if exchange is at $1s.\ 4\frac{15}{16}d.$ per rupee ?

(*b*) A man placed Rs. 1,500 as a fixed deposit in a bank paying 5 per cent. interest, the interest being added to the capital every year. Four years afterwards the bank failed and paid a dividend of 2 annas in the rupee. What did he get from the bank ?

3. If a metre is 39·9708 in., find, correct to the nearest foot, the difference between 83 kilometres and 51 miles.

4. Solve the following equations algebraically and check the results by means of a graph :—

$$3x - \tfrac{2}{3}y = 10\,;\ \tfrac{3}{4}x - 1\tfrac{1}{3}y + 1 = 0.$$

5. In a certain bicycle the hind wheel revolves two and a half times for one revolution of a pedal. If the outside diameter of the hind wheel is 28 ins., express the distance travelled in terms of the number of revolutions of a pedal ; and find the distance when the number of revolutions is 500.

6. (i) Of two cylindrical vessels the diameter of the second is double that of the first, and the height of the second is half that of the first. Compare the capacity of the second with that of the first.

(ii) If each has some water in it and a stone is immersed, first in one and then in the other, compare the height by which the water would rise in the second with the height by which it would rise in the first.

7. (i) Draw four parallel straight lines such that the distance between the first and second is 1 in., between the second and third $\sqrt{2}$ in., and between the third and fourth $\sqrt{3}$ in., finding $\sqrt{2}$ and $\sqrt{3}$ in. geometrically.

(ii) Draw a straight line cutting these four parallels and making an angle of 45° with them. What length of this line is intercepted between the second and third parallels ?

8. (i) Draw a triangle whose sides are 1·8, 2·4 and 3 in. respectively ; give the measurement of each angle and state what is the area of the circumscribing circle.

(ii) Draw a right-angled triangle and draw the square on each side ; then show by dotted lines and by instructions given below your figure, how to cut up the squares on two sides and by superposition of the cut pieces upon the third square, how to prove that the area of two of the squares is equal to that of the third square.

9. Omit ; sphere and frustum of cone.

10. For every acre of wet land a man pays a land revenue tax of Rs. 8 ; and for every rupee of land revenue

he pays a local cess of 1 a. 6 ps. Draw a graph showing the total tax paid upon land varying in extent from 20 to 40 acres. What would be the total tax on 35 acres?

11. Find the cost of whitewashing a room which is 40 ft. by 18 ft. with walls 14 ft. high, allowing for half the area of the following openings; two doorways each 7 ft. by $4\frac{1}{2}$ ft.; four windows, each $3\frac{1}{2}$ ft. by $4\frac{1}{2}$ ft.; and two circular ventilators, each two feet in diameter. The wall at each end of room is as in the figure: the height of the triangular portion being 8 ft. Whitewashing 2 as. 6 ps. per 100 sq. ft.

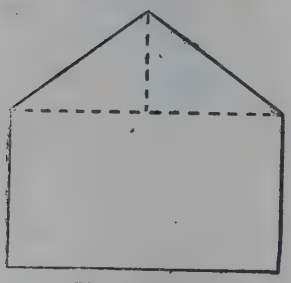


FIG. 315.

12. An open rectangular cistern of which the interior length, breadth and depth are as 4 : 3 : 2, is made of metal 1·5 cm. thick and 8 times as heavy as water. If the capacity of the cistern is 12,288 litres what is its weight in kilogrammes when empty?

13. A bookseller is allowed by a publisher 20 per cent. discount on the nominal price of a book and 5 per cent. on this last amount for cash. For every dozen books he gets an extra copy for nothing. The bookseller allows his cash customer 4 as. in the rupee off the nominal price. Find (to one decimal place) the bookseller's profit on cash sales.

1914.

1. The following table gives the number of workers in Textiles in the Madras Presidency in the years 1901 and 1911. Fill in the percentage column and the row of totals (correct to one decimal place).

Name of textile	No. in 1901	No. in 1911	Percentage of increase (+) or decrease (−)
Cotton (ginning etc.) ...	24,089	32,553	
Cotton (spinning) ...	623,783	580,321	
Jute	533	2,428	
Rope	42,376	58,794	
Other fibres	7,886	9,252	
Total ...			

2. Show by a rough approximation, without working the questions in detail, that the answers affixed to divisions (1) and (2) are obviously wrong and those affixed to divisions (3) and (4) are approximately correct.

(1) Find the cost per metre of cloth at 12 as. a yard. One metre = 39·37 in. (Ans. 12 as. 6 ps.)

(2) Find the income derived from the investment of Rs. 6,220 at 4 per cent. after paying an income-tax of 4 p. in the Re. (Ans. Rs. 249-4-7.)

(3) Simplify $30{\cdot}12 \times 19{\cdot}93 - (10{\cdot}01)^2$. (Ans. 500)

(4) Simplify $\frac{19{\cdot}8 \times 25\frac{1}{14}}{201 \times 398} \times \frac{4042}{1523{\cdot}5}$. $\left\{ \text{Ans. } \frac{1}{60.} \right\}$

3. The average of 'm' numbers is 'x' and the average of 'n' of those numbers is 'y'. What is the average of the remainder?

The area of the Madras city is 17,280 acres and the average population is 30·3 per acre. The area of the thickly populated divisions is 2804 acres and the corresponding average population is 135·3 per acre. Find the average population per acre for the rest of the city.

4. A milk-seller pays 5 as. per measure for the milk he purchases and adds water before selling it. He also defrauds by using, while selling, a false ollock measure which is $\frac{1}{16}$ short of the true ollock. If he sells at $\frac{1}{2}$ anna an ollock and makes 12 per cent. profit, find the proportion of water to milk in his mixture. [A measure = 8 ollocks.]

5. A manufacturer of an article sells it to a merchant at a gain of 'p' per cent. on the cost of making. The merchant sells it for Rs. 'r' at a gain of 'q' per cent. What is the cost of making? Use your result to find the cost of making, when $p = 10$, $q = 12\frac{1}{2}$, and $r = 495$. Verify your result in any way.

6. A circular arch over a river rises 4 ft. at its highest point above the level of the tops of the vertical piers. If the distance between the piers be 28 ft., find, graphically or otherwise, the radius of the circle of which the arch forms a part.

7. ABCD is a quadrilateral field; BC = 20 yds., AD = 24 yds., the diagonal BD = 32 yds., the angles A and C are right angles. Draw a figure to scale (1 in. to 10 yds.) and determine the area.

8. How many square inches of metal are required to make a tray of the shape shown in the diagram? The adjacent sides of the base measure 9 in. and 15 in. and the corresponding sides of the top measure 10 in. and 16 in. and the width of the strip making the sides is 3 in.

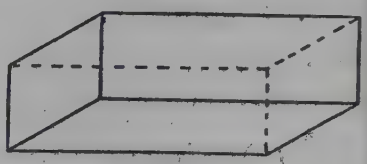


FIG. 316.

9. In order to find the diameter of a tube of uniform bore, some mercury was poured into it and the height of the column measured. The weight of the mercury was 19·72 gm.; the height of the column 20·3 cm. What was the diameter of the tube? (A cubic cm. of mercury weighs 13·6 gm. and $\pi = \frac{22}{7}$).

10. In 1911 the population of the Madras City was 551,660. Since then the birth rate per annum has been 42 per thousand and the death rate 39 per thousand; and the town has attracted people from outside at the rate of 17 per thousand every year. What will be the population in 1914?

11. The following table gives the water level in the Red Hills Lake on the fifteenth of every month.

1911.			
Month	Level	Month	Level
April ...	... 42·1	October	... 37·3
May ...	... 40·5	November	... 39·1
June ...	... 39·0	December	... 44·0
July ...	... 37·9	1912.	
August	... 36·9	January	... 44·3
September	... 36·6	February	... 43·4
		March ...	... 43·0

Exhibit the variations in level by a smoothly drawn graph.

1915.

(*N.B.*)—Only *nine* questions are to be attempted.

1. In a certain year the United Kingdom imported 61,325,694 cwt. of wheat valued at £22,934,523 from foreign countries and 41,568,705 cwt. of wheat valued at £16,837,425 from British possessions. Find, to the nearest penny, the value per cwt. of imported wheat (1) from foreign countries (2) from British possessions. Also find, to the nearest integer, what percentage of the total value of the wheat is the value of the quantity imported from British possessions.

2. An English sovereign weighs 123·3 grains and contains 91·67 per cent. of pure gold. Find the value, to the nearest tenth of a penny, of the gold in a sovereign at £3-17-6 *d.* for 480 grains.

3. I want to buy 20 yds. of certain wire which is 0·14 of an inch in diameter, but I find it is sold only by the pound. Calculate for me the weight of the wire which I want, given that iron weighs 470 lb. a cubic foot. [Use the formula $v = 0{\cdot}785\ d^2\ l$; where 'v' is the volume in cubic inches of a piece of wire 'l' inches long and 'd' inches in diameter.]

4. At a show the following kinds of tickets are sold: green tickets at 9 pies, white at 3 pies, pink at 6 pies, and yellow at one anna. If 'g' green, 'w' white, 'p' pink and 'y' yellow tickets were sold on a day, construct a formula for 'T', the total collection in annas for the day.

Use your formula to find 'T' if $g = 360$, $w = 1760$, $p = 480$, $y = 80$. Also find 'g' when $T = 762$, $w = 920$, $p = 440$, $y = 72$.

5. A steamer goes 'a' miles up stream in 'b' hours and 'c' miles down stream in 'd' hours. The stream flows 'e' miles an hour and the vessel could go 'f' miles an hour in still water. Express a and c in terms of b, d, e and f. If $a = 16$, $b = 2$ $c = 18$, $d = \frac{3}{2}$ what are e and f.

6. A ladder 12 ft. long leans against a wall making with it an angle of 25°. Draw the ladder in position representing a foot by one-fifth of an inch. Then draw the ladder in a new position with its lower end 2 ft. farther from the wall and find, by measurement, how much lower the top of the ladder will then be.

7. Take any straight line and mark a point A at a distance of 4 cm. from it. Now find another point B which is 3 cm. from A and 3 cm. from the line. Find also a point C 2 cm. from A at which the line AB subtends a rightangle. Find by measurement, the distance of C from the first line.

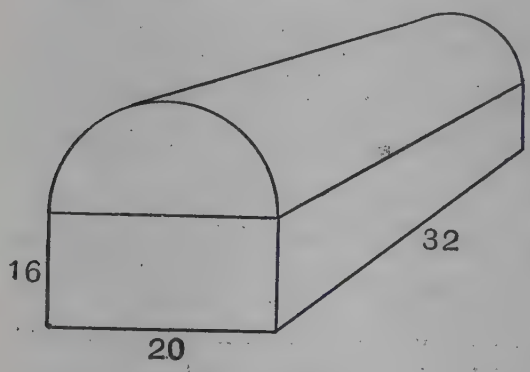


FIG. 317.

8. A hay shed (shown in the adjoining figure) consists of a rectangular space 32 ft. long, 16 ft. high, and 20 ft. wide, surmounted by a covering roof whose section is semicircular. Calculate how many tons of hay can be stored in it, if a ton of hay occupies 300 cu. ft.

9. A merchant sells silk of two qualities which cost him Rs. 4 and Rs. 3-4-0 per yard respectively. The selling price of the latter is $\frac{2}{3}$ that of the former, but the quantity sold is double. He gains 25 per cent. on the whole. Calculate the selling price per yard of each.

10. In a certain factory the machinery was valued at Rs. 32,500. If it depreciates each year at the rate of 8 per cent. of the value which it had at the beginning of the year, find, to the nearest rupee, what its value would be after four years.

11. I live $\frac{3}{4}$ mile from my railway station and take 15 minutes to walk the distance. I am accustomed to meet at a certain point some children going to school in the

opposite direction. The children take 30 minutes to the mile. One day when I am at my usual time I am 220 yds. further on when I meet them. How much later will the children arrive at school that day than on ordinary days, provided they keep their ordinary pace ?

12. Passengers on the railway are allowed a certain weight of luggage free and the rest is paid for at 5*s.* 7*d.* per cwt. If a traveller with luggage weighing 256 lbs. had to pay 8*s.* 4½*d.* for his luggage, find the weight of luggage allowed free and draw a graph connecting the amount of luggage carried and the charges paid.

1916.

N.B.—Only NINE questions to be attempted.

1. The following is a classification of people supported by agriculturists and actual workers in agriculture out of 1000 for the years 1910, 1911. Find the increase or decrease per cent. in the figures for 1911 as compared with those of 1910 under each head :—

Heading	Supported by agriculturists		Actual workers in agriculture	
	1910	1911	1910	1911
Non-cultivating land owners.	34	30	23	19
Cultivating land owners ...	461	512	426	484
Non-cultivating tenants ...	6	2	4	1
Cultivating tenants ...	225	167	207	151
Farm labourers	279	284	340	345

2. On March 1, 1913, a person borrowed from A Rs. 1,450 at 8 per cent. simple interest, and lent the sum on

the same day to B at 8 per cent. compound interest: if both the transactions were closed on March 1, 1915, find his gain.

3. Stocks and shares.

4. A school grants its games association Rs. 216 a year which is distributed among its three branches, Tennis, Foot-ball, and Badminton, in the proportion of 14 : 6 : 1. The tennis branch is 14 strong and each member pays a subscription of Rs. 2 per half-year. If the branch pays Rs. 6 a month to the pickers and spends Rs. 32 per annum on repairs to the court, find how many balls a month at Rs. 12 per dozen the branch can supply itself with so as not to work at a loss.

5. If a book-seller fixes his prices so as to include a profit of 30 per cent. on the cost price and offers a reduction of 10 per cent. on the published price for cash payments, find his gain per cent. If the cost price of 34 copies of a book be Rs. 50, what is his average gain per copy when all are sold for cash?

FIG. 318.

6. Find the volume of a gate pillar, consisting of a cylindrical column 6 feet high and 1 foot in diameter, placed on a cube of 2 feet edge and surmounted by a square slab 1 foot thick and 2 feet side with a sphere of 6 inches radius placed on the top of it (Fig. 318).

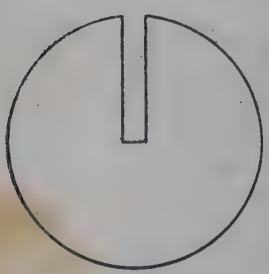


FIG. 319.

7. A cast iron cylindrical weight is 18 inches in diameter and has a slot 2 inches wide and 10 inches long as in figure 319. Find, correct to two places of decimals, its thickness in inches when its weight is 100 lbs. (1 cub. in. of cast iron weighs 0·26 lbs.)

8. The following table gives the power (P lbs.) to be

applied in order to move the weight (W lbs.) for a certain machine.

W.	50	100	200	300	400	500
P.	106	122	152	184	216	246

Draw a graph showing the relation between P and W, and find, to the nearest integer, the value of P when W = 350, and of W when P = 158.

9. If a person buys 'p' dozen mangoes at 'a' as. per dozen and 'q' dozen mangoes at 'b' as. per dozen, and sells $\frac{2}{3}$ of the whole lot at 'd' as. per dozen and the rest at $\frac{1}{2}$a. per dozen, find the profit. If the profit is Rs. 4-6-0 when $p = 12$, $q = 5$, $r = 13$, $a = 20$, $b = 12$, $c = 10$, find 'd'.

10. Solve :—$2x + 3y = 12$.

$5x - y = 4$.

Compare your answers with results obtained graphically.

11. Construct a quadrilateral ABCD having AB = AD = 4 cm., AC = 8 cm., and CB = CD = 6 cm. Measure BD and the angles B and D.

12. Draw an equilateral triangle and a regular hexagon so as to lie between the same parallels and compare the areas of the two figures.

1917.

1. (*a*) The H.C.F. of three numbers is 7 and their L.C.M. is 12012. Two of the numbers are 364 and 84. Find the least value of the third number.

(*b*) You are given that $461010 = 77 \times 5987\frac{1}{7}$. Show how you can write down (correct to 5 places of decimals) the quotient when 0·046101 is divided by $(14 \times \cdot 055 \times 0 \cdot 001)$ without actually going through the ordinary process of division.

2. The following statement gives the figures under each head of contingent charges, both estimated and actual, for 1913–14 :—

Items	Estimated			Actually		
	RS	A	P	RS	A	P
Printing	15,650	0	0	10,595	6	0
Stationery	12,550	0	0	6,610	2	0
Travelling Expenses	10,630	0	0	12,939	7	0
Postal and other charges	6,670	0	0	7,483	9	7
Examination charges	3,450	0	0	3,362	10	1
Contribution to the library ...	5,550	0	0	5,550	0	0
Cost of University Lectures ...	6,600	0	0	6,434	8	0
Total ...						

Fill in the row of totals and find by what percentage (correct to one decimal place) the total of actuals had fallen short of the total of estimated charges.

3. Two partners invest Rs. 2,700 and Rs. 1,500 respectively in their business and arrange that 40 per cent. of the profits should be divided equally between them and the remaining profits treated as interest on the capital. If one partner's share of the profits is Rs. 90 more than the other's share, find what was the whole amount of the profits.

4. A man buys a field for Rs. 3,750 and lets it at a rent of Rs. 150 payable at the end of each year. The rent is invested at 5 per cent. compound interest per annum. At the close of three years he sells the field for Rs. 4,000 and draws out the accumulated rents and interest. Find the total he will have and the annual percentage (simple interest) he makes by the whole transaction.

5. A railway track starts from station A with a gradient of 1 in 200, i.e. a vertical rise of 1 yd. in 200 yds. measured along the track. The gradient remains the same for 1 mile and is followed by rising gradients of 1 in 150 for

$\frac{3}{4}$ mile, 1 in 120 for $\frac{1}{2}$ mile, level for 1 mile and falling gradients of 1 in 110 for $\frac{1}{2}$ mile, and 1 in 330 for $\frac{3}{4}$ mile, when station B is reached. If station A is 179 ft. above sea-level, find, to the nearest foot, the elevation of station B.

6. The average heights of a man and a woman and their corresponding weights are given in the following table:—

Height	5′0″	5′2″	5′4″	5′6″	5′8″	5′9″	5′10″	6′0″
Average weight of a man in pounds.	...	126	139	145	155	162	169	178
Average weight of a woman in pounds.	103	112	126	139	152	156	...	...

Draw graphs in one and the same figure showing the relation between the average height and weight for each sex. Find from your graph the difference between the average heights of a man and woman who each weigh 10 stone, and the difference in weight if they are 5′6″ in height.

7. A cylindrical oil-tank is 12 ft. in diameter and has ends which are hemispheres of the same diameter. Its total length is 40 ft. If it is lying in a horizontal position, how many gallons of oil will be required to fill it to a depth of 6 ft.? (1 gallon is equivalent to $277\frac{1}{2}$ cub. in. nearly.)

8. A child is instructed as follows: 'Think of a number, double it, add 11 to the result, multiply the sum by 3, subtract 12, divide the remainder by 3, give me the answer.'

Find a simple rule by which the number thought of can be readily deduced from the answer given by the child.

9. A and B engage in a play: when A had lost Rs. 20 he had only one-third of the money which B had; but by

continuing the play he not only won back his Rs. 20, but also Rs. 50 more with it, and then found he possessed as much again as B. With what sum did they respectively begin?

Reduce the conditions of the problem to simultaneous equations in two unknowns and solve the problem.

10. A is the centre of a circle 5 cm. radius, and B is the centre of a circle 2 cm. radius. CD is a direct common tangent to the two circles, and touches them at C and D respectively. If CD is 4 cm. long, find the distance between the centres. Draw a neat figure to scale.

11. Divide a straight line AB, 6·4 cm. long, internally at X and externally at Y, in the ratio of 3 : 5. Verify the result of your construction by measuring the lengths of the segments.

Also show that they verify the formula :

$$2/AB = 1/AX + 1/AY.$$

12. Show by a carefully drawn diagram how you will verify by paper cutting the truth of the proposition that the square on the hypotenuse is equal to the sum of the squares on the other two sides of a right-angled triangle.

1918.

Form	Strength	Amount		
		RS	A	P
VI	79	414	0	0
V	80	404	4	0
IV	78	383	8	0
III	76	240	9	0
II	74	191	3	0
I	125	326	8	0

1. The fee income from the secondary department of a school in a certain month is given in the adjoining table.

Find (*a*) the average fee paid by each boy in the whole department; and

(*b*) the percentage of the I form collections to the whole income.

2. A merchant bought at Narasapur 40 candies of paddy at Rs. 38 a candy and sent the same to Tanjore, paying Rs. 10 towards boat-hire as far as Nidadavole and 17 times as much for railway charges from there to Tanjore. His agent sold the whole quantity at Tanjore at Rs. 2-4-0 a kalam. Find the amount of gain or loss to the Narasapur merchant, if he paid a commission of 5 per cent. on the sale proceeds.

[1 kalam = 24 Madras measures. A candy = 800 seers ; each $\frac{2}{3}$ of a Madras measure.]

3. The following is a rough extract from a railway guide :—

Trains from Madura

Distance from Madras in miles	Stations	Boat mail to Ceylon
348	Madura *dep.*	11.45
448	Rameswaram *arr.*	16.30

Trains to Madura

Distance from Madras in miles	Stations	Boat Mail from Ceylon	Rameswaram Express
448	Rameswaram *dep.*	9.45	12.30
348	Madura *arr.*	14.30	19.30

Represent the motions of the three trains graphically. From the graph find when they are 40 miles from Madura

and when and at what distance from Madura the Boat Mail from Madura meets the other two trains.

Note.—Time is calculated for 24 hrs. from midnight to midnight.

4. On each page of an atlas the border lines are $7\frac{1}{2}$ in. lengthwise and 6 in. breadthwise. The map of the Indian Empire is drawn to a scale of 300 miles to an inch and occupies $\frac{4}{9}$ of the space within the border lines. Find the area of the Empire in sq. miles.

On another page of the same atlas with similar border lines, there is the map of the Madras Presidency, whose area is 153,000 square miles, excluding the native states occupying $\frac{17}{72}$ of the space enclosed by the border lines. Find the scale to which this map is drawn.

5. In the Post Office, War Loan Certificates were issued on the following terms:—For a payment of Rs. 7-12-0 a certificate was issued which entitled the holder to get Rs. 8-1-0, Rs. 8-14-0 and Rs. 10, at the end of one, three and five years respectively. What rate per annum simple interest does it come to in each of the three cases?

Given also that the certificate holder is entitled to receive Rs. 8-7-0 and Rs. 9-6-0 at the end of two, and four years respectively, show graphically how the money grows during the five years.

6. A cylindrical granary with a conical top is 14 ft. in diameter on the inside. The wall is 11 ft. high, and the total height of the top is 14 ft. It was full of grain. Half of it was sold at Rs. 5-12-0 a bag and the other half at Rs. 5-4-0 a bag. Find the amount realized, given that a bag of grain measures 8·25 cub. ft.

7. Describe a regular octagon on a straight line 2 cm. long and find its area by any method.

8. At a Public Examination in the Madras Presidency, x candidates who had taken optional mathematics scored in elementary mathematics an average of 'm' marks and the remaining 'y' candidates for the examination scored an

average of 'n' marks. What was the Presidency average in the subject.

If $x = 2{,}500$, $y = 6{,}428$, $m = 43$, and $n = 30$, find the Presidency average ?

9. A village postman attached to the Post Office at P has to go to 5 villages, A, B, C, D, and E and return to P. A is 5 miles due north of P ; B, C and D are on the eastern side of the road from P to A : B is 8 miles from P and 6 miles from A ; C and D are 5 and 6 miles respectively from P on straight road PCD which is at right angles to PB. E is 3 miles from P, its position being to the west of the road PA and the roads PE and AE being at right angles to each other. Draw a diagram (scale, 1 cm. = 1 mile) and indicate the shortest route for the postman. What is the length of his circuit ?

10. (1) Construct a quadrilateral ABCD, given that AB = 3 cm., DA = 4 cm., A = 120°, C = 70°, and AB is parallel to CD. Measure BC.

(2) The middle points of the sides of a scalene triangle are joined so as to form a new triangle. What fraction is this of the given triangle ? Explain from a figure how your answer can be shown to be true experimentally by superposition.

11. The distance from Madras to Ootacamund by railway is 358 miles, part of which is ordinary railway and the other part mountain railway. The second class fare over the former is 9 ps. per mile and over the latter 1 a. 9. ps. per mile. If the total be Rs. 18-10-6, find the length of the mountain railway.

12. A reservoir which supplies water to a town has two inlets whose diameters are 30 in. and 16 in. respectively, through which water pours in throughout day and night. Water is to be taken into the town through six pipes, all of uniform diameter. But they will remain open only from 5 a.m. to 9 p.m. daily. What should be the diameter of each of the latter, if the quantity conveyed to the town

every day is to be equal to the daily supply into the reservoir, assuming that the rate of flow is the same through all the 8 pipes ?

1919.

1. In form I of a certain school there are four divisions. The adjoining table gives the strength and the total marks gained by the students of each division in a certain subject.

Division	Strength	Total Marks
A	34	1372
B	35	1339
C	36	1489
D	37	1257

Find, correct to one decimal place, the average marks for the whole form, and ascertain which division gets the highest average and which the lowest.

2. A merchant cleared his goods by selling them at reduced prices. He sold certain goods whose catalogued sale price was Rs. 760 at a reduction of 15 per cent., another set of goods whose sale price was Rs. 880 at a reduction of $12\frac{1}{2}$ per cent., and the remaining goods whose sale price was Rs. 480 at a reduction of 25 per cent. He however gained in all 11 per cent. on his original cost price. Find the original cost price, and also what percentage he would have gained had he not made any reduction in the catalogued sale prices.

3. ABCD is a quadrilateral field in which AB = 198 yds., BC = 264 yds. and AC = 330 yds. D is equidistant from A and C and at a distance of 214·5 yds. from the diagonal AC. (1) Draw a plan of the field to a scale of 1 in. = 33 yds. (2) Find the area of the field in acres.

4. The distance from P to Q along a road is 65 miles. A person A starts from P at 7 a.m. and travels to Q on his bicycle at the rate of 10 miles an hour, taking rest for 10 minutes at the end of every hour of cycling. B starts from P at 8.45 a.m. and goes on to Q in his motor car at a

uniform rate of 15 miles an hour. Represent their journeys graphically. Find from the graph

(1) when each of them will reach Q, (2) when and where B will overtake A and (3) by what distance they will be apart at 9.45 a.m.

5. A person borrows Rs. 1,800 at 8 per cent. per annum simple interest and forthwith lends it as follows :—(1) Rs. 800 to A at $10\frac{5}{8}$ per cent. per annum simple interest ; (2) Rs. 600 to B at 10 per cent. per annum compound interest ; and (3) the remaining sum to C at $12\frac{1}{2}$ per cent. per annum compound interest during the first year and 12 per cent. compound interest per annum during the second year. At the end of two years, he collects the amount due to him from A, B and C and pays his dues to his creditor. How much does he gain ?

6. (1) Show by geometrical construction how a length which is $\sqrt{3}$ times a given length may be found.

(2) Describe a regular hexagon on a straight line 1·5 in. long and find its area.

7. A stone road roller should be 4 ft. long, and should weigh 4 tons 8 cwt. What should be its diameter given that the weight of the stone is 441 lbs. per cubic foot ?

8. There are some beggars consisting of men and boys. If the former were given 5 as. 6 ps. each and the latter 3 as. 6 ps. each, a sum of Rs. 12-10-0 will be required. If each man is given 6 ps. less and each boy 6 ps. more the total sum required is 2 as. less. Reduce the above conditions to equations and find the number of men and of boys.

Represent the equations graphically and verify the result from the graphs.

9. A garden is 240 yds. long and 180 yds. broad. A moat 5 yards wide and 2 yards deep is dug close to it on the outside all round, and the earth dug out spread evenly over the garden.

(1) Find in inches, correct to one decimal place, the height by which the level of the garden will be raised thereby.

(2) A certain contractor undertook to finish the work in 30 days and employed 36 men. At the end of 10 days, only 1 ft. 6 in. of the depth of the moat had been dug all round. How many additional men should he employ from the eleventh day to finish the work in the stipulated time?

10. A certain boy took a circular piece of card-board having a radius of 17·5 in. and made an angle of 144° at the centre. He then cut out and removed from it the sector piece containing the angle 144°, and made the bounding radii of the remaining piece coincide. What shape did he get? Calculate its height?

11. A merchant buys m maunds of coffee seeds at Rs. a per viss and n maunds at Rs. b per viss and mixes them. He sells the mixture at a uniform price and gains c per cent. on his outlay. Find the selling price of each viss of the mixture.

If $m = 8$, $n = 10$, $a = 1\frac{1}{4}$, $b = 1\frac{1}{2}$, $c = 12\frac{1}{2}$, find the selling price of the mixture per viss.

12. In a certain meadow there is a hut whose ground-plan is a square each side of which is 28 ft. A cow is tethered to a post at one of its corners by a rope 35 ft. long. Find the area that the cow has for grazing.

Re-Examination, 1919.

1. The strength of Form VI in a certain school is 105. The form average in Elementary Science was exactly 39 in a certain year. The boys scored marks thus in the subject:—15 boys 54 marks each; 22 boys 45 marks each; 28 boys 38 marks each; 25 boys 33 marks each; and 13 boys 24 each. Of the remaining 2 boys A and B, A scored 20 marks more than B. How many marks did each of them get? Also find their rank.

2. A grocer wants to mix 80 lbs. of tea worth 15 as. a lb. with some tea worth Re. 1-4-0 a lb., so that by selling the mixture he may gain 20 per cent. on his outlay. How

many pounds of the superior sort should he take to form the mixture ?

3. In a 100 yards race A beats B by 20 yds. and C by 8 yds. If B and C run over the same course of 100 yds., find who will win and by how many yards to the nearest integer. Verifiy the result graphically.

4. An open rectangular box is to be made out of a rectangular piece of card-board 21 in. long and 10·5 in. broad, by cutting out four equal squares at the corners of the rectangle and folding up the four rectangles that result so as to form the vertical sides and the ends of the box. If the ends of the box are squares, find the cubical contents of the box.

Draw a plan of the card-board to a scale of 1 cm. to 1 in., as it will look after the square corners are removed and before the rectangles are folded up.

Find the maximum number of such card-board boxes that can be packed in a cubical box whose edge on the inner side is 2 ft. 4 in. The thickness of the card-board to be neglected throughout.

5. According to the catalogue of a London firm the cost of the books that are to be bought for a High School in Madras is £60, of which 25 per cent. is marked net. In the case of the books so marked the packing is free. For the remaining books a discount of 20 per cent. is allowed, but £1-4-0 is charged for packing. The freight to Madras for the whole consignment is £2-16-0. A Madras book-seller undertakes to supply the books marked net at Re. 1 a shilling and the rest at 14 as. a shilling. Which is more profitable, to get the books from London or to buy them in Madras, and by how many rupees. [Current market rate £1 = Rs. 19.]

6. Men, women, and children are employed in a factory in the proportion of 1 : 2 : 3, and the wages of a man, a woman, and a child are as 6 : 3 : 2. When the number of men employed is 50, the total weekly wages of all the

hands amount to £225. Find the weekly wages paid to a man, a woman and a child respectively.

7. Describe a quadrilateral ABCD with the following data :—AB = 8·5 cm., BC = 7·5 cm., AC = 6 cm., CD = 5 cm. and angle D = 90°. Find its area correct to two decimal places.

If it represents a field drawn to a scale of 1 cm. = 30 yds., find the area of the field in square yards correct to one decimal place.

8. A person borrows a certain sum of money at $13\frac{1}{3}$ per cent. per annum simple interest, promising to repay the amount at the end of 4 years and 6 months. But the loan had to be extended for a period of 1 year and 6 months more. Thereupon the creditor demanded not only that the interests that had accumulated till then should be added to the principal but also the rates should be raised to 16 per cent. per annum for the period extended. On account of this the person had to pay Rs. 107-5-4 more than he would have paid had the sum continued at the simple interest for the whole period at the original rate. Find the sum borrowed.

9. The following table gives the population, in thousands, of two towns A and B at the beginning of each of the year specified :—

Year	1835	1845	1855	1865	1875	1880	1890
A	24·4	26	29·5	34	40	43	50
B	34	36	38·4	41·1	43	44·8	46·7

Draw graphs on the same diagrams to represent the above and estimate therefrom the population of each of the towns at the beginning of 1870. In which year the population was approximately the same in both towns, and what was it ?

10. An Indian merchant arranged to buy tweed in France at 4·4 francs per metre. The cost of getting the same to India averages 0·1927 francs per metre. If he sells at Rs. 3 per yard, find his gain or loss per cent. [Re. 1 = 1·68 francs ; 1 metre = 1·0935 yards.]

11. The length of a park is to its breadth as 4 : 3, and its area is 77 acres, 88¾ square yards. Find the length of its diagonal in yards.

12. To walk one mile A takes 'm' minutes and B 'n' minutes. (1) What distance does A walk more than B in one hour? (2) Find in terms of 'm' and 'n' what time A takes to walk 3 miles more than B. (3) Express to the nearest minute in what time A walks 3 miles more than B, when $m = 16$ and $n = 21$.

1920.

1. The sea-borne trade of the Madras Presidency for the year 1918–19 shows that the value of foreign imports under 'Tobacco, raw and manufactured,' was Rx. 188,652 for 1918–19, Rx. 113,000 for 1917–18, the average of the preceding 4 years up to 1916–17 inclusive being Rx. 49,394. Find (1) the increase per cent. in the value of imports in 1918–19 over that in 1917–18, and (2) by how many rupees the value of imports in 1918–19 was in excess of the average value of the preceding 5 years up to 1917–18 inclusive. [1 Rx. = 10 rupees.]

2. The cross-section of the canal running across the Godavari at the Gunnavaram Aqueduct is a trapezium (a quadrilateral with two of its sides parallel) 24 ft. at the top and 19 ft. at the bottom, the depth being 6 ft. If the slopes are of equal inclination, draw the cross-section to a scale of 1 cm. = 2 ft. Measure the angle at which the sides are inclined to the bottom of the canal.

Given that the length of the aqueduct is 810 yds., find in cub. yds. the maximum quantity of water that can be contained in the whole length of the aqueduct and also in

how many minutes a boat 15 yds. long and going down th canal at $3\frac{3}{4}$ miles an hour will pass the aqueduct.

3. To find the height of a certain tower a surveyor fixe a pole vertically at a distance of 80 yds. from it, the heigh of the pole above the ground being 15 ft. On going 5 yds from the pole and away from the tower, he sees the top of the pole and of the tower in the same straight line wit his eye. Find the height of the tower, given that th height of his eye is 5 ft. from the ground.

4. In the catalogue of a certain merchant the selling prices are given as shown in the accompanying table. He usually allows a discount of 5 per cent. on silks and 10 per cent. on checks. Owing to the war conditions he has raised his prices on all his goods by 20 per cent. but allows still the same rate of discounts on the increased prices. A customer buys 6 yds. of Tassure silk, 7 yds. of Nagur silk, 8 yds. of Turban silk, 14 yds. of Calicut check and 16 yds. of Holland check. What sum has he to pay the merchant ?

Goods	Price per yard
	RS A
Tassure silk ...	4 11
Nagur silk ...	3 2
Turban silk ...	2 5
Calicut check ...	1 9
Holland check ...	0 12

5. A person bought two horses for different prices. By selling the first at a profit of 10 per cent. and the second at a profit of 15 per cent. he gained 13 per cent. of the total cost price. Had he sold the first at a profit of 15 per cent. and the second at a profit of 10 per cent. he would have got Rs. 6 less on the whole. Find the cost price of each of the horses ?

6. A road runs from P to S passing through Q and R. PQ is $10\frac{1}{2}$ miles and QR 7 miles. A starts from P at 6 a.m. and walks to S at a certain uniform rate, halting at Q for half an hour and at R for one hour. B starts from P at 7 a.m. on his cycle, his rate of cycling being 7 miles an

hour. He passes A at 7.45 a.m. and goes on to R where he stops for one hour. He returns to Q where business detains him for an hour and a half. He then goes straight to S, which he reaches at 4 p.m.

Represent the above graphically and find from the graph.

(*a*) the distance from P to S.

(*b*) A's rate of walking.

(*c*) the time when A reaches S.

(*d*) when and where A and B are together for the third time.

7. Describe an acute-angled triangle ABC, given that AB = 3·2 in., BC = 2·6 in., and angle A = 50°. Measure AC and the angle C. Find the area of the triangle by any method.

If it represents a field drawn to a scale of 1 in. = 1 furlong, find the area of the field in acres.

8. When performing a certain experiment in Science the temperature at the end of 10, 20, 30, 40, 50 and 60 seconds was found to be 15°, 30°, 43°, 56°, 58° and 45° respectively, it being observed also that 60° was the maximum reached. Draw a smooth curve, showing the changes in the temperature and find from it the temperature at the end of 25 seconds, and also the time when the temperature was 50°.

9. If 'm' men can do $\frac{1}{n}$ of a piece of work in p days find an expression for the number of men required to do the work in q days.

Find the number when 'm' = 15, $n = 4$, $p = 12\frac{1}{2}$ and $q = 20$.

10. When a certain weight was cast it was found to weigh 1·85 lbs. more than was intended. Thereupon a cube, with an edge of 1·5 in., was scooped out from the bottom of it. But thereby the weight was diminished only by 1·08 lbs. It is proposed to diminish out the rest by

scooping out a cone with a bottom circumference of $5\frac{1}{2}$". What should be the depth of the cone ?

11. One litre of water weighs 1 kilogramme (kg.), and one litre of pure milk 1·032 kg. A person bought 12 litres of milk and found it weighed only 12·288 kg. Find in cu. cm. the quantity of water in the milk. (1 litre = 1 cubic decimetre.)

12. A war savings bank certificate costs 15*s.* 6*d.*, and is worth at the end of the fifth year £1. After that period one penny is added to it every month and at the end of the tenth year from the commencement a bonus of 1*s.* is added to the total value of the certificate. What rate of simple interest per annum does this work out for the whole period of 10 years ?

Forty-eight Englishmen employed in India saved each a sum of Rs. 2-8-4 daily from March 1 to September 2, both inclusive ; and with the total sum thus saved arranged to buy war savings certificates in England. How many certificates did they buy in all ; what total sum would they get in England by converting the certificates into cash at the end of 10 years after their purchase ? [Re. 1 = 1*s.* 4*d.*]

1921.

1. A metre is a ten-millionth part of the arc of a meridian of the Earth from the Equator to the Pole. The Earth's semi-circumference measured along a meridian is 10,800 nautical miles. Find the number of feet in a nautical mile, if 1 metre = 39,370 inches.

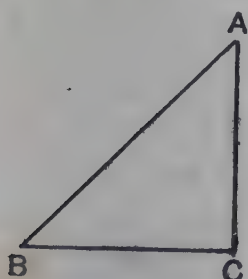


FIG. 320.

2. How far is the following statement correct ? Three measurements are necessary and sufficient to prepare an exact copy of a given triangle.

Measure the sides, correct to the nearest tenth of an inch, and the angles, correct to the nearest degree, of the given triangle ABC.

Construct a triangle PQR of the same shape as ABC but of perimeter 9·6 inches.

3. AB is a diameter of a circle, whose centre is C. The circle is folded so that the points A, B are brought together at the centre C, thus causing two creases. Show that the points A, B and the four ends of the two creases are the corners of a regular hexagon.

Calculate the area of the hexagon to the nearest hundredth of a square inch, if the radius of the circle is exactly 2·4 inches.

Check by drawing an accurate figure.

4. Here is an extract from a statement of Educational Progress in the Madras Presidency for three years.

—	1914–15	1915–16	1916–17
Total population ...	41,404,405	No change	No change
Total scholars in all institutions ...	1,542,955	1,615,159	1,661,012
Grand total of expenditure in thousands of rupees ...	19,353	20,652	21,688

Find from this statement :—

(1) The ratio of the number of scholars to the total population for 1915–16 expressed as a percentage.

(2) The increase per cent. in the number of scholars in 1916–17 over that of 1914–15.

(3) The increase in the average expenditure per scholar for 1916–17 over that for 1914–15. Answer to the nearest anna.

5. When sovereigns were selling at Rs. 18-12-0, a gentleman purchased a watch-chain made of 12 sovereigns (including a wastage of $6\frac{1}{4}$ per cent.). The making charges were 14 as. per sovereign. A few days later he had to sell it when the price of the sovereign fell to Rs. 16-4-0. The

purchaser would take the chain at its net weight only and pay only 75 per cent. of the making charges per sovereign on the net weight of the chain. What did the gentleman lose by the transaction?

6. A High School Co-operative Society was supplied by a Bombay firm with mathematical books at a discount of $33\frac{1}{3}$ per cent. and History books at a discount of 40 per cent. off the published prices. Subsequently it was found that the society would have paid $2\frac{1}{2}$ per cent. of the published prices less, if all the books had been purchased from a Madras book-seller, who offered a uniform discount of $37\frac{1}{2}$ per cent. What percentage of the purchases, calculated at the published prices, was of mathematical books?

7. The Secretary of a Chit Fund was authorized to buy a cash box. Every year of use was supposed to decrease the value of the box by $12\frac{1}{2}$ per cent. of what it was worth at the beginning of that year. At the end of two years, the Fund was dissolved, and the box was sold by auction for half its original value. By how much per cent. was this less than its value, as calculated at the above rate of depreciation?

8. At the time of the German reverse (1914) a body of Germans began their retreat at the rate of 24 kilometres a day, from a place 36 kilometres distant from the Allies' Battle Front. The Allies began their pursuit a day later at the rate of 32 kilometres a day. After three days' marching they were delayed in their pursuit for a day to construct a bridge across a river; they then continued their march for three days more and were again delayed for half a day to repair another bridge blown up by the enemy. After how many days from the beginning of the retreat were the Germans overtaken?

Draw a graph to illustrate the movements of the armies. Find from the graph

(1) how far the armies were apart seven days after the beginning of the retreat.

(2) the time when the armies were 32 kilometres from one another.

9. A circus tent is in the form of a cylinder surmounted by a cone. The cylindrical portion of the tent is of diameter 96 feet, the height of the top of the tent is 36 feet from the ground, and the height of the cylindrical part 16 feet. Find the total area of the canvas required for the tent. Answer to the nearest square foot. [$\pi = 3{\cdot}1416$.]

10. The alloy of which the rupee is made consists of silver and copper melted together in the ratio of $r : 1$. The alloy is worth a rupees per lb. and pure silver s rupees per lb. Express the value of copper per lb. in terms of a, s and r. What does copper cost per lb., if $a = 36\frac{3}{4}$, $s = 40$, and $r = 11$?

11. For an entertainment to be given to the principal of a college on the eve of his transfer, the student of the B.A. classes agreed to raise among them a certain sum, levying a uniform rate of subscription. The Intermediate students joining them subsequently, the subscription per student was reduced in the ratio 7 : 4 although the estimated expenses were increased by 50 per cent. Compare the strengths of the B.A. and the Intermediate classes.

From your result find the strength of the college, given that the number of students in the Intermediate classes was 182.

12. The following table gives a series of soundings (measures of depth) taken across a river 42 yards broad at a ford:—

Distance in yards from a bank	0	5	8	10	14	16	19	23	26	30	32	35	38	42
Corresponding depth in feet	0	2·8	4·4	5·0	7·2	8·0	10·2	9·8	8·4	7·3	6·0	4·8	3·1	0

Draw a diagram to represent a section of the river at the ford. From it find

(1) the probable depth in the middle of the stream.

(2) how far a man, who will be safe up to a depth of 5 ft. 2 inches, can venture into the river from either bank without danger.

1922.

1. Here is an extract from Indian Railway Statistics :—

Year	Miles of State Railways	Gross earnings in rupees.
1903	21,131	36,00,82,035
1912	25,125	61,65,07,005

Calculate (1) the earning capacity of the Railways per mile in the two cases, correct to the nearest rupee, (2) the percentage of increase in that capacity for the period 1903–12, correct to the first decimal place.

2. Two persons standing at P and Q, 300 yards apart on a straight sea-shore, wish to find the distance of a boat B from the shore. They observe the angles BPQ and BQP to be 50° and 70° respectively. Obtain the distance of the boat from the shore.

3. Assuming that 15 centimetres = 5·2 inches, draw a graph to convert inches into centimetres and *vice versa*. From the graph, get (1) the number of inches (to the nearest tenth) in (i) 4 cm., (ii) 10 cm. ; and (2) the number of centimetres (to the nearest mm.) in (iii) 2·8 inches and (iv) 4·5 inches.

4. The following is one of the rules of a Sangeetha Sabha : Each member shall pay Rs. 1-4-0 every time he attends a performance, or pay an advance annual subscription of Rs. 15 and also 10 annas at each performance—performances being weekly.

A member X elects the first method of payment and another member Y the second method. Both attend the performances regularly. After how many performances does Y gain an advantage over X? Verify the solution of the question by drawing two graphs (in the same figure) to show the amounts paid by X and Y.

5. In connection with a street improvement scheme, a Municipal Corporation purchased the whole block of houses on one side of the street and the ground (708 ft. by 108 ft.) on which they stood. The houses alone cost Rs. 254,000 and the land cost Rs. 45 per Kuli (= 144 square feet). The houses were pulled down and the materials auctioned away for Rs. 95,000. The street (708 feet long) was then widened by 12 ft. and the remaining land was sold at Rs. 185 per Kuli. Find the net cost of the improvement scheme.

6. From a rectangular sheet of brass 32 inches long and 20 inches broad, a circular piece of diameter 20 inches is cut out, and the remaining part is melted and beaten into a rectangular piece of the same shape as the original sheet and of the same thickness. Find its length and breadth to the nearest inch ($\pi = 3{\cdot}1416$).

7. Pure gold is said to be 24 carats fine. Mint gold contains 22 parts of pure gold to every 2 parts of alloy and is hence said to be 22 carats fine. The gold nibs of Waterman Fountain pens are 14 carats fine. To make such nibs 89 ounces of pure gold and 48 ounces of mint gold are melted together with a certain quantity of alloy. Find this quantity of alloy.

8. At a grand clearance sale a firm say that they have reduced their catalogue prices 20 per cent. and also allow their customers 5 per cent. off their bills for immediate cash payment. If their profits even after such reductions on cash sales come to 14 per cent., find with what rate of profit in view were the original catalogue prices fixed ?

9. I purchased a bungalow two years back for Rs. 50,000, and spent 10 per cent. of this sum to effect certain alterations and repairs. The bungalow has remained unoccupied till now and during this interval I reckon 5 per cent. per annum compound interest on my outlay. If I want to

realize a profit of 20 per cent. on the total amount, at what price should I sell the bungalow now?

10. The owner of a lead mine collected 216·15 tons of stuff from which lead was to be extracted. It contained X per cent. of lead. After washing, in which process no lead was lost, the stuff was found to contain Y per cent. of lead. Show that $\frac{Y - X}{Y}$ of 216·15 tons was washed away. Find the quantity of lead obtained from the stuff, if X = 15·9 and Y = 87·45.

11. Draw a circle of radius 3 inches and divide it into six sectors whose areas are proportional to the areas of the six continents given in the adjoining tabular statement.

Names of continents	Areas in miles to the nearest tenth of million
Asia	17·0
Europe	3·8
Africa	11·4
Australia	3·0
North America ...	8·0
South America ...	6·8

(1) Indicate in your figure the values of the angles of the sectors to the nearest degree.

(2) Enter neatly the names of the continents in the corresponding sectors.

12. Two cylindrical rollers of radii a and b lie side by side on a level ground, touching each other along their lengths. Express in terms of a and b the distance between the lines along which the rollers touch the ground. Find the distance, if $a = 2{\cdot}25$, $b = 1{\cdot}69$.

1923.

1. The following statistics are taken from the Report on Public Instruction in Madras for 1920–21 :—

Year	Grants from Provincial funds to		
	Schools under private management	Local Board schools	Municipal schools
	RS	RS	RS
1919-20 ...	40,54,293	33,39,703	6,53,466
1920-21 ...	42,96,756	34,61,324	4,31,880

Find (*a*) the grand total of grants-in-aid in 1919–20 and 1920–21 ;

(*b*) the increase per cent. in grants for 1920–21 as compared with those for 1919–20 for Local Board schools ;

(*c*) what percentage the grants in 1920–21 for private institutions was of the total grant for the same year ?

2. (*a*) If a piece of iron plate 2·1 metres long 1·13 metres broad and 2·3 cms. thick weighs 425 Kilogrammes find, to the nearest gramme, the weight of a square metre of sheet iron 2 mms. thick.

(*b*) Two cyclists start abreast at 3 o'clock to race on a circular track. If one goes round in 3 minutes 12 seconds and the other in 3 minutes 30 seconds, find at what time they will again be riding abreast past the starting point. Find also the number of complete circuits they will then have made.

3. A tradesman marks certain goods 30 per cent. above cost price ; he allows cash customers a discount of

12 per cent. and credit customers a discount of 5 per cent.; five-eighths of his goods thus marked are sold for cash and the rest on credit. When the whole stock has been sold and paid for, what is his percentage of profit?

4. The Boy Scouts of a High School have a Savings Bank of their own and one of them had a balance of Rs. 6 at the bank on the 1st January, 1922. At the beginning of February and of each alternate month following, he deposited Rs. 3. In the middle of April when the school closed, he withdrew Rs. 5 for his journey. If the interest be allowed at $6\frac{1}{4}$ per cent. per annum, what interest should be added to his account at the end of June 1922, each month being calculated as one-twelfth of a year?

5. A Railway contractor purchased 360 tons of cement at Rs. 95 per ton two years before it was actually required, and in conveying the cement from Madras to the nearest Railway Station, paid a railway freight of Rs. 9-14-0 on each ton. The unloading and conveying charges amounted to 4 annas for a bag of cement (10 bags of cement weighed one ton). On the above outlay he reckoned 5 per cent. compound interest per annum. During the two years the cement lay idle, he had to stock it in a building paying a rent of Rs. 25 per mensem. When after two years the cement was wanted, he charged the railway a higher rate and thus realized a profit of 60 per cent. on the total amount spent. Find, to the nearest rupee the rate he charged for a ton.

6. A contractor employs 20 men working 8 hours a day in order to finish building a house in 25 days. At the end of 14 days only one-third of the work is finished. He engages more men and all his men now work 9 hours a day. What is the least number of additional men that he must engage if the work is to be done by the specified time?

7. Two boys A and B had their heights measured on

the first days of six consecutive years and their measurements are given below :—

—	1916	1917	1918	1919	1920	1921
	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.
A	3 0	3 $4\frac{1}{2}$	3 $7\frac{3}{4}$	3 $10\frac{3}{4}$	4 1	4 $2\frac{3}{4}$
B.	3 2	3 $3\frac{1}{2}$	3 $5\frac{3}{4}$	3 $8\frac{1}{2}$	3 $11\frac{3}{4}$	4 4

Draw, on one diagram, graphs to illustrate their growth, and find the dates when they were of the same height.

8. If the speed of an aeroplane is v miles per hour, find its average speed from A to B and back, if the wind is blowing all the time from A to B at u miles an hour.

Find the average speed of the aeroplane, if $v = 60$ and $u = 5$.

9. I stand on the road 6 ft. from a lamp-post and notice that the end of my shadow just reaches a point at the foot of a wall. Find, by drawing to scale (1 cm. to represent 1 foot), the distance of this point of the wall from the lamp-post, if the light is 10 ft. above the ground, and my height is 5 ft. 6 in. Find the distance also by calculation.

10. In the accompanying figure LMNR is a rectangle ; a quadrant of a circle is drawn with M as centre and ML as radius, meeting NM at P: another quadrant with N as centre and NP as radius meets RN at Q, P being between N and M and Q being between R and N. If the curved path LPQ from L to Q is 20 ft. less than the path LMNQ and 10 ft. less than the path LRQ, find the sides of the rectangle ($\pi = 3{\cdot}1416$).

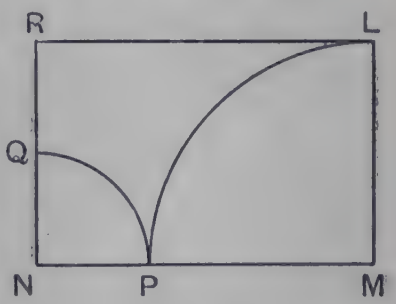


FIG. 321.

11. Draw a right-angled triangle with equal sides and with hypotenuse equal to 2·3 in. On the other side of the hypotenuse construct a square.

Compare the area of the triangle with that of the square: If the figure thus drawn represents the end view of a hay stack whose breadth is b ft. and length l ft., what is the volume of the stack in cubic feet?

Calculate the volume in cubic feet, when $b = 20$ and $l = 50$.

12. A boy fills with moist sand a cylindrical bucket which measures internally 12 in. in diameter and 16 in. in height. He empties the bucket and builds a conical mound with the sand on a base whose diameter is 18 in. Find the height and volume of the mound and the area of its curved surface.

1924.

1. The following statistics taken from the census of 1921 give the population, in thousands, of Hindus, Muhammadans and Christians in six districts :—

	Population in thousands			
	Hindus	Muhammadans	Christians	Total
Guntur (M) ...	1,514	131	153	
North Arcot ...	1,893	118	36	
Tanjore (M) ...	2,105	130	89	
Trichinopoly (M) ...	1,746	63	92	
Ramnad	1,521	116	85	
Tinnevelly (M) ...	1,600	109	192	

(*a*) Find the total population for each district (horizontally) and for each community (vertically).

(*b*) Express the Muhammadan population in each of the districts marked (M) as a percentage of the total population of that district, correct to two significant figures, and state which of them has the highest percentage.

2. (*a*) Find, in pounds and ounces (correct to the nearest tenth of an ounce), the weight of a cubic foot of water, given that 1 metre = 39·371 in., 1 kilogram = 2·2046 lbs. and 1 cubic metre of water weighs 1,000 kilograms.

(*b*) A railway company increased its railway fare by one-half and again increased it by $\frac{3}{4}$ of its new value. Find the original cost, to the nearest anna, of a ticket which after the two increases costs Rs. 19-4-0.

3. (*a*) The capital of a Co-operative Publishing House is fifteen thousand rupees ; one-third of it is borrowed from a Bank at 6 per cent., and the remainder is held in shares. The working expenses of the House are 40 per cent. of the gross receipts in a year. Find what the gross receipts must be in order to pay the shareholders 9 per cent. at the end of the year.

(*b*) Three vessels, each capable of holding 32 gallons, contain mixture of wine and water. In the first, which is half full, the quantities of wine and water are equal ; in the second which is three-quarters full, there are 9 gallons of wine ; and in the third, which is full, there are only 4 gallons of wine. If the second is now filled up from the third, and then the first filled from the second, how many gallons of wine and water will be found in the first vessel ?

4. For a certain football tournament, twelve teams (each consisting of six players) entered, paying an entrance fee of Rs. 10 each. During each of the first four days of the tournament the spectators who numbered 500 on an average were charged 2 annas each, and on the fifth day, when the finals were played, the attendance was half as much again as on any previous day and tickets were sold at 4 annas

each. The committee in charge spent 30 per cent. of the total income in fencing the ground, 25 per cent. of the remainder for ground-rent and Rs. 30-7-0 on miscellaneous charges. They further set apart Rs. 200 for Reserve Fund after purchasing a silver cup for the winning team and six silver medals for the runners-up, the cost of each medal being one-twelfth of that of the cup. Determine the cost of the cup and of a medal.

5. A dealer offers to sell a motor-car for Rs. 5,000 cash or for twelve monthly instalments of Rs. 500 each, the first being payable at once. Calculate the rate of simple interest charged on the supposition that the Rs. 5,000 invested at that rate for 11 months gives the same amount as the total of the amounts of the instalments invested at the same rate.

6. Among the elected members in a certain Legislative Council, the ratio of the ministers' party to the rest was 5 : 2. Eighteen members left the ministerial party and joined the opposition and consequently the ratio was reduced to 26 : 23. Find the total number of elected members.

7. The following table gives the times of sunrise and sunset at a certain place from July to December 1923 :—

Date.	July 1.	July 15.	Aug. 1.	Aug. 15.	Sep. 1.	Sep. 15.	Oct. 1.	Oct. 15.	Nov. 1.	Nov. 15.	Dec. 1.	Dec. 15.	Dec. 31.
	H.M.	H.M.	H.M.	H.M.	H.M.	H.M.	H.M.	H.M.	H.M.	H.M.	H.M.	H.M.	H.M
Sunrise ...	5 47	5 50	5 54	5 56	5 58	5 58	5 58	5 59	6 2	6 8	6 16	6 23	5 31
Sunset ...	6 39	6 39	6 36	6 31	6 21	6 12	0	5 51	5 44	5 39	5 40	5 45	6 53

(1) Plot graphs (in two different diagrams) showing for the given period (*a*) the time of sunrise, (*b*) the duration of sunshine, on successive days.

(2) Deduce from the graphs the time of sunrise on November 22, and the duration of sunshine on August 8.

8. A man standing 12 yds. from a river bank observes a

post on the further bank directly opposite to him. He walks 30 yds. parallel to the bank and then walks directly towards the post, reaching the bank after walking 15 yds. Find, by drawing to scale (1 c.m. to represent 3 yds.), the breadth of the river, assuming the banks to be straight. Find this distance also by calculation.

9. A surveyor gives the following measurements of a quadrilateral field ABCD (having no re-entrant angle): AB = 1,980 links, BC = 920 links, AD = 1,700 links, $\angle BAC = 35°$, $\angle BAD = 61°$, C and D being on the same side of AB. Draw a plan of the field to a scale of 1 in. for 400 links, stating the steps of your construction. Find the area of the field in acres, given 100 links = 22 yds.

10. A shop-keeper uses a badly made balance so that a body really weighs $a + b$ Q lbs. when the balance shows only Q lbs., a and b being numbers which are the same for all weights. If the balance shows 1 lb. for a real weight of 1·1 lb. and 2 lbs. for a real weight of 1·9 lb., what are the values of a and b and what will the balance show for a real weight of 2 lbs.? Draw a graph to show the relation between the real weights and the weights shown by the balance.

11. A canal whose cross-section is a trapezium is 8 ft. deep, 18 ft. wide at the surface and 12 ft. wide at the bottom. Find the area of the cross-section and the number of gallons of water that pass any spot in an hour when the canal is full and water flows at 2 miles per hour. [Given one cubic foot of water = 6·25 gallons.]

12. A right circular cylindrical log of teak is 12 ft. long and of diameter 14 in. Find its volume and curved surface.

If it is planed down to the least amount necessary to form a prism whose cross-section is a regular hexagon, what would be the volume of the prism in cubic feet, given $\pi = \frac{22}{7}$ and $\sqrt{3} = 1{\cdot}73$.

What percentage (correct to an integer) of the whole log would be wasted in planing down?

1925.

1. The following table shows the population, in thousands of five Indian provinces according to the census taken in the years 1911 and 1921 :—

	Population in thousands	
	1911	1921
Bengal	45,483	46,696
Madras	41,405	42,319
Bombay	19,626	19,348
United Provinces	46,807	45,376
Punjab	19,579	20,685

Find, correct to one decimal place

(*a*) the percentage of increase or decrease of the population in each of the first three provinces.

(*b*) the percentage of increase in all the five provinces taken together.

2. Three men, A, B and C, have together completed a piece of work for which they receive Rs. 64-14-0. A has worked for 4 days of $10\frac{1}{2}$ hours, B for 11 days of 7 hours, and C for 6 days of 9 hours. How much money ought each to receive ?

3. A goldsmith undertook to make a jewel casket out of certain quantities of gold and silver, together weighing 104 ounces and costing £230-4-0. Instead of using the quantities of gold and silver agreed upon, he interchanges them. If gold costs £3-17-6 and silver 5*s*. 6*d*. per ounce, find by how much the value of the casket has been reduced.

4. A clerk's salary was £80 for each of the first 3 years. Then it was raised at the rate of £10 a year for 12 years, after which it remained stationary. When he retired from

work he found that his average salary per year had been £174-10-0. How many years did he work ?

5. Exactly 3 years ago a man borrowed Rs. 1,890 from a bank at $6\frac{1}{4}$ per cent. per annum. At the end of a year he paid the interest for that year and a part of the loan, amounting altogether to Rs. 500. Similarly he paid Rs. 700 at the end of the second year. What sum must he now pay to clear off the debt ? (Answer to the nearest anna.)

6. Three cyclists, A, B and C, ride round a circular track at 8, 9 and 10 miles per hour respectively. They start together from the same point, but C rides in an opposite direction to A and B, and meets A one minute after passing B. What is the length of the track ?

7. If the manufacturer of a machine sells 36 machines to a wholesale dealer at 25 per cent. above cost price, and gives one extra machine free ; and if the wholesale dealer sells each machine to a shopkeeper at 5 per cent. more than what it cost him, taking into account the machine given free ; and if the shopkeeper sells it to a customer for £34-13-0, making a profit of 10 per cent., what did it cost to make ?

8. A rectangular metal plate has a circular hole in it. The length of the plate is 1·3 metres, the width is 80 centimetres and the thickness is 2 millimetres. If the plate weighs 32·25 lbs., find the diameter of the hole, correct to a centimetre, given 1 c. cm. of the metal weighs 8 grams and 1 lb. = 453·6 grams.

9. The following table shows the number of grams of alum which can be dissolved in 100 grams of water at different temperatures :—

Temperature in degrees C	0°	10°	20°	30°	40°	50°	60°	70°
Grams soluble in 100 grams of water.	3·9	9·5	15·1	22·0	30·9	44·1	66·6	90·7

Draw a graph and find at what temperature 50 gms. of alum could be dissolved in 100 gms. of water.

10. A straight rod AB, 2 ft. 8 in. long, is held under water, A being 2 ft. 6 in. and B 1 ft. 2 in. below the surface. Find, by drawing to scale, the distance below the surface of a point C on the rod which divides AB so that $AC = \frac{3}{5}$ BC. Find also the angle which the rod makes with the surface of the water.

11. The cost of making a certain machine consists of an initial expense plus a fixed sum for each machine. To make 25 machines costs £885; to make 40 machines costs £1,335. Find the initial expense and the fixed sum.

12. ABCD is a quadrilateral field in which the diagonals AC and AD are at right angles. AB, BC, CD and AC are 600, 700, 800 and 1,000 links respectively. Draw the quadrilateral to scale and hence find the area of the field in acres. (100 links = 22 yards.)

13. A vessel in the form of a hollow right circular cone, whose height is 1 ft. 6 in. and radius of base 6 in., is placed with its axis vertical and vertex downwards. Find how many gallons of water it will hold (1 cubic foot = 6·24 gallons). Find also the cost of tinning the inner surface of the vessel at 4 as. per square foot.

APPENDIX

I. Tables.

Indian Money.

12 Pies (p.) make 1 Anna (a.).
16 Annas ,, 1 Rupee (Re.).

English Money.

4 Farthings (*q*) make 1 Penny (*d.*).
12 Pence ,, 1 Shilling (*s.*).
20 Shillings ,, 1 Pound (£).
Note.—2 Shillings ,, 1 Florin.
5 Shillings ,, 1 Crown.
21 Shillings ,, 1 Guinea.

Madras Government Weights.

3 Tolas (Tol.) make 1 Palam (pal.).
40 Palams ,, 1 Viss (v.).
8 Viss. ,, 1 Maund (md.).
20 Mds. ,, 1 Candy or baram.

1 Tola = the weight of a rupee = 180 Grains.
1 Viss = 5 Seers, 1 Seer = 8 pal.
1 Md. = 25 lbs. Avoirdupois.

Avoirdupois Weights.

16 Drams make 1 Ounce (oz.).
16 Ounces ,, 1 Pound (lb.).
14 Pounds ,, 1 Stone (st.).
28 Pounds ,, 1 Quarter (qr.).
4 Quarters ,, 1 Hundred weight (cwt.).
20 Hundred weights make 1 Ton (ton).

Note.—Since pound occurs in this and one of the above tables £ should be read as pound (money) and lb. ,, ,, (weight).

Troy Weights.

24 Grains = 1 Pennyweight (dwt.).
20 Pennyweights = 1 Ounce (oz.).
12 Ounces = 1 Pound (lb.).

Apothecaries' Weights.

20 Grains (grs.) make 1 Scruple.
3 Scruples ,, 1 Drachm.
8 Drachms ,, 1 Ounce.
12 Ounces ,, 1 Pound.

Note.—The grain is the same in the above three systems of weight.

1 lb. Avoirdupois = 7,000 Grains.
1 lb. Troy = 5,760 Grains.

Indian Weights.

5 Tolas make 1 Chatak (chk.).
16 Chataks ,, 1 Seer (sr.).
40 Seers ,, 1 Maund (md.).
1 Maund = $82\frac{2}{7}$ lbs. avoirdupois.
or 100 lbs. troy.

Madras Measures of Capacity.

8 Ollocks = 1 Measure (meas.).
8 Measures = 1 Markal (mar.).
12 Markals = 1 Kalam (kal.).
5 Markals = 1 Parah,
80 Parahs or 400 Markals = 1 Garce.

British Measures of Capacity.

2 Pints = 1 Quart.
4 Quarts = 1 Gallon.
2 Gallons = 1 Peck.
4 Pecks = 1 Bushel.
8 Bushels = 1 Quarter.

Note.—1 Gallon = 277·45 c. in. or 0·16 c. ft. A cubic foot of water weighs 62·5 lbs.

Miscellaneous Measures.

12 Articles make 1 Dozen.
12 Dozen ,, 1 Gross.
20 Articles ,, 1 Score.

Paper Measure.

24 Sheets make 1 Quire.
20 Quires ,, 1 Ream.
10 Reams ,, 1 Bale.

British Measurement of Length.

12 Inches make 1 Foot (ft.).
3 Feet ,, 1 Yard (yd.).
220 Yards ,, 1 Furlong (fur.).
8 Furlongs (1760 yds.) make 1 Statute mile (mi.).
1 Rod, Pole, Perch = 5½ yds.
1 League = 3 Miles.

For Land Surveying.

1 Chain (ch.) = 100 Links = 22 yds.
1 Chain = 4 Poles, and 80 Chains = 1 Statute mile.

For Measuring Distances on the Sea.

1 Fathom = 6 feet.
1 Cable = 100 Fathoms = 220 yds.

A degree of latitude = 69·1 statute miles if the radius of the earth is 3959 miles. A nautical mile is $\frac{1}{6}$ part of a degree of latitude and varies from 6,046 ft. to 6,107 ft. It is usually taken to be 6,080 ft. A knot is a velocity of 1 nautical mile per hour.

Area.

144 Square inches (sq. in.) = 1 Square foot (sq. ft.).
9 Square feet = 1 Square yard (sq. yd.).
4840 Square yards = 1 Acre (ac. or a).
640 Acres = 1 Square mile (sq. m.).

Note.—1 Sq. Pole = $30\frac{1}{4}$ Sq. yds.
40 Poles = 1 Rood.
4 Roods = 1 Acre.

For Land Surveying.

100^2 (square) Links = 1 Square Chain.
10 Square Chains = 1 Acre.
1 Manai or Ground = $60' \times 40'$.
24 Manais or Grounds = 1 Cawnie.

Table of Time.

60 Seconds = 1 Minute.
60 Minutes = 1 Hour.
24 Hours = 1 Day.
7 Days = 1 Week.
365 Days = 1 Year.
366 Days = 1 Leap year.

Note.—1894 is not a leap year because it is not divisible by 4; 1896 is a leap year because it is divisible by 4. Again if the date is a multiple of 100, it is a leap year if the date is divisible by 400 exactly. 1,900 is not a leap year but 1,600 is a leap year.

Metric System.

Lengths.

A millimetre (mm.) $= \frac{1}{1000}$ metre.
A centimetre (cm.) $= \frac{1}{100}$ metre.
A decimetre (dm.) $= \frac{1}{10}$ metre.
A Decametre (Dm.) = 10 metres.
A Hectometre (Hm.) = 100 metres.
A Kilometre (Km.) = 1000 metres.

1 metre = 39·370113 in. or 3·280843 ft. or 1·0936143 yd.
1 Km. = ·0621372 mi.
1 in. = 25·4000 mm.
1 mile = 1·609343 Km.

A metre $= 1\frac{1}{10}$ yd.
A foot = 30 cm.
A Km. $= \frac{5}{8}$ m.
} approximately.

Weights.

A milligram (mg.) $= \frac{1}{1000}$ gram. (gm.).
A centigram (cg.) $= \frac{1}{100}$ gm.
A decigram (dg.) $= \frac{1}{10}$ gm.
A Decagram (Dg.) = 10 gm.
A Hectogram (Hg.) = 100 gm.
A Kilogram (Kg.) = 1000 gm.
A Tonne (metric ton) = 1000 kg.
A quintal = 100 kg.
1 gm. = 15·432356 grains.
1 Kg. = 2·2046223 lbs.
1 metric ton = 0·984206 ton (avoirdupois).
1 ounce = 28·34953 gm.
1 pound = 0·45359243 Kg.
1 ton = 1·016047 metric tons.
A Kg. = $2\frac{1}{2}$ lbs. approximately.

Metric Area

1 are (a)	= 1 sq. D.M.
1 centiare (ca)	= $\frac{1}{100}$ are.
1 hectare (Ha)	= 100 ares.

Note.—1 sq. cm.	= 0·1550006 sq. in
1 sq. m.	= 1·195992 sq. yd.
1 sq. in.	= 6·45159 sq. cm.
1 sq. yd.	= 0·836126 sq. m.
1 are	= 119·5992 sq. yd.
1 hectare	= 2·47106 ares.
1 acre	= 0·404685 Ha.

Metric Volume.

1 cubic centimetre	= 0·0610239 c. in.
1 cubic metre	= 1·307954 c. yd.
1 cubic in.	= 16·3870 c. c.
1 cubic yd.	= 0·764553 c. m.

Metric Capacity.

1 litre	= 1 cubic decimetre.
A centilitre (cl.)	= $\frac{1}{100}$ litre (L.)
A decilitre (de.)	= $\frac{1}{10}$ litre.
A Decalitre (Dl.)	= 10 litres.
A Hectolitre (Hl.)	= 100 litres.
1 litre	= 1·75980 pt.
1 Hectolitre	= 2·74969 bushels.
1 pt.	= 0·568245 L.
1 gallon	= 4·545963 L.
A litre	= $1\frac{3}{4}$ pt. approximately.

II. Mensuration Formulæ.

1. Area of a rectangle or parallelogram = base × height.

2. (*a*) Area of a triangle = (i) $\frac{1}{2}\ ah$. where a is the base and h the corresponding altitude.

= (ii) $\sqrt{s\ (s-a)\ (s-b)\ (s-c)}$

where a, b, and c are the sides and $s = \frac{1}{2}\ (a + b + c)$.

(*b*) Area of a right-angled triangle = $\frac{1}{2}\ ab$ where a and b are the sides containing the right angle.

3. Area of a quadrilateral = (i) $\frac{1}{2}$ × a diagonal × the sum of the offsets (i.e. the perpendiculars on the diagonal from the opposite angular points.

= (ii) or $\frac{1}{2}$ the area of the parallelogram formed by drawing parallels to the diagonals through the angular points of the quadrilateral.

= (iii) $\frac{1}{2}$ the product of the diagonals when they are at right angles.

4. Area of a trapezium = $\frac{1}{2}\ h\ (a + b)$ where a and b are the parallel sides and h the perpendicular distance between them.

5. Circumference of a circle = $2\ \pi\ r = \pi\ d$. where r is the radius and d the diameter.

π = 3·14159265. . . . If the result is required to 4 figures, then π = 3·1416. If π is taken as 3·14, the third figure of the result is unreliable. $3\frac{1}{7}$ is a convenient value but the answer is unreliable after the third figure.

6. The area of a circle = (i) πr^2

where r is the radius.

= (ii) 0·78539814 × d^2 where d is the diameter.

7. The area of a sector = (i) $\frac{1}{2}\ l\ r$ where l is the arc and r the radius.

= (ii) $\frac{n}{360}$ of the area of the circle where n is the angle of the sector in degrees.

8. The area of the curved surface of a right circular cylinder $= 2\pi rh$

where h is the height and r the radius of the cross section.

9. The volume of a right prism or circular cylinder $= A\,h$ where A is the cross section and h the length at right angles to the cross section.

$= \pi r^2 h$ where r is the radius of the base and h the height.

Volume of a hollow cylinder $= (\pi R^2 - \pi r^2)\,h$ where R and r are the external and internal radii and h the height of the cylinder.

10. The area of the slant surface of a right circular cone $= \frac{1}{2} l.\,c.$ where l is the slant height and c the circumference of the base $= \pi rl.$

$= \pi r \sqrt{r^2 + h^2}$ where r is the radius of the base and h the height of the cone.

11. The volume of a cone or pyramid $= \frac{1}{3} A\,h$ where A is the area of the base and h the height.

Volume of a cone (alone) $= \frac{1}{3} \pi r^2 h$ where r is the radius of the base and h the height.

12. The surface of a cube $= 6a^2$ where a is an edge of the cube.

13. The volume of a cube $= a^3$. . . . where a is an edge of the cube.

14. The volume of a cuboid = length $\times$ breadth $\times$ height.

15. If a is a side of an equilateral triangle,

$$\text{its height} = \frac{\sqrt{3}}{2} a = \cdot 866a = \cdot 87a.$$

16. If h is the height of an equilateral triangle, then its side $\frac{2h}{\sqrt{3}} = 1{\cdot}1547h = 1{\cdot}15h.$

17. If a is a side of a square, its diagonal $= \sqrt{2}\cdot a = 1{\cdot}414a.$

18. If d is a diagonal of a square, its side $= \frac{d}{\sqrt{2}} =$ $\cdot 7071a$.

19. If a is a side of an isosceles right-angled triangle, then its hypotenuse $= a\sqrt{2} = 1 \cdot 414a$.

20. If a is the hypotenuse of a right-angled triangle, then its side $= \frac{a}{\sqrt{2}} = \cdot 7071a$.

21. The area of an equilateral triangle of side a $= \frac{\sqrt{3}}{4} a^2 = \cdot 433a^2$.

22. The area of a regular pentagon of side $a = 5a^2 \times \cdot 344$.

23. The area of a regular hexagon $= \frac{6\sqrt{3}}{4} a^2 = 6a^2$ $\times \cdot 433$.

24. The area of a regular octagon $= 2 (1 + \sqrt{2}) a^2$ $= 8 a^2 \times \cdot 6035$.

25. The area of an equilateral triangle inscribed in a circle of a radius $r = \frac{3\sqrt{3}}{4} r^2 = 3r^2 \times \cdot 433$ (side of the triangle $= r\sqrt{3}$).

26. The area of a square inscribed in a circle of a radius $r = 4 r^2 \times \cdot 5 = 2r^2$ (side $= r\sqrt{2}$).

27. The area of a regular pentagon inscribed in a circle of radius $r = 5 r^2 \times \cdot 476$.

28. The area of a regular hexagon inscribed in a circle of radius $= 6 \cdot \frac{\sqrt{3}}{4} r^2 = 6r^2 \times \cdot 433$ (side $= r$).

29. The area of a regular octagon inscribed in a circle of radius $= 2\sqrt{2}r^2 = 8 r^2 \times \cdot 354$.

30. The area of an equilateral triangle described about a circle of radius $r = 3r^2 \times \sqrt{3} = 5 \cdot 196 r^2$ (side $= 2\sqrt{3}.r$).

31. The area of a square described about a circle of radius $r = 4r^2$ (side $= 2r$).

32. The area of a regular pentagon described about a circle of radius $r = 5\ r^2 \times \cdot 727$.

33. The area of a regular hexagon described about a circle of radius $r = \frac{4}{3}$ of the inscribed hexagon $= 2\sqrt{3}\ r^2 = 6r^2 \times \cdot 577$.

34. The area of a regular octagon described about a circle of radius $r = 8r^2 \times \cdot 414$.

FORMULAE.

35. $(x + a)(x + b) = x^2 + (a + b)\ x + ab$.

36. $(x - a)(x - b) = x^2 - (a + b)\ x + ab$.

37. $(a + b)^2 = a^2 + 2ab + b^2$; $(a - b)^2 = a^2 - 2ab + b^2$.

38. $a^2 - b^2 = (a + b)(a - b)$.

39. Simple interest formula. $I = \frac{P \times n \times r}{100} =$

$A = P\left(1 + \frac{nr.}{100}\right)$

40. Compound interest formula. $A = P\left(1 + \frac{r}{100}\right)^n$.

APPROXIMATIONS.

41. $\frac{1}{1 + x} = 1 - x$ when x is very small

and $\frac{1}{1 - x} = 1 + x$ when x is very small.

$(1 + x)^n = 1 + nx$ when x is very small.

$(1 + x)(1 + y) = 1 + x + y$ when x and y are very small.

ANSWERS.

Exercise XXI (A) (Page 6).

1. (*a*) Rs. 5 lakhs ; (*b*) 70 million miles ; (*c*) ·03 km., hundredths; (*d*) 900 lbs., hundreds ; (*e*) 5,000 gallons, thousands ; (*f*) ·08 lb., hundredths. **2.** (*a*) ·641 ; (*b*) 488000 ; (*c*) 123000 ; (*d*) 9·07 ; (*e*) ·0008 ; (*f*) 8·48 ; (*g*) ·0008 ; (*h*) 864000 ; (*i*) 470000. **3.** (*a*) 1, 0, 3 ; (*b*) 4, 1 ; (*c*) 6, 2 ; (*d*) 2, 2 ; **4.** 2,500 gms. **5.** 246·9 acres. **6.** 280 cub. in. **7.** (*a*) Rs. 767·3855 and 767·3845 ; (*b*) £380·3095 and £380·3085 ; (*c*) 460·0535 miles and 460·0525 miles ; (*d*) 851·9865 kg. and 851·9855 kg. **8.** (*a*) Rs. 30·045 and Rs. 30·035 ; (*b*) Rs. 300·45 and 300·35 ; (*c*) Re. ·030045 and Re. ·030035 ; (*d*) Rs. 30,04,500 and Rs. 30,03,500 ; (*e*) 8·6345 kg. and 8·6335 kg. ; (*f*) 86·345 Hg. and 86·335 Hg. ; (*g*) 8634·5 gm. and 8633·5 gm. ; (*h*) 8634500 mm. and 8633500 mm. **9.** (1) (*a*) Rs. 80·35 and Rs. 80·25 ; (*b*) Rs. 8·035 and Rs. 8·025 ; (*c*) Rs. 80,350 and Rs. 80,250 ; (*d*) Rs. 8,035 and Rs. 8,025 ; (2) (*a*) Rs. 80·305 and 80·295 ; (*b*) Rs. 8·0305 and Rs. 8·0295 ; (*c*) Rs. 80,305 and Rs. 80,295 ; (*d*) Rs. 8,030·5 and Rs. 8,029·5. **10.** (*a*) + 500 and — 500 ; (*b*) + 5 and — 5 ; (*c*) + 50000 and — 50000 ; (*d*) + ·005 in. and — ·005 in. ; (*e*) + 5 sq. miles and — 5 sq. miles ; (*f*) + ·005 yd. and — ·005 yd. ; (*g*) + ·05 gm. and — ·05 gm. **11.** (*a*) (1) 3 significant figures, (2) 1 significant figure ; (*b*) 1 significant figure ; (*c*) 2 significant figures. **12.** (1) (*a*) ·0001 nearly ; (*b*) ·0003 nearly ; (*c*) ·03 nearly ; (*d*) ·00044 nearly ; (*e*) ·000005 nearly ; (*f*) ·000002 nearly ; (*g*) ·00005 nearly ; (2) (*a*) ·01 nearly ; (*b*) ·03 nearly ; (*c*) 3 nearly ; (*d*) ·044 nearly ; (*e*) ·0005 nearly ; (*f*) ·0002 nearly ; (*g*) ·005 nearly.

Exercise XXI (B) (Page 10).

1. 1652·080. **2.** 2394·256. **3.** 574·520. **4.** 122831716 km. **5.** 215. **6.** 330. **7.** 1·04. **8.** 859. **9.** (1) 50 million sq. miles ; (2) 502 hundred thousand sq. miles ; (3) 50,203 thousand sq. miles. **10.** (1) Great Britain, 3851 million dollars ; France, 2277 million dollars ; Russia, 336 million dollars ; Italy 676 million dollars ; Belgium, 93 million dollars ; Serbia, 9 million dollars · Cuba, 15 million dollars ; (2) (*a*) 2043 million dollars ;

(*b*) 5214 million dollars. **11.** (2) (*a*) Correct to 3 significant figures; (*b*) Correct to 3 significant figures. **12.** 13456 thousand sq. miles. **13.** (1) (*a*) 1916—£50,400,000, 1917—£67,800,000; (*b*) 1916—£50 millions, 1917—£68 millions; (2) (*a*) £17 millions; (*b*) 17,400,000. **14.** Correct to 3 significant figures (ten lakhs), (*a*) Rs. 33,04,00,000; (*b*) 33 crores. **15.** £190,000. **16.** 36,550,000. **17.** Between 15·73 m. and ·43 m.; between 15·74 m. and 15·72 m.; ·44 m; and ·42 m. **18.** 9,000 ft. correct to 3 significant figures. **19.** 1,800 gms. (correct to 2 significant figures). **20.** (*a*) (1) Between 18·2385 and 18·2358, (2) 5·3925 and 5·3895, (3) 1·5885 and 1·5855. (4) 11·2605 and 11·2575; (*b*) Rs. 14,000 and Rs. 12,000; correct to the first significant figure (correct to the nearest ten thousand rupees).

Exercise XXI (C) (Page 17).

1. 52·13. **2.** 2·14. **3.** 1293·47. **4.** ·22. **5.** ·33. **6.** 6·53 **7.** 7300·56. **8.** 575·56. **9.** ·086. **10.** 1·911. **11.** 2·218. **12.** ·524 **13.** ·271. **14.** ·003. **15.** ·003 **16.** ·086. **17.** 1526 ten millions or 1526×10^7. **18.** 1157 thousands. **19.** ·008435. **20.** 1·476. **21.** 1259 ten millions. **22.** 3303 hundred millions **23.** 460200. **24.** ·03121. **25.** 3444 millions. **26.** 89722 lakhs. **27.** 265 hundred thousands. **28.** 449 thousands. **29.** 10 millions **30.** 9 hundreds. **31.** 100868. **32.** ·001. **33.** 2726 in. **34.** 70264 yds. **35.** 1905 sq. miles. **36.** 320 miles. **37.** 84 sq. miles. **38.** 41 millions. **39.** 1347 thousands. **40.** £43,107 thousands. **41.** 10 lbs. **42.** 7987 thousands. **43.** 1673 metres per minute. **44.** 176×10^{11} miles. **45.** 1314872 days. **46.** 25000 miles. **47.** 134000 candies. **48.** 34079700 tons. **49.** Rs. 1,640. **50.** 8·1 + ·3175 and — ·3125 or ± ·315 nearly. **51.** 6·12, ± ·26 nearly. **52.** ·86 sq. units. **53.** 1·11 sq. cm. **54.** (*a*) 6 sq. ft. 9·20 sq. in., 6 sq. ft. 3·3 sq. in.; (*b*) 16 sq. ft. 4·8 sq. in., 15 sq. ft. 139·2 sq. in.; (*c*) 9 sq. ft. 10·82 sq. in., 9 sq. ft. 3·6 sq. in.; (*d*) 1 sq. ft. 139·92 sq. in., 1 sq. ft. 136·56 sq. in. **55.** 2726 gms. **56.** 12·17 ft. **57.** 9093·30804 lbs. correct to 3 significant figures 9090 lbs. **58.** Rs. 1,12,500 correct to a lakh.

Exercise XXI (D) (Page 23).

1. 69·6. **2.** ·483 **3.** ·90535. **4.** ·000408. **5.** 8·898. **6.** ·17. **7.** ·07. **8.** 1·58. **9.** ·79. **10.** ·04. **11.** 1·0583. **12.** ·3183. **13.** ·146. **14.** ·017. **15.** ·8. **16.** 2381. **17.** 40000. **18.** 5·657. **19.** ·3937 in.; 39·37 in.

20. 700 yds. **21.** 494 sec. **22.** 7·3. **23.** ·9144 m. **24.** 2 lbs.
25. Rs. 6-13-7. **26.** 229 ; 211 ; 224. **27.** 1609 m.
28. ·093 sq. m. **29.** 247·12 ac. **30.** 28·317 litres.
31. 28317 gm. **32.** ·002 lbs. **33.** 5 litres nearly. **35.** 1*s.* 6*d.*
36. ·3183. **37.** 7957·5. **38.** ·3679 ; 3·0862 ; 2·3504.
39. ·3 per cent. **40.** 13·6 per cent. **41.** (*a*) 1 mg. ; (*b*) $\frac{1}{100}$ mg.
42. (*a*) 1000 ; (*b*) 100 ; (*c*) ·01. **43.** ·322 lb. nearly.
44. 7 as. 7 ps. nearly. **45.** 3400 lbs. **46.** 39·4 inches. **47.** 11,
48. Not more than 3 sec. **49.** Correct to 1 decimal place ; there is no error. **50.** ·9993 nearly. **51.** ·997269672 nearly.
52. (*a*) 5·5 ; (*b*) 21·06 ; (*c*) 42000 ; (*d*) 1140.

Exercise XXI (E) (Page 37).

1. 8·0. **2.** ·9. **3.** 405·3. **4.** 77·6. **5.** 2·98.
6. ·04. **7.** ·86. **8.** ·04. **9.** 106×10^9. **10.** 817×10^7.
11. 757. **12.** ·170. **13.** 418. **14.** 99·4. **15.** 5086·86.
16. ·09. **17.** 263·25. **18.** ·04. **19.** 51160000.
20. $1·01 \times 10^8$ or 101 millions. **21.** ·999. **22.** ·5000.
23. 1·2974. **24.** 1·6667. **25.** Rs. 3-7-11 **26.** Rs. 4-11-4
27. Rs. 16-4-1. **28.** Rs. 39-0-5. **29.** £5-14-1. **30.** £6-2-0.
31. £4-15-4. **32.** £864-0-8. **33.** £19·896. **34.** Rs. 530·969.
35. £20·691. **36.** Rs. 438·932. **37.** 4·594. **38.** Rs. 16·490.
39. £0·447. **40.** Rs. 8·365. **41.** 1·9 miles. **42.** (*a*) 9·870 ; (*b*) 33·01. **43.** 65587000 sq. miles. **44.** 72×10^8 tons.
45. 19×10^{26} tons. **46.** 23000 times the radius of the earth.
47. 18 miles nearly. **48.** ·64 miles. **49.** (*a*) 1·005 ; (*b*) 1·009 ; (*c*) 1·013 ; (*d*) 6·066 ; (*e*) ·001017 ; (*f*) ·001013 ; (*g*) 1·026 ; (*h*) 1·048 ; (*k*) 1·15 ; (*l*) ·996 ; (*m*) 3·8452 ; (*n*) ·996.
50. (*a*) 7·52 ; (*b*) 8·56 ; (*c*) 27·115 ; (*d*) 7·07 ; (*e*) 18 ; (*f*) 1·623 ; (*g*) 1·6176 ; (*h*) 1·4 nearly ; (*i*) 3·451 ; (*j*) ·093 ; (*k*) 5·934 ; (*l*) ·006 ; (*m*) 1·147 ; (*n*) ·88 ; (*p*) Rs. 12-5-9 ; (*q*) £57-14-6.
51. 1014 thousands. **52.** 455 hundred millions. **53.** 15·255.
54. 15·246. **55.** 5000 (correct to a thousand). **56.** Rs. 423563 ± Rs. 11875. **57.** Rs. 420000. **58.** (*a*) Rs. 5373-14-0 ; (*b*) Rs. 2295 ; (*c*) Rs. 191¼ ; (*d*) Rs. 2-13-0 ; (*e*) Rs. 144. **60.** 2576×10^{13} sq. miles.
61. (*a*) £72-2-8 ; (*b*) £709-12-8 ; (*c*) £7,084-12-8 ; (*d*) £709,643-15-6.
62. 16090 and 15484 gms. **63.** ·074 and ·072 cm. **64.** 5·683.
65. 1·3 per cent. nearly. **66.** 57°17 min. 44 sec. nearly.
69. (*a*) to 4 decimal places ; (*b*) 5 places ; (*c*) 6 places.
70. 2 decimal places. **71.** (*a*) 3 decimal places ; (*b*) 4 places; (*c*) 6 places.
73. 1 significant figure. **74.** 580 and 577. **75.** 2 significant figures (100 yds.) ; 7⅞ yds. **76.** (49·296 ± 862) : 1. **77.** 321 c.ft.
78. 983·15. **79.** 99·118. **80.** Re. ¼.

Exercise XXII (A) (Page 46).

1. 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14. **2.** 2×3. **3.** 3×4.
4. 4×5. **5.** 5×6. **6.** $2 \times 4 \times 3$. **7.** $2 \times 6 \times 8$. **8.** $2 \times 9 \times 10$.
9. 4×6^2. **10.** $3^2 \times 8^2$. **11.** $3^2 \times 6^2 \times 8$. **12.** $5a^2b^2c$. **13.** $6a^3b^2c$.
14. $7xy^2z$. **15.** $10x^3y^3z^2$. **16.** $5m^2p^2q^2r$. **17.** 15. **18.** 17.
19. 18. **20.** 19. **21.** 22. **22.** 24. **23.** 25. **24.** 26.
25. not a square. **26.** 48. **27.** 54. **28.** 56. **29.** 64.
30. 72. **31.** 84. **32.** 102. **33.** 96. **34.** 103. **35.** 10.
36. 53. **37.** 109. **38.** 257. **39.** 266 **40.** 95.
41. $3 \times 2 \times 5 \times 101$. **42.** $2 \times 19 \times 13$. **43.** 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13. **44.** 25, 64, 72, 96.

Exercise XXII (B) (Page 60).

1. 21. **2.** 26. **3.** 22. **4.** 39. **5.** 43. **6.** 47. **7.** 56.
8. 63. **9.** 69. **10.** 73. **11.** 82. **12.** 92. **13.** 99. **14.** 105.
15. 237. **16.** 481. **17.** 627. **18.** 803. **19.** 961. **20.** 6401.
21. 8002. **22.** 9451. **23.** 16105. **24.** 46124. **25.** 99997. **26.** 9·83.
27. ·431. **28.** 5·05. **29.** 331·1. **30.** 6·0483 **31.** ·0682. **32.** 4·281.
33. 64·005. **34.** 64·0098. **35.** ·007001. **36.** 8064·43. **37.** 1·732.
38. 2·236. **39.** 2·449. **40.** 2·828. **41.** 3·162. **42.** 1·844.
43. 2·960. **44.** 3·354. **45.** 14·878. **46.** ·224. **47.** ·288.
48 ·408. **49.** 12·003. **50.** ·823. **51.** 18·311. **52.** 31·623.
53. $3\frac{1}{2}$. **54.** $1\frac{1}{3}$. **55.** $3\frac{4}{7}$. **56.** $\frac{121}{149}$. **57.** $\frac{176}{413}$.
58. $6\frac{14}{143}$. **59.** $1\frac{182}{843}$. **60.** $46\frac{13}{41}$. **61.** ·378. **62.** ·943.
63. 2·183. **64.** ·522. **65.** 1·826. **66.** 2·760. **67.** ·293939.
68. 37·8668. **69.** ·6417. **70.** 6·171520. **71.** 2·8868. **72.** ·2236.
73 ·2817. **74.** 1·0328. **75.** 2·5981. **76.** 3·1749. **77.** ·1443.
78. ·2361. **79.** ·3178. **80.** 1·1805. **81.** 13·9282. **82.** 9·8990.
83. 4·5559. **84.** 7·8730. **85.** 21. **86.** 143. **87.** 3·528.
88. 6·881. **89.** 39·05. **90.** 44·75. **91.** 55·5. **92.** 31·94.
93. 47·96. **94.** 50·01. **95.** 59·97. **96.** 17·33. **97.** (*a*) 36; (*b*) 45; (*c*) 56; (*d*) 104; (*e*) 30; (*f*) 144; (*g*) 96; (*h*) 175; (*i*) 400. **98.** (*a*) 1763; 84 (*b*) 224; 30 (*c*) 728; 54 (*d*) 2303; 96 (*e*) 2499; 100 (*f*) 3599; 120 (*g*) 2600; 102.
100. 81·6 yds. nearly. **101.** 10001 yds. **102.** 1203 yds., 1604 yds.
103. 1248 km., 520 km. **104.** 98 yds. **105.** 269 yds.
106. $b = 74$, $l = 3362$. **107.** B = 71 yds., L = 212 yds. nearly.
108. 717 m. **109.** 35 ft. **110.** 47 yds. 6 in. nearly. **111.** 4220 m.
112. (*a*) 204; (*b*) 1023; (*c*) 80; (*d*) 198. **113.** (*a*) 28; (*b*) 44.
114. 624 : 355. **115.** (*a*) 13 : 12; (*b*) 15 : 7. **116.** $\frac{13}{4}$ in.
117. (*a*) 10 sec.; (*b*) 11 sec. **118.** 48. **119.** 1·06 : 1. **120.** 1·85
121. 74. **122.** 1175 sq. units. **123.** 3146 sq. units.
124. 20·27 in. **125.** Q = 728.

126. 3595216 c. ft. **127.** $r = 6$. **128** 4·38 ft.
129. 18·4″, 18·1″, 18·1″ nearly. **130.** 10·8″ nearly, 20·8″ nearly.
131. 6 in. ; 216 c. in. **132.** (*a*) 89·44 ; (*b*) 31·176.
133. (*a*) 2·4 nearly ; (*b*) 2·88 nearly. **135.** 4·04 ft. **136.** 32.
137. $l = 179{\cdot}5$ in. nearly. **138.** 360 ft. **139.** 32, 22.
140. common rafter = 8 ft. 5 in. : hip rafter 11 ft. 11 inches nearly.
141. 12390 miles. **142.** 4·69 : 1. **143.** 3·142. **144.** $d = 12'$.
145. 31 ; 29. **146.** 393 yds. **147.** 98·4 cm., 38·7 cm. **148.** 11 ft. 4 in.
149. 7826 yds. **150.** The square. **151.** 59. **152.** 1936.
153. 50. **154.** 500·01 ft. **156.** 2000·004 ft. **157.** 50·5 cm.
158. 68 in. **159.** 101 units. **160.** 25.

Exercise XXIII (Page 81).

1. (*a*) Rs. 280-14-5 ; (*b*) Rs. 4,520-4-0 ; (*c*) Rs. 440-5-5 ; (*d*) £842-14-0 ; (*e*) Rs. 994-4-10 ; (*f*) Rs. 3,061-12-10 ; (*g*) Rs. 423-8-4 ; (*h*) Rs. 1,794-1-1 ; (*i*) Rs. 3,821-11-5. **2.** (*a*) Rs. 75-12-3 ; (*b*) £126-2-0 ; (*c*) Rs. 34-4-8 ; (*d*) Rs. 541-0-0 ; (*e*) Rs. 47-7-9 ; (*f*) Rs. 652-1-4 ; (*g*) Rs. 345-0-7. **3.** Rs. 360-7-10.
4. Rs. 1,226-14-9. **5.** Rs. 450-1-5. **6.** Rs. 12-7-7.
7. Rs. 6-14-2. **8.** Rs. 3,374-9-10. **9.** Rs. 1,144-12-10.
10. Rs. 13,576-2-11. **11.** He has lost, Rs. 1,601-14-7.
12. Rs. 2-15-4. **13.** Rs. 752-1-6. **14.** Rs. 116-7-3.
15. 5856138. **16.** 7420845. **17.** Rs. 2,896-1-7.
18. Rs. 4,283-6-4. **19.** Rs. 471-9-4. **20.** Rs. 22,819-3-2.
21. 5 per cent. **22.** 5 per cent. **23.** 1477.
24. $\frac{16384}{78125}$. **25.** $4\frac{191}{256}$ ft. **26.** (*a*) Rs. 819-0-4 ; (*b*) Rs. 12,393-0-0 ; (*c*) Rs. 1,350 ; (*d*) Rs. 10,000 ; (*e*) 2 yrs. ; (*f*) $2\frac{1}{2}$ yrs. ; (*g*) $1\frac{1}{2}$ yrs. ; (*h*) 5 per cent. **27.** 4·4 per cent.
28. Rs. 6173. **29.** Rs. 404-11-0. **30.** Rs. 6,377-13-5.
31. 7 per cent. **32.** 5·96 per cent. **33.** Rs. 362-9-5.
34. Rs. 242-15-7. **35.** 5 per cent., Rs. 851-0-0. **36.** 622·962 and 587·7.
37. 178 lbs. correct to the nearest pound. **38.** The second by Rs. 4·7025. **39.** Rs. 324-2-11 loss. **41.** Rs. 55-2-5 ; Rs. 1,255-2-8. **42.** Rs. 2,844-6-4. **43.** 6 yrs.
44. The bank will pay him Rs. 1,078·29. **45.** (*a*) Rs. 4-6-2 ; (*b*) Rs. 338-6-3 ; (*c*) Rs. 219-15-3. **46.** Rs. 5-13-8.
47. Rs. 6,545-4-0. **48.** 3·26203 times. **49.** (*a*) Rs. 150-9-9 ; (*b*) 25·509 per cent. **50.** Rs. 11,634-15-4.

Exercise XXIV (A) (Page 92).

1. $2\frac{1}{8}$. **2.** $\frac{1}{2}$. **3.** 20. **4.** 24. **5.** 5. **6.** 24. **7.** $\frac{108}{139}$.
8. $\frac{7}{118}$. **9.** $\frac{33}{52}$. **10.** 15. **11.** 10. **12.** 240. **13** $2\frac{74}{547}$.

14. $\frac{356}{551}$. **15.** $6\frac{16}{17}$. **16.** $6\frac{9}{19}$. **17.** $8\frac{4}{7}$. **18.** $5\frac{893}{3335}$. **19.** $7\frac{2}{3}$.
20. 30. **21.** $8\frac{113}{881}$. **22.** 26. **23.** $1\frac{19}{23}$. **24.** -432. **25.** $1\frac{20}{23}$.
26. $-1\frac{25}{49}$. **27.** $\frac{5}{12}$. **28.** $\frac{1}{7}$. **29.** 2. **30.** 6. **31.** 6. **32.** $\frac{1}{3}$.
33. 2. **34.** 2. **35.** $\frac{1}{3}$. **36.** $-\frac{8}{9}$. **37.** -5.
38. 0. **39.** $\frac{11}{17}$. **40.** $3\frac{2}{3}$.

Exercise XXIV (B) (Page 108).

1. Rs. 98, 63 half-rupees, 35 quarter-rupees, 121 two anna pieces, 186 annas. **2.** 100 sovereigns, 23 guineas, 65 crowns, 83 *sh.*
3. 40 ten rupee notes, 12 five rupee notes, 83 rupee notes and 15 half-rupees. **4.** Down hill 3240 yds., up hill 1680 yds., level 580 yds.
5. L = 80 ft., B = 20 ft. **6.** B = 73 yds., L = 100 yds.
7. L = 165 yds., B = 65 yds.
8. L = 90 ft., B = 40 ft. **9.** 83. **10.** 39. **11.** 27. **12.** 85
13. 73. **14.** 185. **15.** 320. **16.** 640. **17.** 200.
18. £2-9-$0\frac{24}{77}$. **19.** Rs. 56. **20.** Rs. 120. **21.** £33-6-8.
22. 512. **23.** Rs. 140. **24.** Rs. 52-1-4. **25.** 2240 lbs.
26. Rs. $120\frac{10}{27}$. **27.** 200 mangoes. **28.** 150 apples and 300 pine apples.
29. 600 books of each sort. **30.** 24 bullocks of each sort.
31. Cost of the house Rs. 5,750 and that of the garden Rs. 4,600.
32. Motor = Rs. 3,966-10-8 ; coach = Rs. 963-5-4. **33.** 500.
34. 200. **35.** 125. **36.** 30. **37.** 6. **38.** 0 or $-\frac{1}{4}$.
39. $-\frac{24}{29}$. **40.** Rs. 50. **41.** 6 yrs. **42.** $10\frac{1}{2}$ per cent.
43. Rs. 3,000, Rs. 2,000. **44.** Rs. 600, Rs. 200
45. A = 40, B = 24. **46.** Father's = 40, Son's = 15. **47.** 48 and 45.
48. House = Rs, 15,000, garden = Rs. 27,000, farm = Rs. 60,000.
49. Leather Rs. 3,500, Indigo Rs. 10,500, groundnut Rs. 16,500.
50. Hyp. = 13 units, other side = 5 units. **51.** Hyp. = 15 units, other side 9 units. **53.** 40 units and 9 units. **54.** $6\frac{3}{4}$ sec.
55. 9 sec. **56.** $22\frac{1}{2}$ sec. **57.** 5′ $8\frac{3}{4}$″. **58.** $16\frac{1}{2}$ sec.
59. (*a*) $4\frac{11}{16}$ sec. ; (*b*) $3\frac{33}{64}$ sec. **60.** (*a*) $7\frac{2}{19}$ sec. ; (*b*) $3\frac{6}{7}$ sec.
61. (*a*) $14\frac{13}{28}$ sec. ; (*b*) $4\frac{37}{92}$ sec. **62.** 4 sec. **63.** (*a*) $3\frac{6}{7}$ sec. ; (*b*) $18\frac{3}{4}$ sec. **64.** (*a*) $78\frac{3}{4}$ sec. ; (*b*) $5\frac{1}{4}$ sec.
65. (*a*) $10\frac{10}{11}$ min. past 8, (*b*) $27\frac{3}{11}$ min. past 8, or at 9 o'clock, (*c*) $43\frac{7}{11}$ min. past 8. **66.** (*a*) $5\frac{5}{11}$ min. past 7, (*b*) $21\frac{9}{11}$ min. past 7 or $54\frac{6}{11}$ min. past 7, (*c*) $38\frac{2}{11}$ min. past 7.
67. $14\frac{9}{44}$ min. past 1. **68.** 10 min. past 8.
69. (*a*) (1) $23\frac{11}{23}$ min. past 3 or $54\frac{18}{23}$ min. past 3, (2) $39\frac{3}{23}$ min. past 3, (3) $7\frac{19}{23}$ min. past 3 ; (*b*) (1) $23\frac{14}{23}$ min. past 15 or $54\frac{8}{23}$ min. past 15, (2) $7\frac{19}{23}$ min. past 15, (3) $39\frac{3}{23}$ min. past 15.

70. 1 hr. and $2\frac{14}{23}$ min.; 17 times; 1st time $31\frac{7}{23}$ min. past 12, 2nd time $33\frac{21}{23}$ min. past 13, 3rd time $36\frac{12}{23}$ min. past 14 and so on. **71.** 1 hr. $2\frac{14}{23}$ min. six times, 1st time $46\frac{22}{23}$ min. past 6, 2nd time $49\frac{13}{23}$ min. past 7, 3rd time, $52\frac{4}{23}$ min. past 8 and so on.

72. $31\frac{7}{23}$ min., eleven times, (I) $23\frac{11}{23}$ min. past 3, (II) $54\frac{18}{23}$ min. past 3. Adding $31\frac{7}{23}$ min. successively. **73.** 1 hr. and $5\frac{5}{11}$ min.

74. 1 hr. and $5\frac{5}{11}$ min. **75.** $32\frac{8}{11}$ min.

Exercise XXV (Page 123).

1. $x = 1\frac{4}{25}$, $y = \frac{14}{25}$. **2.** $x = 8\frac{8}{19}$, $y = 3\frac{16}{19}$. **3.** $x = 6$, $y = 5$.

4. $x = 2$, $y = 3$. **5.** $x = 1\frac{1}{2}$, $y = -\frac{1}{3}$. **6.** $x = -6$, $y = 3$.

7. $x = 18$, $y = 1\frac{1}{2}$. **8.** $x = 20$, $y = 20$. **9.** $x = 15$, $y = 14$.

10. $x = 2$, $y = 1$. **11.** $x = 4$, $y = -2$. **12.** $x = 1$, $y = \frac{1}{2}$.

13. $x = \frac{6}{5}$, $y = \frac{7}{5}$. **14.** $x = 5$, $y = 1$. **15.** $x = 20$, $y = 10$.

16. $x = 1$, $y = 1$. **17.** $x = 2$, $y = 2$. **18.** $x = 12$, $y = 1$.

19. $x = 11$, $y = 6$. **20.** $x = -20$, $y = 7$. **21.** $x = 36\frac{1}{2}$, $y = 25$.

22. $x = 21$, $y = 7$. **23.** $x = 1\frac{1}{2}$, $y = 5$. **24.** $x = 6$, $y = -5$.

25. $x = 0$, $y = 0$. **26.** $x = 0$, $y = 0$.

27. $x = -3\frac{11}{16}$, $y = -3\frac{1}{4}$. **28.** $x = \frac{17}{21}$, $y = \frac{31}{63}$.

29. $x = 1\frac{33}{43}$, $y = \frac{26}{43}$. **30.** $x = 5\frac{14}{19}$, $y = 14\frac{12}{19}$. **31.** $x = 1$, $y = \frac{2}{3}$,

32. $x = \dfrac{bf - ce}{ae - bd}$, $y = \dfrac{cd - af}{ae - bd}$. **33.** $x = \dfrac{mr - nq}{lq - mp}$, $y = \dfrac{np - br}{lq - mp}$.

34. $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$, $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$. **35.** $x = 0$, $y = 6$.

36. $x = 10$, $y = 4$. **37.** $x = \dfrac{ad(bc - ef)}{bd - ae}$, $y = \dfrac{be(df - ae)}{bd - ae}$.

38. $x = \dfrac{ab(b^2e - c^2d)}{b^4 - a^2c^2}$, $y = \dfrac{bc(b^2d - a^2e)}{b^4 - a^2c^2}$.

39. $p = 32\frac{44}{83}$, $q = 21\frac{117}{166}$. **40.** $p = 3\frac{27}{47}$, $q = -\frac{45}{47}$.

41. $A = 2\frac{7}{19}$, $B = \dfrac{-35}{57}$. **42.** $x = 2\frac{21}{23}$, $y = 1\frac{28}{69}$.

43. $u = 5\frac{17}{57}$, $v = -1\frac{38}{57}$. **44.** $x = 1$, $y = 1$.

45. $x = 1$, $y = 1$. **46.** $x = 1$, $y = 1$.

47. $u = \dfrac{8533}{1128}$, $v = \dfrac{-6601}{1128}$. **48.** $x = \dfrac{-461}{59}$, $y = \dfrac{182}{59}$.

49. $x = 16$, $y = 22$. **50.** $v = \frac{304}{75}$, $y = \frac{366}{75}$.

51. $x = -3\frac{20}{27}$, $y = 2\frac{8}{27}$. **52.** $x = \dfrac{-56}{95}$, $y = -\dfrac{4}{95}$.

53. $x = \frac{352}{349}$, $y = -\frac{4}{95}$. **54.** $x = 56\frac{1}{4}$, $y = 21\frac{3}{4}$.

55. $x = -580$, $y = 442$. **56.** $x = 1$, $y = 1$.

57. $x = -3\frac{1}{2}$, $y = -3\frac{7}{8}$. **58.** $x = -\frac{4}{29}$, $y = \frac{3}{22}$.

59. $x = 1, y = 1$. **60.** $x = 1, y = 1$.
61. Cycle Rs. 800, car Rs. 3,200. **62.** H. Rs. 1,000, C = Rs. 750.
63. 18 and 13. **64.** 4 and 3. **65.** III Form Re. 1, IV Rs. 7.
66. Short Rs. 2-12-0, 1 pair of shorts Re. 1-8-0.
67. R = Rs. 60, jutca = Rs. 120. **68.** Cot Rs. 5, table Rs. 4-12-0.
69. A Rs. 5, B Rs. 4. **70.** Rs. 150 and Rs. 200
71. 500 pl. and 50 mangoes **72.** B at Rs. 24, Press Rs. 3.
73. 12 teachers and 12 boys. **74.** Rs. 120 and 90.
75. 1st yr. 240, 2nd yr. 120. **76.** Consulting 4050, other branches 4000
77. Consulting Re. 1, visit Rs. 3. **78.** 58 or 85. **79.** 29 or 92.
80. 81. **81.** 38. **82.** 864. **83.** F = 35 yrs., S = 18 yrs.
84. G. F = 63 yrs. G. S = 14 yrs. **85.** F = 48 yrs., son = 24 yrs.
86. 12 cm. and 13 cm. **87.** AC = $20\frac{1}{2}$ in., BC = $8\frac{1}{2}$ in.
88. AC = $4\frac{2}{9}$ cm., CD = $5\frac{5}{9}$ cm., BD = $\frac{2}{9}$ cm.
89. AX = 2·8 in., XB = 2·4 in. **90.** AX = 11·2 in., BX = 4·8 in
91. $\frac{2pq}{h^2q^2} \times 9{\cdot}6$ cm. **92.** $m = 35$ yrs., $d = 13$ yrs. **93.** $\frac{5}{9}$.
94. $\frac{20}{48}$. **95.** $\frac{7}{17}$. **96.** $\frac{72}{125}$. **97.** Ghee Rs. 2-8-0, oil Re. 1-12-0.
98. Man 15 as., woman 6 as. **99.** Half-rupees 39, rupees 19.
100. Rs. 20, anna coins Rs. 400. **101.** Bench Rs. 23, chair Rs. 13.
102. $a = 35, b = 18$. **103.** $a = 100, b = 30$.
104. Side of the sq. 56 yds.; 64 yds., 49 yds.
105. L = 160 yds., B = 128 yds. **106.** L = 64 yds., B = 36 yds.
107. $x = 7, y = 15$. **108.** $x = 2, y = 4$, L = 20, B = 3.
109. 3, 4, 2. **110.** $6\frac{1}{4}$ per cent., 7 per cent.
111. 12 as. per sh., 14 as. per sh. **112.** $y = 5x + 3$.
113. $\frac{x}{15} + \frac{y}{20} = 1$. **114.** $y = -x + 5$. **115.** $\frac{x}{10} - \frac{y}{3} = 1$.
116. $5x^2 + 6x + 7 = 7$; when $x = \frac{1}{2}$ expression = $11\frac{1}{4}$, when $x = -3$, expression = 34.
117. $p = \frac{-24}{11}, q = \frac{39}{11}, r = 8$ $\therefore$ expression = $-\frac{24}{11}x^2 + \frac{39}{11}r + 8$; when $x = \frac{2}{3}$, expression = $\frac{310}{33}$, when $x = -8$, expression = -160.
118. A = 1, B = 2. **119.** A = $\frac{54}{19}$, B = $\frac{25}{19}$. **120.** $14x + 21y = 82$.
121. $y(y - a) = 4(x + 3)$. **122.** Not consistent.
123. (*a*) Yes, $x = 1, y = 2$, satisfy all; (*b*) yes, $x = 10, y = -4$. satisfy all the equations and hence consistent.
124. *a* and *b*. The two equations are identical and hence cannot be solved. **125.** 3 : 4. **126.** 104 oz. **127.** 2 : 1.
128. 40 oz. of gold and 20 oz. of silver. **129.** 63. **130.** 55 : 92.
131. Gold $40\frac{8}{15}$, silver $23\frac{7}{15}$. **132.** $9\frac{9}{16}$ oz. **133.** 18 men and 40 days.

134. 24 days. **135.** 15 days. **136.** $x = 3$, $y = 4$.
137. 20 miles and 15 hours. **138.** $x = 5$, $y = 4$. **139.** 15.
140. 24 beggars Rs. 3-12-0. **141.** 1 ton = 12 sacks. C.P. = £86.
142. 1 sack = 112 lbs. ; C.P. = £150. **143.** Rs. 200 and Rs. 300.
144. Free = $\frac{1}{2}$ md., Re. 1. **145.** Scouts 12, buns = 50.
146. 20 seers, 40 seers. **147.** Wet = Rs. 36,000, dry = Rs. 12,000
148. 60 and 85. **149.** Tram 300, motor 50. **150.** Walking 352 yds.
151. 6 miles and 3 miles stream. **152.** 37 and $47\frac{4}{7}$ miles.
153. $1\frac{1}{4}$ miles and 5 miles. **154.** 60 miles and 20 miles.
155. $31\frac{1}{2}$ miles and $13\frac{1}{2}$ miles. **156.** $30\frac{90}{143}$ minutes past 1.
157. 2 hrs. $46\frac{2}{13}$ min. **158.** A = $185\frac{5}{19}$ yds. per min., B = $152\frac{8}{21}$ yds. per min. **159.** A = 245 yds., B = 236.
160. $3\frac{5}{16}$ miles, $1\frac{5}{16}$ miles. **161.** Rs. 36-9-0. **162.** 10 miles.
163. 210 miles. **164.** Several answers but the first of these positions occurs at $9\frac{1}{19}$ a.m. **165.** 4 miles. **166.** 3600 and 37260.
167. 2000 men and 30 days. **168.** 6,100,280,420,840.
169. $a = 11$, $c = 9$. **170.** $b = 15$.

Exercise XXVI (A) (Page 148).

1. (1) $y = 3$, (2) $y = 4$, (3) $y = 2$, (4) $y = 5$, (5) $y = 7$; (*a*) They are parallel $m = 1$; (*b*) They are parallel $m = 2$; (*c*) Straight lines, the m of whose equations is the same are parallel ; (*d*) 1 passes through the origin while others do not ; if there is no absolute term then the graph passes through 0.

2. (*a*) -7, $-10{\cdot}5$, -14, $-17{\cdot}5$, -4, $-14{\cdot}5$, -19, $-23{\cdot}5$; $x = 5$, $3\frac{1}{3}$, $2\frac{1}{2}$, 2, $3\frac{1}{2}$, $4\frac{2}{3}$, $3\frac{3}{4}$, $3\frac{1}{5}$; (*b*) 1st and 3rd quadrants ; (*c*) 1 and 5, 2 and 6, 3 and 7, 4 and 8 ; (*d*) 1, 2, 3, 4 pass through 0 ; (*e*) In the first 4 equations $c = 0$, 3, -4, -5, -6. These are also true from graphs.

3. $y = 2{\cdot}5$, 5, $7{\cdot}5$, 10, $3{\cdot}5$, 6, $2{\cdot}5$, 4 ; $x = -12$, -6, -4, -3, -11, $-5{\cdot}5$, $-\frac{17}{3}$, $-\frac{9}{2}$; (*b*) In the 2nd and 4th quadrants ; (*c*) $m = 1$, -2, -3, -4 ; (*d*) in question ; (2) m is positive ; in question (3) m is negative. In question 2 they pass through the 1st and 3rd quadrants. In question 3 they pass through the 2nd and 4th quadrants.

4. (*a*) $y = x$, $y = 2x$; $y = 4x$ (*b*) $y = -x$, $y = -2x$, $y = -3x$, $y = -4x$; (*d*) $y = 2x$; (*e*) $y = nx$. **5.** $x - y = 0$, $y - 2x = 0$, $y - \frac{1}{2}x = 0$, $3x - y = 0$, $2x - 3y = 0$, $3x - 5y = 0$. These pass through the 1st and 3rd quadrants. The rest pass through the 2nd and 4th quadrants. **8.** $y = 1{\cdot}2$, **9.** They are parallel to x axis. **10.** They are parallel to y axis.

Exercise XXVI (B) (Page 157).

11. $x = -2$, $y = -9$. **12.** $x = \cdot 28$, $y = -4\cdot 85$.
13. $x = 2\cdot 82$, $y = 2\cdot 91$. **14.** $x = 2\cdot 43$, $y = 2\cdot 02$.
15. $x = 34\cdot 4$, $y = 30$. **16.** $x = \cdot 79$, $y = -3\cdot 07$. **17.** $x = -\cdot 03$.
18. $x = 1$. **19.** 19·8. **20.** $x = 20\cdot 9$. **21.** $x = 5\cdot 4$ nearly.
22. $x = 1\cdot 8$. **23.** $x = \cdot 12$. **24.** $x = \cdot 49$ nearly. **25.** 17·4 nearly.
26. $x = 1\cdot 9$, $y = \cdot 3$. **27.** $x = 3$ nearly, $y = 1\cdot 1$. **28.** $x = 3$, $y = 4$.
29. $x = -2$, $y = -7$, **30.** $x = 6\cdot 5$, $y = -2\cdot 5$. **31.** $x = 8$ nearly, $y = 12$. **32.** $x = 6\cdot 4$, $y = 1\cdot 8$. **33.** $x = 2\cdot 5$, $y = 2$.
34. $x = 3$, $y = 4$. **35.** Chair = Rs. 6-8-0, table = 25. **36.** Bull = Rs. 80, buffalo = Rs. 120. **37.** Gramaphone = Rs. 90, record = Rs. 6.
38. Boat = 3 miles per hour, stream = 2 miles per hour. **39.** 33
40. $l = 87$ yds., $b = 73$ yds. **41.** 6·82 kg., 17·6 lbs.
42. 41 metres, 38·5 yds. **43.** 18 viss, $12\frac{1}{2}$ lbs. **44.** 12·4 mi., 26 km.
45. 1·4 in. **46.** 29·3 ft., 51·3 ft., 63 ft.; 23·86 miles; 32 miles nearly.
47. 60° R, 43·7° C. **48.** 167° F., 89·6° F., 113° F., 57·2° F., 35° C., 44·4° C. **49.** $m = \frac{9}{5}$, $p = 32$. **50.** Rs. 193, Rs. 73, Rs. 313.
51. Rs. 320, Rs. 704, Rs. 808. **52.** Rs. 400, Rs. 425. **53.** ·26, ·57.
54. Re. 0-12-6, Re. 1-2-6, Rs. 2-3-0. **55.** 484 ft. per sec., 388 ft. per sec., downward velocity = 156 ft. per sec., 20·1 sec.
56. Rs. 135, Rs. $172\frac{1}{2}$, Rs. $112\frac{1}{2}$. **57.** $\frac{1}{2}$, ·30 nearly, 16 nearly, 6 hrs.
58. 2,500 ft., 2,200 ft., 700 ft., above sea-level. **59.** Rs. 1,500, Rs. 1,750, 23 days. **60.** 30 days. **61.** Rs. 19-0-6. **62.** 740.
63. 8. **64.** Rs. 9,000. **65.** 20. **66.** 24. **67.** Rs. 2. **68.** 53, 38, 19.
69. 17 and 23. **70.** 97, 82 and 64. **71.** 78. **72.** 48.
73. Rs. 10-7-11, Rs. 10-11-9 nearly, 301, 302. **74.** 8 as.
75. 47, 56, 49 and 45 nearly. **76.** 54, 38, 17, 20. **77.** 50·24, 35·17, 47·1.
78. 6·93, 8·23, 6·58. **79.** 21, 14·7, 12·0″. **80.** $\frac{3}{7}$, $\frac{1}{20}$, $\frac{11}{15}$, $\frac{4}{5}$.
81. 42·8 per cent., 55 per cent., 73·3 per cent., 80 per cent.
82. 48, 24. **83.** 15 miles per hour., 25 miles, 35 miles per hour.
84. 1st investment. **85.** 50 ft. per sec., after 11 sec. **86.** Rs. 5-12-0.
87. Rs. 8. **88.** $\frac{1}{2}$ a maund, Rs. 14-4-0. **89.** Yes, $y = 5x + 6$, $4y + 6x = 5$. **90.** $m = 2$, $63\frac{1}{2}°$. **91.** $m = -2$, $116\frac{1}{2}°$.
92. Yes. **93.** Yes, No. 94; (*a*) $m = 5$, $c = -3$; (*b*) $m = \frac{8}{3}$, $c = -2$.
95. (*a*) Yes, $y = 4x - 7$; (*b*) $5y = 6x - 5$. **96.** $y = 4x$, $5y = 6x$.
97. $4y + x = 0$, $6y + 5x = 0$. **98.** $y = 4x - 10$, $5y = 6x + 120$.
99. $4y + x = 45$, $6y + 5x = 85$. **100.** (*a*) Not concurrent; (*b*) Concurrent 49° and 70°. **101.** (*a*) $x = y$; (*b*) $y = 2x$; (*c*) $x + y = 2$; (*d*) 2 lines, $y = x + 1\cdot 5$, $x = y + 1\cdot 5$; (*e*) $2y + 3x = 5$. **102.** No. 15 travels at 16·4 miles per hour, No. 16, 20 miles per hour, at 12.53; at 129 miles from Madras.

103. No. 7 travels at 24·4 miles per hour, at 1.35 ; at 120 miles from Madras. **104.** At 5.54 ; 14·8 miles from Madras, No. 16 and No. 7 do not meet. **105.** Madras to Arakonam 25 miles per hour. **106.** (*a*) Raichur to Guntakal 25 miles per hour ; (*b*) at 13 hr. 54 min. 152½ miles from Madras, 5 miles distant, at 14.7. **107.** At 1·55 ; 136 miles from Madras. Maximum, speed between Arakonam and Madras 39·7 and 30·4.
108. It goes to Bitragunta at 15.50 ; to Tenali at 22.18. Rajamundry 369 miles from Madras, 427 miles. **109.** (*a*) Tuni 18.41, Ellore 2-20 ; (*b*) 303 miles and 459 miles from Madras ; (*c*) at 2 o'clock at 311th mile. **110.** (*a*) The two mails meet at 1.56, 164 miles from Madras ; (*b*) 4 miles and 116 miles ; (*c*) 2 hr. 28 min., 45 sec. **111.** (*a*) 6.34 a.m. ; 5⅔ miles from Chengleput ; (*b*) 28½ miles from Madras. **112.** 6.54 a.m. ; 9 miles from Chengleput. **113.** 6 days. **114.** 4⅘ days.
115. ·74 days. **116.** ·54 days. **117.** 2 hr. 12 min. **118.** 1 hr. 40 min.
119. 20 hr. **120.** 2 hr. 36 min. **121.** (*a*) 2 hr. 5 min. ; (*b*) 1 foot ; (*c*) 6 hr. 15 min. **122.** (*a*) 10 hr. ; (*b*) 6 hr. **123.** (*a*) 5·5′ past 1. (*b*) 10·9′ past 2 ; (*c*) 12 ; (*d*) 43·6′ past 8.
124. (*a*) 22′ past 1 nearly, 1 hr. 55′ nearly ; (*b*) 27′ past 2, also at 3 o'clock ; (*c*) 11′ past 11, 43′ past 11 ; (*d*) 27′ past 8 ; at 9.
125. (*a*) 38′ past 1 ; (*b*) 43′ past 2 ; (*c*) 27′ past 11 ; (*d*) 11′ past 8.
126. Rs. 52-8-0. **127.** Rs. 270 ; (*a*) Rs. 250 ; (*b*) Rs. 112½.
128. (*a*) Rs. 585 ; (*b*) Rs. 300. **129.** (*a*) Rs. 1,000 ; (*b*) Rs. 324.
130. Rs. 45 ; Rs. 120.

Exercise XXVII (Page 177).

1. 1857—38·25″ ; 1860 — 31·2″ ; 1866— 49·8″. **5.** 4500. **6.** 26.
7. 101·09, 100·2, 102·85. **8.** 484000. **9.** 29·73. **10.** 3·9, 3·88.
11. 67, 11·5 ; 66°. **12.** Elongation varies as weight or $\frac{\text{elongation}}{\text{load}}$ = 0·06 cm. per gm., 66 gm., 326, 69 cm. **13.** 93 cm., 2·07.
14. (*a*) 67·3 ; (*b*) 64·2 ; (*c*) 56·5 ; (*d*) 29·7 ; (*e*) 18·1 ; (*a*) 59·5 ; (*b*) 80 ; (*c*) 32·5. **15.** 204°·4, 205°·3, 206°·56, 213°·3, 213°·5. **16.** No.
18. 2·5. **19.** 43·4. **20.** 125. **21.** 39 and 7000. **22.** 564.
23. 53·4. **24.** 5·45. **25.** 10·6. **26.** 19388. **27.** 88.
28. 32·3. **29.** 17·1. **30.** 7·38. **31.** 225. **32.** April 7th.
33. 42. **35.** 212, 196. **37.** 478, 790 **38.** Rs. 27-15-0
39. 22·05, 14·18. **42.** 400, 4·33. **43.** $V = 56·6$, $h = 56·25$.

Exercise XXVIII (Page 193).

1. 587500 sq. yds. **2.** 23·8 acres. **3.** Half. **4.** 98000 sq. yds **5.** 213750 sq. yds. **6.** AD = 2·42 cm., BC = 3·4 cm., ∠A = 143°, ∠B = 65°, ∠C = 85°, ∠D = 67°. **7.** AD = 64 km., ∠A = 129·5°, ∠D = 57·5°, area = 350200 hectares.
8. BC = 2·2″, DA = 1·8″, CD = 1·3″, ∠C = 78°, ∠D = 98°.
9. ∠A = 123°, AD = 1·3″, CD = ·43″. **10.** ∠A = 108°, AB = 6·0 cm., AD = 1 cm. **11.** CD = 50 yds.
12. AB = 174 yds., BC = 210 yds., CD = 130 yds., AD = 138 yds. and BD = 170 yds. **13.** AB = 3·5″, BC = 2·8″, CD = 2·8″, DA = 2″, area = 15 acres 2041 sq. yds.
14. CD = 67·99 yds., area = 1 acre 254 sq. yds. **15.** 4·62 sq. in.
16. AD = 330 yds., CD = 550 yds., area = 42·0633 acres.
17. One side = 5 cm., area = 24 sq. cm. **18.** AD = 1·01″, DC = 1·95″, ∠A = 98·1°, ∠C = 58·6°, ∠D = 113·3°. Area = 2·4 sq. in. **19.** CD = 119·5 yds., area = 1·3770 acres.
20. Area = 49 sq. cm.

Exercise XXIX (A) (Page 196).

1. (1) Only with respect to a and c, $3b$ to be changed into $5b$.
(2) ,, ,, a and b, $-3c^2$ to be changed into $+2c^2$.
(3) ,, ,, b and c, $3bc$,, ,, $7bc$.
(4) ,, ,, b and c, $-bc$, ,, ,, $+2bc$.
(5) Is perfectly symmetrical.
(6) ,, ,,
(7) Is symmetrical with repsect to b and c, $b^2 + c^2$ should be added.
(8) Is symmetrical with respect to b and c, $+3b^2 + 3c^2$ should be added.
(9) Symmetrical with respect to x and y, $-6xz - 6yz$ should be added.
(10) Symmetrical with respect to b and c, . . . $5a^2$ should be changed into $5a^3$, and $-7bc$ should be added.
(11) Not symmetrical $-4ab^2$ should be changed into $-3ab^2$ or $-3a^2b$ into $-4a^2b$.
(12) Not symmetrical. The expression either $a + b + c$ or $a^2 + b^2 + c^2$ or $a^3 + b^3 + c^3$.

2. (1) 5; (2) −3; (3) 7; (4) Any value; (5) p; (6) 9; (7) −7; (8) −3; (9) 2; (10) Any value.

Exercise XXIX-B (Page 198).

16. 1·1 cm., ·9 cm., 1·5 cm. **23.** 250 yds. **24.** $\frac{8}{10}$ of a mile
27. 3·6 in.

Exercise XXIX (C) (Page 205).

4. 33·8 ft. **5.** 1·3 in. **6.** 1·95 ft. **7.** 16 ft. **8.** (*a*) 15·6 in.; (*b*) 1·6 ft. **9.** 5·15 ft. **10.** 2·9 ft. **11.** 10·37 ft. **12.** 6 ft. 2·6 in. **13.** EF = 11·8 ft. nearly, AB = 14 ft. nearly.

Exercise XXIX (D) (Page 212).

1. 4 in.; 6·9 sq. in. **2.** 10·4 cm.; 47·25 sq. cm. **3.** $1{\cdot}73r$; $1{\cdot}3r^2$. **4.** 3 in.; 9 sq. in. **5.** 7·1 cm.; 50 sq. cm. **6.** $1{\cdot}4r$; $2r^2$. **7.** 2·5 in.; 16·125 sq. in. **8.** 7 cms., 126·42 sq. cm. **9.** r. **10.** $2{\cdot}58r^2$. **11.** ·75 in., 2·8 sq. in. **12.** 1·5 in.; 11·2 sq. in. **13.** $\cdot 75r$. **18.** Rs. 45-13-10.

Exercise XXX (A) (Page 214).

1. 120°, 60°. **2.** 144°, 72°, 144°. **3.** 120° each. **4.** ∠ CPD = 45°, ∠ APD = 45°, ∠ BPC = 45°, ∠ APB = 135°. **23.** 5 yds. **24.** 30°. **25.** 12·6 ft.

Exercise XXX (B) (Page 220).

1. 2 in. **2.** 7·2 cm. **3.** 3 in. **19.** $3\frac{1}{2}$ cm. **20.** 2 in. **21.** 24 ft. **25.** (*a*) Less than 10 ft.; (*b*) Exactly 10 ft.; (*c*) More than 10 ft. **26.** (*a*) Less than 2·2″; (*b*) Equal to 2·2″; (*c*) More than 2·2″. **30.** 3 tangents, each = diameter. **34.** $d = \sqrt{(a+b)^2-(a-b)^2} = 2\sqrt{ab}$. **36.** Tangents are boundaries. **49.** 3·1 in. **51.** 13·8 cm.; 25°. **59.** 0, −4. **60.** 3·8 in.

Exercise XXXI (Page 234).

1. $1\frac{1}{7}$ days. **2.** $1\frac{1}{8}$ days. **3.** After 3 days. **4.** 2nd day. **5.** $\frac{80}{47}$ days. **6.** After 2 days. **7.** At 8 a.m. **8.** At 1.30 p.m. **9.** A at 7 a.m. **10.** At 9.12 a.m. **11.** At 11.25 a.m. **12.** At 4.5 p.m. **13.** 9.30 a.m. on the following day. **14.** $12\frac{1}{2}$ days. **15.** 4 hrs. after the commencement. **16.** After 1 year. **17.** (*a*) 4·08; (*b*) 8·28; (*c*) 6·48; (*d*) 1·38. **18.** (*a*) 2·83; (*b*) 1·92; (*c*) 2·35; (*a*) 2·4. **19.** (*a*) 2·1; (*b*) 1·8; (*c*) 3·57; (*d*) 3·26. **20.** (*a*) 20 — 36, 14 miles from Egmore; (*b*) 1 — 0, 70 miles from Egmore; (3) Pallavaram Olakur. **21.** (1) 6 — 9, 236 miles from Egmore; (2) 9 — 50, 180 miles from Egmore; (3) 8 — 21, 204 miles; (4) (*a*) 20 — 14, 27 miles; (*b*) 20 — 48, $18\frac{2}{3}$ miles.

22. 4 — 35. **23.** 12 — 20. **24.** 40, 35, 25. **25.** 500, 300, 200
26. 4 : 1. **27.** 3 : 1. **28.** 6 days. **29.** 5 days.
30. 24, 15, cyclist 9 — 27, motor cyclist 9 — 9. **31.** (*a*) $1\frac{1}{4}$ miles; (*b*) 1 mile from the village; (*c*) $7\frac{1}{2}$ miles an hour.
32. $4\frac{1}{2}$ miles an hour. **33.** $2\frac{1}{2}$ miles. **34.** Midway between P and Q
35. $4\frac{1}{2}$ miles, 6 miles an hour. **36.** At 11.12 a.m. **37.** 84 miles.
38. $7\frac{1}{3}$ miles per hour nearly. **39.** $118\frac{2}{11}$ yds. **40.** 603 yds. per minute. **41.** $79\frac{1}{11}$ yds. **42.** $\frac{1}{11}$ hr. past 12, $\frac{2}{11}$ hr. past 1, $\frac{3}{11}$ hr. past 2, $\frac{4}{11}$ hr. past 3 and so on. **43.** $\frac{2}{11}$ hr. past 12, $\frac{3}{11}$ hr. past 1, $\frac{4}{11}$ hr. past 2, $\frac{5}{11}$ hr. past 3, and so on.
44. 144 hrs. **45.** 5 hrs. $10\frac{10}{19}$ minutes.

Exercise XXXII (A) (Page 241).

1. $75\frac{3}{7}$ ft. **2.** 26 in. nearly. **4.** 13·1 in. **5.** 1680 ft. **6.** 1120 yds.
7. 3 ft. **8.** 2 ft. 5·83 in. **9.** 7·32 in. nearly **10.** $12\frac{1}{2}$ in.
11. 7 ft. **12.** $34971\frac{2}{11}$ ft. **13.** 12500 **14.** $2\frac{1}{2}$. **15.** $203\frac{7}{11}$ rev.
16. $5\frac{1}{7}$ miles per hr. **17.** 15·32 in. **18.** $707\frac{1}{7}$ ft. **19.** ·095 in.
20. 1·38 in. **21.** 3·84in. **22.** Second hand 70·4 yds.; hour hand 9·55 in., minute hand, 187 in. **23.** 16 coins may be nearly placed, 6 coins more. **24.** (*a*) $25\frac{1}{7}$ in.; (*b*) 35·2 in.
25. (*a*) $3754\frac{2}{3}$ ft.; (*b*) 3344 ft. **26.** 29·05 in. **28.** (*a*) ·3 in. (*b*) $\frac{1}{12}$ rev.; (*c*) $3\frac{1}{7}$ in. **29.** $171\frac{3}{7}$ yds. **30.** 63 yds.

Exercise XXXII (B) (Page 246).

1. Area by counting = 3·22 sq. in., by calculation area = 3·14 sq. in. Error per cent. = $2\frac{1}{2}$ in. **2.** 587 sq. yds. nearly.
3. 191 sq. miles. **4.** ·875 sq. in. **5.** $3\frac{3}{7}$ sq. in. **6.** 4·83 sq. in.
7. 422400 sq. ft. **8.** 181·7 in. **9.** 393 sq. yds.
10. (*a*) $R = \sqrt{a^2 + b^2}$; (*b*) 21·63 ft. **11.** 15·9 ft. **12.** 28 pieces, $\frac{1}{4}$ sq. in. left. **13.** $22\frac{5}{16}$ sq. ft. **14.** 3·53 in. **15.** 122·3 yds. from the centre. **16.** $585\frac{1}{7}$. **17.** (*a*) $50\frac{2}{7}$ sq. in.; (*b*) 4 sq. in.
19. Diameter of the wheels = 7 ft.; of the plot, 110 ft.
20. $30\frac{6}{7}$ sq. in. **21.** 55·62 ft. **22.** Circle = 1·03 sq. in.; triangle = ·62 sq. in. **23.** The circle has the greater area.
24. $1537\frac{1}{7}$ sq. yds. **25.** (i) 37·34 sq. in.; (ii) 23·16 sq. in.; (iii) 11·03 sq. in. **26.** $7\frac{1}{14}$ sq. in. **27.** $10\frac{5}{7}$ sq. ft.
28. $27\frac{3}{7}$ sq. in. **29.** $\frac{4}{7}$. **30.** (*a*) Circumference and diameter; (ii) Area of radius *a* and a square of side *a*. **31.** $2292\frac{6}{7}$ sq. yds.

32. (*a*) $A = \frac{c^2}{4\pi}$; (*b*) $A = \frac{\pi d^2}{4}$; (*c*) πr^2; (*d*) $A = \frac{cr}{2}$;

(*e*) $A = \frac{\pi dr}{2}$; (*f*) $C = \frac{2A}{r}$; (*g*) $C = 2\sqrt{\pi A}$;

(*h*) $r = \sqrt{\frac{A}{\pi}}$; (*i*) $d = 2\sqrt{\frac{A}{\pi}}$; (*j*) $r = \frac{c}{2\pi}$;

(*k*) $\frac{2A}{C}$. **34.** $9\frac{29}{77}$ sq. yds. **35** $4601\frac{1}{7}$ sq. in.

36. (*a*) 69·4 sq. in.; (*b*) ·83 sq. ft.

Exercise XXXII (C) (Page 253).

1. $123\frac{3}{4}$ sq. in. **2.** $26\frac{4}{21}$ sq. ft. **3.** $17\frac{2}{11}°$. **4.** $45\frac{9}{11}°$. **5.** $10\frac{10}{11}$ in.
6. 7·82 yds. **7.** 7·82 yds. nearly. **8.** 6 ft. **9.** 12·6 in.
10. 2 ft. **11.** 26·2 sq. in. **12.** 33·63 ft.
13. $\frac{9}{40}$ sq. m. **14.** 36·32 sq. in. **15.** 20·63 sq. in.
16. $3\frac{19}{35}$ sq. ft. **17.** 76·3 sq. ft. **18.** $2857\frac{1}{7}$ sq. ft.
19. (*a*) 41·9 cm.; (*b*) 83·8 cm.; (*c*) 1676 sq. cm.; (*d*) 3352 sq. cm.
20. 146·4 sq. in. **21.** $603\frac{3}{7}$ yds. **22.** 168 ft. nearly.
23. $11\frac{1}{3}$ ft.; $168\frac{2}{3}$ sq. ft. **24.** *At.* = 91·3°; angle at the centre, 177·4°; I = 65·2°; Arc = *Ant.* = 13°. **25.** E = 28·2°, Asia = 120°, Af. = 84·7°, N.A = 56·5°, S.A = 49·4°, Aus. = 21·2°. **27.** 83·6 sq. ft. **28.** 22·45 sq. cm.
29. 9·13 sq. ft. **30.** 13·04 sq. in. **31.** 142·63 sq. in., 171·65 sq. in., 1114·51. **32.** 100°; 6897 miles. **33.** 55°, 3793.
34. 6·18 sq. in. **35.** 115·84 sq. ft. **36.** 41·25 sq. yds. **37.** 34·64 ft.
38. 2 ch. 1 link. **39.** 52·35 inches. **40.** 19·66 ft. nearly.
41. 8·28 ft. **42.** 6146·562 cub. ft. **43.** 15·07 miles.
44. 10·56 miles. **45.** 178·88 miles. **46.** 88 yds. **47.** 3971 miles.

Exercise XXXIII (Page 259).

1. (*a*) 4455 c. ft.; (*b*) 78 c. ft., 162 c. in.; (*c*) 675 c. ft.; (*d*) 42 c. ft.; (*e*) $121\frac{1}{2}$ c. in.; (*f*) 324 c. in.
2. 1 cub. ft. 972 c. in. **3.** $2476\frac{11}{16}$ c. ft. **4.** $112\frac{1}{2}$ gallons.
5. $56466\frac{2}{3}$ c. yds. **6.** $546\frac{7}{8}$ lbs. **7.** $919\frac{11}{16}$ c. ft. **8.** 2000 c. ft.
9. *xyz* c. in.; 60 c. in. **10.** (*a*) (i) $l = \frac{V}{bh}$, (ii) $b = \frac{V}{lh}$, (iii) $h = \frac{V}{bl}$; (*b*) $A = \frac{V}{h}$, $h = \frac{V}{A}$ **11.** ·64 in. **12.** ·30 in. **13.** ·35 in.
14. (*a*) 2·678 c. ft.; (*b*) 2·803 c. ft. **15.** $17\frac{23}{25}$ c. ft. **16.** $5593\frac{3}{4}$ c. ft.
17. $2\,prm + 2m\,(q - 2m)\,(r - 2m) + 2pm\,(q - 2m)$ or $pqr - (p - 2m)\,(q - 2m)\,(r - 2m)$. **18.** $85\frac{5}{7}$ c. ft. **19.** $856\frac{1}{4}$ c. ft.
20. $385\frac{1}{2}$ c. ft. **21.** $14701866\frac{2}{3}$ c. yds. **22.** 580 bricks.
23. 3646·84 lbs. **24.** 2894·4 million sq. ft. **25.** Rs. 0-14-5; 188 lbs.
26. 50400 cub. ft. **27.** 6·4 in. nearly. **28.** Rs. 31-8-0. **29.** $\frac{65}{144}$ sq. mile.
30. Side 9·4 in.; diagonal 16·28 in. **31.** 4·97 ft. **32.** 624 ft. per hr.

33 $908\frac{11}{12}$ sq. ft. **34.** 2583 cub. ft. **35.** 2 c. ft. 684 c. in.; 2 c. ft. 504 c. in. **36.** 17386 c. in. **37.** 3·308 c. ft.; cot = 1529·95 lbs. **38.** 108 c. in. **39.** 28 in.
40. 126 boys, 144 c. ft. **41.** 116 days nearly. **42.** 17·7 in.
43. Rs. 9-0-5 if there is no lid, Rs. 13-3-1 if there is lid.
44. Rs. 42-11-4. **45.** (1) a^6-b^6; (2) x^4-y^4; (3) $x^6+7x^5+7x^4-35x^3-56x^2+28x+48$. **46.** $(x+p)(x+q)$.
47. $\frac{xpqr}{ny}$ units of area. **48.** 7 ft. $6\frac{2}{3}$ in. **49.** $16\frac{2}{9}$ cub. ft.
50. 256 books and 4 cub. ft. **51.** (*a*) 35 books; (*b*) 350 books; (*c*) either 320 or 336. **52.** 1066 blocks. **53.** 5688.
54. Rs. 630-14-6 nearly, Rs. 68-1-10. **55.** 4 in., 8 sq. in.
56 91238400 c. ft. **57.** 3683 nearly. **58.** 15840.
59. $\frac{e(a-2d)(b-2d)(c-d)}{(f-e)\{abc-(a-2d)(b-2d)(c-2d)\}}$; ·49 : 1.
60. 760·5 oz. **61.** 3705 lbs. **62.** 999·9 oz. **63.** Rs. 552-10-6; 31440 bricks. **64.** $(a+b)^3=a^3+3a^2b+3ab^2+b^3$.
65. 486 measures. **66.** 277·2 cub. in., 5 41 gallons.
68. $4\times8\times10+8\times8\times2$ or 448; $960-(8)^3=448$. **69.** 13 in.
70. 162 cub. ft.

Exercise XXXIV (A) (Page 270).

1. 9 c. ft., 1696 c. in. **2.** 91 c. ft., 1512 c. in. **3.** 3·62 in.
4. 11 ft. 6 in. **5.** 1 sq. ft., 139 sq. in. nearly. **6.** 1 sq. ft. 65 sq. in.
7. 10·1 c. in.; 41·68 sq. in. **8.** 84 c. in., 150 sq. in.
9. 120 c. in., 920 sq. in. **10.** 360 c. in., 348 sq. in.
11. 35·28 c. in., 66·08 sq. in. **12.** 175·5 c. in., 307·8 sq. in.
13. 78·51 c. in., 130·70 sq. in. **14.** 70·146 c. in., 119·69 sq. in.
15. 265·75 c. in.; 435·8 sq. in. **16.** 14289·82 lbs. **17.** 3·95 in.
18. 2227·918 c. ft. **19.** 38·624 c. ft. **20.** 44000 c. yds.
21. $\frac{\text{Height of triangle}}{\text{Height of the square}}=\frac{625}{519}$. **22.** rect.: $\triangle$ :: 5 : 9.
23. 8400 c. ft. **24.** 1170 c. ft. **25.** 1802240 c. yds. **26.** 2·598 c. ft.
27. 450 c. in. **28.** 2·07 c. yds. **29.** 161·591 sq. ft.; 3231·82 c. ft.
30. 192 c. ft. **31.** 3840 c. ft., below the verandah; 2720 c. ft. above the verandah. **32.** 380160 c. ft. **33.** 506880 c. ft.
34. 22320 c. ft. **35.** 816 c. ft.

Exercise XXXIV (B) (Page 277).

1. 443·23 c. in. **2.** 1897·28 c. in. **3.** 342·65 c. cm. **4.** 358·52 c. cm.
5. 345 c. ft., 128 c. in. **6.** ·166 in. **7.** 9·27 cm. **8.** 6·27 in.
9. 6·396 in. **10.** 43·8. **11.** 1·64 cm. **12.** 8·15 in. **13.** 7·74 cm.

14. $3\frac{1}{7}$. **15.** 3·1416. **16.** 3·1416. **17.** 13·26 c. ft. nearly. **18.** $125\frac{1}{8}$ c. ft.
19. 301·6 c. ft. **20.** $56\frac{4}{7}$ c. in. ; $1095\frac{3}{7}$ c. in. **21.** 624·2 c. in.
22. 59·17 c. in. **23.** 961·142 c. cm. **24.** 371·76 c. in. **25.** 9822ac.
26. rect. : cyl. = 1·27 : 1. **27.** ·88. **28.** $2545\frac{5}{7}$ c. in. **29.** 21·85 ft.
30. 13552 gall. **31.** Diameter of 1 measure = 4·34 in. diameter of $\frac{1}{8}$ measure = 2·34 in. **32.** 3·96 in., 2·74 in., 2·14 in.

33. (*a*) $\pi b^2 a$; (*b*) $\pi \frac{b^2}{4} a$; (*c*) 4 times. **34.** dia. 2·86, ht. 5 in. ; 1·6 in. ht. 9 in. **35.** 34·5 in. **36.** 1165 c. ft. **37.** 10 c. ft., $1172\frac{4}{7}$ c. in. **38.** $112\frac{7}{8}$ c. ft. **39.** 22·9 c. ft. ; 4136·8 lbs
40. $38\frac{1}{2}$ c. ft. **41.** 1096945 gallons. **42.** Rs. 330
43. 9 c. ft., 866 c. in., 1085 lb. **44.** $26\frac{5}{7}$ c. in.
45. 3072 yds. per hr. **46.** 7 wooden cylinders. **47.** 1·93 lbs.
48. 4·4 in. nearly. **49.** $3\frac{1}{3}$ jars. **50.** 6·3 cm. nearly.
51. 1·43 c. in. **52.** 6·15 ft. **53.** 1·06 ft. ; 16·06 ft.
54. $160\frac{5}{7}$ c. ft. **55.** 124 persons ; nearly 10 persons.
56. (*a*) 1 pair of opposite sides ; (*b*) Another pair of opposite sides ; (*c*) A pair of adjacent sides.
(i) 46917c. ft. ; (ii) $23892\frac{3}{4}$ c. ft., (iii) 35405 c. ft.
57. ·488 in. **58.** Vol. of lower : Cylinder = $282\frac{6}{7}$ c. in, vol. of upper cylinder = $75\frac{3}{7}$ c. in. **59.** ·2064 c. in. **60.** Rs. 16-9-11 nearly.
61. $\frac{\text{Cir.}}{\text{sq.}} = \frac{11}{14}$, the circular one is the cheaper.
62. 17 sq. ft. 11 sq. in. nearly. **63.** Rs. 202-13-4. **64.** $1(r = h)$.
65. $\sqrt{2}$: 1. **66.** 429 c. in. **67.** Rs. 9-8-2.
68. $s = \frac{\pi}{4}$ w. ^2l. **69.** $78\frac{3}{4}$. **70.** ·54 c. in. **71.** ·902 sq. in.
72. 2078 in. **73.** 2·1 cm. **74.** $6\frac{5}{14}$ c. in. **75.** 112 cubes.
76. 18208 gallons, 6·2 in. **77.** (*a*) 151 in., (*b*) ·823 cub. in.
78. 68·64 kg. ; $62562\frac{1}{2}$ c.c.

Exercise XXXIV (C) (Page 288).

1. 770 c. in. **2.** $377\frac{1}{7}$ c. in. **3.** 292 c. in. nearly.
4. 1·82 in. **5.** 2·1 in. **6.** 5·9 in.
7. 4·46 in. **8.** 4·76 in. **9.** 4·2 in.
10. ·37 in. **11.** ·227 c. ft. **12.** 10·214 c. ft.
13. 1928·5 c. in. **14.** 392 c. in. **15.** 189·627 c. in.
16. $29333\frac{1}{3}$ c. ft. **17.** $490\frac{2}{7}$ c. in. **18.** $314\frac{2}{7}$ c. in., $754\frac{2}{7}$ c. in
19. 14·51 c. in. **20.** 1·77 c. in. **21.** $h = 30$ ft., $d = 19{\cdot}54$ ft.
22. Vol. of con. portion = $4\frac{4}{21}$ c. in. ; of the other portion $108\frac{20}{21}$ c. in.
23. $\sqrt[3]{93{\cdot}3} = 4$ 6 in. from the vertex. **24.** 4·9 in. **25.** ·214 c. ft.

26. 2 in. nearly. **27.** 34·802 sq. in. **28.** 1936·9 c. in.

29. Area of base $= \frac{\pi a^2}{4}$; of slant surface $= \frac{\pi a^2}{2}$; a right angle

30. 80·52 sq. in. **31.** 12215·7 sq. ft.

32. $\sqrt{3} : 2$. **33.** $l = 6{\cdot}45$ in. nearly.

34. $h = 5{\cdot}25$ in. **35.** 3·7 in. (this involves a quadratic).

36. 14 in. nearly (quadratic). **37.** 56·17 sq. in. **38.** 19·98 c. ft.

40. 9·38 in. **41.** 1·5. **42.** Rs. 9-3-4; 9429 c. ft. nearly.

43. $\frac{\text{ht. of 8 ollock measure}}{\text{ht. of cone}} = \frac{1}{3}$.

44. $\frac{\pi d^2 h}{6}$ where d and h is the diameter and height of the cylinder.

45. $\frac{4}{9}$ in., $\frac{4}{2}$ in. **46.** (*a*) $452\frac{4}{7}$ c. in., (*b*) 209·5 c. in. **47.** 48·1 paras.

48. 56 c. ft. **49.** 241·37 c. in. **50.** Vol. of big cone : Vol. of cyl.: Vol. of small cone :: 8 : 3 : 1

Miscellaneous Examples.

Simplification of Fractions.

1. 4·23, 2·23, 1·309, ·691, ·763, 1·447, ·236, ·448, 1·896, ·690, 1·896, −·690. **2.** $\frac{1}{3}, \frac{1}{3}$. **3.** $\frac{1}{7}$. **4.** 11. **5.** $\frac{1}{16}$. **6.** 1.

7. 112. **8.** 2. **9.** $4\frac{28885}{35473}$.

1`. $\frac{acde - acdf + ac + ae - af + b\,(de - df + 1)}{cde - cdf + c + e - f}$.

G.C.M. and L.C.M.

1. mc. **2.** 8 times. **3.** Either 224 or 56. **4.** 127.

5. $k = 20$, $c = 10$. **6.** 160 min. **7.** $17\frac{7}{9}$ min.; $35\frac{5}{9}$ min.; 160 miles, 15 miles, 12 miles. **8.** B reaches earlier. 197 yds. nearly.

9. 40 min. after starting, 60 yds. from the starting point.

10. A's 104·8 in., B's 120·8 in.

Decimals.

1. 486548 Rem. ·000152. **2.** ·92. **3.** 1·51. **4.** 2·72. **5.** ·367880.

6. 204·272. **7.** 1188·54205. **8.** 354·764772. **9.** 417·307 tons.

10. 28·8075. **11.** 150·66719. **12.** 13641·709.

Scale Representation.

1. $26\frac{7}{33}$ miles. **2.** 2498 acres. **3.** 25·45 ft. **6.** 1439 ft.

8. From B, 65·5 in.; from D, ·94 in. **9.** 59·148 acres.

10. 4 ac. 1244 sq. yds.

PARTNERSHIP.

1. Rs. 960, A gets Rs. 460, B Rs. 200, C Rs. 300.
2. After 4 months. **3.** A Rs. 18,400, B 12, 266⅔. **4.** 20 per cent.
5. A's $\frac{12pa}{12a + (12 - m)\,b}$, B's $\frac{(12 - m)\,bp}{12a + (12 - m)\,b}$, $m = 4$.
6. 6750. **7.** Rs. 600, Rs. 900, Rs. 500. **8.** Rs. 45-2-7.
9. Rs. 160, Rs. 168, Rs. 100. **10.** Rs. 8,000, Rs. 5,000.
11. Rs. 490. **12.** Rs. 510. **13.** A Rs. 189, B Rs. 84; A Rs. 189; B Rs. 126. **14.** A's Rs. 32,400. **15.** At the end of 7 months.
16. £2,310. **17.** Rs. 5,200 **18.** A's Rs. 840 and B's Rs. 420.
19. A's Rs. 303-12-0; B's Rs. 506-4-0. **20.** A gets Rs. 450, B Rs. 1,200.
21. Rs. 125, Rs. 125. **22.** A's Rs. 750, B's 700; A's Rs. 1,650, B's Rs. 1,540. **23.** Rs. 270. **24.** Rs. 525.
25. A for 14 days, B for 16 days. **26.** A Rs. 450, B Rs. 747, C Rs. 120. **27.** A Rs. 2,240, B Rs. 1,692, C Rs. 2,364.
28. Rs. 480, Rs. 533⅓, Rs. 466⅔, ⅓ per cent. **29.** Rs. 4,860.
30. £33, £210. **31.** 7⅛ month. **32.** 70 cattle, 9 months.

PROPORTION.

1. y varies universely as x^2, $y = 12$ nearly. **2.** 9·05 nearly; $\frac{25}{8}r^2$. **3.** 1631·373. **4.** 28·6 yrs. **5.** $\frac{1}{40144896000}$.
6. $\frac{1}{50000}$. **7.** 342 lbs. **8.** 1·0075. **9.** 200 c. ft.
10. 6 sq. miles 453 ac. **11.** pay : bqx. **12.** Rs. 30,250, Rs. 42,250, Rs. 56,250. **13.** 8400. **14.** 11900. **15.** $\frac{aq + ac - gh}{c}$
$c = 1sh.\ 6d.$ **16.** 1 cwt. **17.** $\frac{1}{16000}\,y(x - 20)$.
18. (*a*) 2000; (*b*) 6 ps. in the rupee; (*c*) Rs. 42-3-0; (*d*) Rs. $\frac{1}{32}$ (R − 2000). **19.** Rs. 28-2-0. **20.** Tax $= 3\frac{2}{3} + \frac{17b}{60}$.
21. 70 men. **22.** 114 men. **23.** Rs. 8,78,850. **24.** Rs. 48-2-0.

PERCENTAGES.

1. $t\left(1 - \frac{p}{100}\right)\left(1 - \frac{s}{100}\right)$; $\frac{y}{\left(1 - \frac{p}{100}\right)\left(1 - \frac{s}{100}\right)}$. **2.** 167 millions.
3. 70,848,000; 76,161,600. **4.** 12·986. **5.** 427½ lakhs.
6. $\left(\frac{x - y}{100}\right)\left\{\frac{lz}{100} + m\left(1 - \frac{z}{100}\right)\right\}$. **7.** ¼$d$, $\frac{11}{40}d$.
8. $a\left(1 - \frac{b}{100}\right)^3$; 17·1475. **9.** E $= 36\frac{4}{11}$ per cent.; V $= 42\frac{8}{11}$ per cent. **10.** 38s. **11.** 7·5 per cent.; 42·7 per cent.; 11·2 per cent.; 46·1 per cent. **12.** Rs. 12,20,40,450; 45 per cent.; 1737·85350; 64 per cent.; 19549330; 66 per cent.

13. $y = \frac{p(100-m)-mq}{n-m}$, $x = \frac{-p(100-n)+nq}{n-m}$.

14. $y - x - \frac{xy}{100}$. **15.** $56\frac{1}{4}$ per cent. **16.** $\frac{p^3}{10000} + 3p\left(1+\frac{p}{100}\right)$.

17. 75·5 per cent. **18.** $p\left(1+\frac{p}{100}\right)\left(1-\frac{l}{100}\right)$; $p - l - \frac{pl}{100}$.

AVERAGES.

1. $\frac{144ax-20by}{144a-20b} \times 12d.$; $13\frac{1}{5}d.$ **2.** (*a*) £4-4-2; £3-2-8; £3-19-8, (*b*) 71 per cent.; 72 per cent.; 40 per cent. **3.** Illiterates: H = 195774, ch = 737106; M = 2052725; Average No.: H = 6, Ch = 4, M = 9; All sects.: Li = 512947. Illiterates: 2900155; Total 3413102; Average 6. **4.** $a + 2b$. **5.** 481.

6. Rs. 10-3-10. **7.** Re. 1-3-3; Re. 1-8-1. **8.** 20 days.

9. $\left(\frac{d}{s+x}+\frac{d'}{s-y}\right)$ hrs.; $\frac{2}{1/(s+x)+1/(s-y)}$ miles an hour.

19. $\frac{ag+rs}{p}$.

INTEREST.

1. Rs. 60-3-10 (nearly) if present worth is calculated or Rs. 62-8-0 if interest is calculated. **2.** £271. **3.** Rs. 1,041-10-8.

4. $9\frac{5}{11}$ per cent. **5.** £2,000. **6.** 4 per cent. **7.** £556-1-4.

8. £976-11 3. **9.** Rs. 32,000. **10.** £2394.

11. Bank owes him Rs. 576-5-4. **12.** 3500000.

FORMULAE AND SUBSTITUTION.

1. 9·4843. **2.** $105\frac{5}{9}$. **3.** 42925. **4.** 4·49004; 111·5136.

5. 456·476. **6.** 4·225 sq. in.; ·3014. **7.** 2·33. **8.** $\frac{d+34}{V}$.

9. $K\left(\frac{1}{s_2}-\frac{1}{s_1}\right) - A\left(\frac{1}{s_2}-1\right)$. **10.** $\frac{AE^3}{1950\ (40+E^2)}$.

SHORT METHODS.

1. 372123504. **2.** 6317, rem. 3120. **3.** Rs. 5,916-13-9.

10. £350-3-4. **11.** Rs. 2,049-11-11.

SQUARE ROOT.

1. 50·65 ft.; 51 ft. **2.** 11·314 miles. **3.** 460 yds. 1 ft. 6 in.

4. 72 men, 288 women. **5.** left. **6.** 4·47 in. **7.** 3·354

8. $34\frac{2}{9}$ c. in. **10.** 53·85 ft.

Equations and Problems.

1. $x = 1$, $y = 1$. **2.** $y = -\frac{30}{17}$, $x = -\frac{11}{17}$; $x = \frac{67}{26}$, $y = -\frac{3}{13}$.
3. 12 sides. **4.** 5 : 6. **5.** $13933\frac{1}{4}$. **6.** Advertisement = Rs. 3,000, Expenditure Rs. 12,250. **7.** Rs. $5,062\frac{1}{2}$, Rs. 5,325.
8. Man's wages 9 as., boy's wages 4 as. **9.** (*a*) A Rs. 100, B Rs. 90; (*b*) A's increment Rs. 8, B's Rs. 10; (*c*) 5 yrs.
10. $\frac{101}{135}$, $\frac{34}{135}$. **11.** 480.

Practical Geometry.

1. $6\frac{1}{4}$ sq. cm. **2.** 3 in. 5·2; 7·79 sq. in. **4.** They are equal; they are equal. **5.** 8 cm.; 48 sq. cm. **6.** 181·86 yds.
7. 17·4 ft. **8.** 6 cm. $\angle BAC = 41°$; 42°; a parallelogram which is $\frac{1}{2}$ the quadrilateral. **9.** 55 yds. 1 ft. 6 in.
10. 9·66 sq. units. **11.** (0, − 1), (0, 25); (5, 0), (− 5, 0).

Graphs.

2. 7.36 p.m.; 3 p.m.; 5 p.m. **3.** 10·46 a.m.; 62 miles from Madras; 7.11 a.m.; 12 miles. **4.** 94. **5.** 1 hr. 36 min.; 1·8 miles. **6.** 7.30 a.m.; 11.12 a.m.; 103°. **7.** 20 acres and 25 acres. **8.** $21\frac{3}{5}$ miles from Madras, 8.12 a.m.
9. (*a*) 1·4; (*b*) 6·1; (*c*) 9·1. **10.** Rs. 95-5-0, Rs. 118-2-0.

Mensuration.

1. Rs. 748-11-8. **2.** 10·1 in. **3.** 24 ft., $2\frac{2}{3}$ hrs. **4.** 66·90 sq. in., 42·412 c. in. **5.** ·6 of an inch. **6.** 381 ft. **7.** ·79 of an in.
8. 3·1. **9.** 7 ft. 7 in. **10.** 6156 c. ft. per hour; 164·16 sq. in.
11. 12 cwt. 1 qr. 3 lb. 15 oz. **12.** $520\frac{5}{6}$ lbs. water; $16303\frac{4}{7}$ lbs. mercury $5616\frac{76}{189}$ lbs. steel; $8069\frac{4}{9}$ lbs. cast iron.
13. 838·109 c. in. **14.** 3 ft. $7\frac{3}{4}$ in. **15.** 385 : 324.
16. 16 min. nearly. **17.** 327 lbs. **18.** Rs. 173-4-0.
19. 1 ft. 1 in. nearly. **20.** 520344000 c. ft.; $1\frac{47}{88}$ in.

Profit and Loss.

1. $26\frac{2}{3}$ per cent.; Rs. 2-4-0. **2.** Rs. 23,292. **3.** Chicory 9 as.; coffee 16 as. **4.** Rs. 152-9-7. **5.** Increase; 40 : 41.
6. $5\frac{3}{16}$ per cent. **7.** 3 as. 11 ps.; Rs. 48-11-10. **8.** 16·8 per cent.
9. Rs. 23-9-1. **10.** Rs. 1,018-12-0. **11.** 43·675 per cent.
12. Rs. 140-4-0. **13.** 32 tons. **14.** I : II :: 5 : 3.
15. 100 $\left\{\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)\left(1-\frac{c}{100}\right)-1\right\}$ per cent.; 16 nearly.

Time and Distance.

1. Distance = 40 miles, rate = 10 miles per hour.
2. Distance = 70 miles ; rate = 10 miles per hour.
3. Rate of driving = 6 miles an hour, rate of walking = 3 miles an hour.
4. 21 Miles from P. **5.** At a place 7 miles from Q in 28 hrs.
6. Z $\left(\frac{a}{x} - \frac{b}{y}\right)$ miles ; $2\frac{1}{2}$ miles. **7.** $x = c(a + b)$.
8. Rate $= \frac{m + xa}{x}$; $8\frac{1}{2}$ miles an hour. **9.** $\frac{a - my - nz}{x - y - z}$ miles per hour ; 5. **10.** 12 miles 890 yds.
11. Rate of the thief = $40\frac{1}{3}$ miles per day ; that of the detective = $24\frac{1}{3}$ miles per day. **12.** $18\frac{1}{3}$ miles. **13.** $11\frac{1}{2}$ sec.
14. 40 miles ; 20 miles. **15.** $2\frac{8}{11}$ sec. **16.** $3\frac{1}{4}$ miles per hour.
17. 40 miles per hour. **18.** 45 miles per hour ; 132 yds.
19. 3 miles $36\frac{2}{3}$ yds. **20.** $2\frac{26}{35}$ miles per hour.
21. $1\frac{7}{18}$ mile. **22.** S reaches earlier ; $35\frac{5}{9}$ hours and $32\frac{32}{39}$ hours.
23. 12.37$\frac{1}{7}$ p.m. ; $18\frac{8}{7}$ from Q. **24.** 2 miles an hour.
25. 5 miles. **26.** $17\frac{1}{2}$ sec. **27.** $4\frac{1}{2}$ miles an hour.
28. Train = $26\frac{18}{22}$ miles an hour ; man $2\frac{21}{22}$ miles per hour ;
29. (*a*) 6.50 a.m. ; (*b*) 7 a.m. ; 6.57 nearly. **30.** 8.20 p.m.
31. 30 miles ; 6 miles, $7\frac{1}{2}$ miles and 10 miles an hour.
32. 12.22$\frac{1}{2}$ p.m. ; $16\frac{2}{3}$ miles. **33.** 1 min. 48. sec. **34.** $\frac{1}{5}$ of a mile further than the usual meeting place in the direction of the up train. **35.** $9\frac{2}{7}$. **36.** $321\frac{1}{12}$ miles an hour. **37.** 4·56 min. after he heard the second shot. **38.** $118\frac{15}{16}$ yds.
39. 45 miles per hour ; 220 yds. **40.** $8\frac{4}{7}$ min. past 9 ; $34\frac{2}{7}$ miles from the starting point ; A 10 a.m. ; B 9·36 a.m.
41. 45 miles per hour. **42.** 77 yds., 99 yds. ; 45 miles per hour, 35 miles per hour. **43.** $13\frac{5}{7}$ miles.

Approximations, Errors.

1. Rs. 54-7-9. **2.** Rs. 22,800 ; Rs. 20,400. **3.** 36,360 sq. links.
4. 720 times ; $\frac{a(b - c) + 82\frac{1}{2}}{p - d}$. **5.** (*a*) ·1921 oz., ·323 oz. (*b*) ·230 oz. ; (*c*) Rs. 15,00,609-5-7. **6.** 30·3 and 29·2.
7. 217 yrs. **8.** (*a*) Rs. 14-13-6 ; (*b*) 2·0873 ; (*c*) 6249879 ; (*d*) 140.
10. (*a*) 29·7 per cent. increase, 48·1 per cent increase ; (*b*) 55·3.
11. Rs. 57-2-0. **12.** 309375 kg.

TIME AND WORK.

1. 48 men. **2.** 1000 c. ft. **3.** 16 days. **4.** Rs. 13-2-0.
5. Rs. 41-4-0. **6.** R in 75 min. ; Q in 30 min. ; P in $8\frac{4}{7}$ min.
7. 36 days. **8.** $67\frac{1}{2}$ days. **9.** 130 men, 65 women, and 260 boys.
10. $13\frac{1}{13}$ days.

CONSTRUCTION OF QUADRILATERALS.

8. 2·9 miles. **10.** CD = 5 cm., BC = 4·8 cm., $\angle B = 70°$, $\angle C = 100°$, $\angle D = 70°$. **13.** 17·36 sq. cm.
14. 1·7 in., $\angle 88°$, $\angle 92°$. **16.** 65°, 115°. **22.** 858·93 sq. yds.
23. 76·8 sq. cm. **24.** 102 sq. ft. **26.** 598 ac. 3680 sq yds.
27. 37·5 sq. cm. **28.** 495 ac. 420 sq. yds.

BANKRUPTCY.

1. Rs. 1,575. **2.** $6\frac{4}{5}$ as. **3.** Rs. 4,687½. **4.** 12 as. 10 ps.
5. 6 as. 1 p. **6.** Rs. 4,000. **7.** Rs. 12,000. **8.** Rs. 37,350.
9. A Rs. 6,000, B Rs. 5,000, C Rs. 4,000, D Rs. 1,000.
10. £9,309$\frac{1}{11}$, £271$\frac{17}{33}$; £19$\frac{13}{33}$. **11.** Rs. 4,000, Rs, 400, Rs. 50.
12. £576.

REVISION PAPERS—II SERIES (Page 344).

I.

1. (*a*) A cub. metre $> a$ cub. yds. or 27 cub. ft. ; $903 \times 573 = 90300 \times 5{\cdot}73$ which is less than 90300×6 i.e. 541800 ; (*c*) the greatest value $= \frac{800 \times 2400}{5000 \times 3000} = \frac{192}{1500} = \frac{1{\cdot}92}{15} = {\cdot}121$. Hence absurd ; (*d*) $({\cdot}28)^2 = {\cdot}0784$; (*e*) $\sqrt{20 \times 10 \times 6 \times 5} = \sqrt{6000} = 10 \times 8 = 80$ nearly.
2. Rs. 1,467-13-5. **3.** Dry 8 acres, wet 10 acres. **4.** 40 miles from P : $\frac{11}{12}$ hours. **5.** $BC = CD = 2\sqrt{2}$ in. $DA = 2$ in. $AB = 2\sqrt{3}$ in. area = 74640 sq. yds. **6.** ·009211 ; 7763 miles.
7. ·0316. **8.** (*a*) 2·6, (*b*) ·0857.

II.

1. (*a*) $(5x - 3)(5x + 14)$; (*b*) $-12(x + 2)(x + 1)(x - 1)$.
2. 4·62 sq. in. **3.** 13·75 ft., $11\frac{11}{24}$ ft., error in area to the nearest sq. ft. is 30 ; Rel. errors are $\frac{1}{12}$ ft. and $\frac{1}{19}$ ft. percentage error $8\frac{1}{3}$ per cent.
4. At 10.50 p.m. ; 95 miles. **5.** $\frac{\sqrt{3}}{2}$ or ·87 in. and 120°.
6. 60 miles by rail and 90 miles by river. **7.** 1 gallon = 277·27 4 cub. in. **8.** 1·2 in. and 0·6 sq. in.

III.

1. 20,000 sq. miles. **2.** 1·12499. **3.** 2·38 sq. in. **3.** 5 cms.
5. Father 30 yrs., eldest son $7\frac{1}{2}$ yrs., youngest son 6 yrs.
6. 3835 yds., Rs. 6,709½ nearly. **7.** $x = 6$, $y = -1$.
8. $c = 1 + \frac{1}{2}m$ where c is the cost in rupees and m the number of miles ridden.

IV.

1. 231 (hundred thousands). **2.** 387. **4.** 5 cm., 24 sq. cm.
5. (*a*) 19·74; (*b*) 326 ft. **6.** $2\frac{1}{2}$ yds. **7.** 5·5 in.; 54 acres and 2640 sq. yds. **8.** Rs. 300, Rs. $367\frac{1}{2}$.

V.

2. 26·131. **3.** Chicory : coffee :: 4 : 17. **4.** 100 $\frac{(b^2+c^2)}{a^2}$ per cent.
6. $x = y = z$. **7.** 20 cows and 30 sheep. **8.** 9.15 a.m.; 68 miles from P.

VI.

1. 62 lbs. 6·4 oz. **2.** $l = \frac{t^2q^2}{4\pi^2}$; ·986. **3.** 40 ft. **4.** 3·46.
5. Rs. 1,365. **6.** £8-5-7. **7.** 19 men. **8.** $2\frac{2}{9}$.

VII.

1. ·268. **2.** £198-15-0. **3.** L = 27 ft., B = 17 ft.
4. BC = 8 cm. and AD = 8·4 cm. **5.** 2 in.; 73°. **6.** $9\frac{6}{23}$ sec.
7. $115\frac{1}{2}$ sec. **8.** At 6 p.m.; 30 miles from P, $7\frac{1}{2}$ miles per hour.

VIII.

1. £46-2-6. **2.** 265 metres. **3.** £137-10-0. **4.** 4·22 sq. in.
5. $x = 7$, $y = 9$. **6.** DC = 3·4 cm. and BD = 5·4 cm.
7. 45 miles per hour, 37 ft. nearly per sec. **8.** $86\frac{5}{8}$ c. ft.

IX.

1. 15*d*. **2.** 30 gallons per head, 6*s*. per head. **3.** 2 kg. 252 gm.
4. CD = 4·5 cm., ∠C = 105°, ∠D = 78°. **5.** $x = 5$, $y = 7$.
6. 4.43; 4.26. **7.** $6696\frac{2}{3}$ lbs. **8.** 2 in., 1·1 in. : sq. octagon :: 1 : 4·8.

X.

1. 252·96. **2.** ·25000. **3.** ·0000099 per cent.; 97; ·038 per cent.
5. $4\frac{1}{3}$ miles from town. **6.** 239·1 miles. **7.** £16,750. **8.** $2\frac{1}{4}$ ft.

S.S.L.C. Papers.

1911.

1. Q = 3, rem. = 21. **2.** Area of the rectangle = 99·80 sq. cm. nearly. **3.** Rs. 3,295-11-0. **4.** 33·7 per cent. nearly, the greatest percentage is 63·8, the least percentage is 18·9.
5. Rs. 90 more. **6.** 878 men. **7.** 490 yds. **8.** 40·124 in.
9. Rs. 5 for 8 miles. **10.** No. 1 — 100 miles from Madras, No. 2 — 305 miles from Madras, No. 3—275 miles from Madras ; No. 1 passes, No. 2 at 2.15 a.m. on Tuesday and No. 3 at 4.10 a.m. on Tuesday. **11.** $AD = AB\sqrt{3}$. **12.** ·79 sq. in.
13. 27·6 in. **14.** 35 ft. nearly. **15.** $x = P - \frac{P}{40}(y - 30)$; Rs. 165 to the nearest rupee.

1912.

1. Name	A	B	C	D	E	F	G	H	L	M	N	O
Total	105	50	26	145	90	38	21	131	53	45	58	114
Percentage ...	70	33	17	97	60	25	14	87	53	30	39	76
Rank	4	8	11	1	5	10	12	2	6	9	7	3

2. (1) = Rs. 80-12-0.
(2) = ,, 303-3-0.
(3) = ,, 192-11-0.
(4) = ,, 202-9-0.
(5) = Rs. 260-11-0.
(6) = ,, 354-3-0.
(7) = ,, 51-0-0.

3. (1) Omit (stocks and shares) ; (2) Rs. 76-5-10. **4.** 141 ft.
5. 910000 sq. yds. **6.** 1571 sq. ft. nearly. **8.** 103·2, 100·1.
9. 4466·2 tolas. **11.** £32 a year, Rs. 75 a month.
12. x lbs. per sq. ft. = ·489x gm. per sq. cm. **13.** Rs. 4.
14. 45 cm. nearly. **15.** Rs. 0-7-11.

1913.

1. (i) 919366 ; 266,526 ; 67,099 ; 61,571 ; 40,673 ; 31,321.
2. £49-6-4 ; Rs. 236-11-5. **3.** 3035 ft. nearly. **4.** $x = 4$, $y = 3$.
5. 1 m. 5 fur. 195 yds. 1 ft. 8 in. **6.** (1) $\frac{2}{1}$; (2) 4. **7.** (2) 2″.
8. (1) 7·071 sq. in. **10.** Rs. 306-4-0. **11.** Rs. 2-10-7.
12. 3103·788 kg.

1914.

1. 698, 667, 683, 348 ; + 35·1 per cent., — 7 per cent., + 355·5 pe cent., + 38·7 per cent., + 17·3 per cent., — 2·2 per cent.

3. $\frac{mx - ny}{m - n}$; 9·9 nearly. **4.** 5 : 16. **5.** Rs. 400. **6.** $26\frac{1}{2}$ ft.

7. 504 sq. yds. nearly. **8.** 285 sq. in. **9.** 3 min. **10.** 58542

12. Highest 44·6, December 27, lowest 36·4, September 3.

1915.

1. £0-7-0 ; £0-8-1 ; 42 per cent. **2.** £0-18-3. **3.** 3 lbs. nearly.

4. 1330 ; 320. **5.** $a = b(f - e)$; $c = d(f + e)$; $f = 10$, $e = 2$.

6. 1·4 ft. **7.** 5 cm., 2·1 cm. **8.** 51 tons. **9.** Rs. 5-10-0 Rs. 3-12-0. **10.** Rs. 23283 nearly. **11.** $6\frac{1}{4}$ minutes.

12. 88 lbs.

1916.

1. (1) D = 11·8 per cent., D 17·4 per cent. ; (2) I 11·0 per cent. I 13·6 per cent. ; (3) D 66·7 per cent., D 75 per cent. (4) D = 25·8 per cent., D 27·1 per cent. ; (5) I 1·8 per cent. I 1·5 per cent. **2.** Rs. 9-4-6. **4.** 8. **5.** 4 as.

6. 17 c. ft. nearly. **7.** 1·64 in. nearly. **8.** W = $218\frac{3}{4}$, P = 200

9. $\frac{1}{6}(p + q + r)(4d + a) - (ap + bq + cr)$ as., $d = 20$.

10. $x = 6\frac{6}{7}$, $y = -\frac{4}{7}$. **11.** BD = 5·8 cm., B = D = 105°.

12. (1) $\frac{2}{3}$; (2) $\frac{8}{9}$.

1917.

1. (i) 77 ; (ii) 5·98714. **2.** 13·3 per cent. **3.** Rs. 525.

4. Rs. 4,472-14-0 ; 6·4 per cent. **5.** 205 ft. **6.** 6 in. 6 lbs.

7. 12682 nearly. **8.** Subtract 7 from the final result and divide th remainder by 2. **9.** A Rs. 70, B Rs. 130. **10.** 5 cm.

1918.

1. (i) Rs. 3-13-3 ; (ii) 16·7 per cent. **2.** Rs. 200. **3.** (i) 40 min from Madura at 13.38 ; (ii) 12.34 ; (iii) 16.41 ; (iv) 13.8 29 miles ; (v) 14.53 and 65 miles from Madura.

4. 18,000,000 sq. miles ; 120 miles to an inch. **5.** 4·0 per cent. 4·8 per cent., 5·8 per cent. **6.** Rs. 1,232. **7.** ·41 cm.

8. $\frac{mx + ny}{x + y}$; 36·4 per cent. **9.** 29 miles. **10.** $\frac{1}{4}$. **11.** 30 miles

12. 17 inches.

1919.

1. D has the lowest average 34·5, C the highest average 41·4.

2. 32·5. **3.** 12·7 ac. nearly. **4.** (i) 2·30 p.m. ; (ii) 1·5 p.m. $37\frac{1}{2}$ miles from P ; (iii) 9·15 a.m. $12\frac{1}{2}$ miles. **5.** Rs. 112.

6. (ii) 5·85 sq. in. **7.** 2 ft. 8 in. **8.** 24 men and 20 boys.
9. (i) 7·2 in.; (ii) 18 men more. **10.** 1·4 in.
11. Rs. $\frac{am+bn}{m+n}\frac{(100+c)}{100}$; Re. 1-9-0. **12.** 2964·5 ft.

1919 Re-examination.

1. 37, 57; 57, rank 1st, 37, 67th rank. **2.** Cannot be solved; S P. not given. **3.** C will win by 13 yds. **4.** 171½ c. in.; 128 boxes.
5. Direct from London; Rs. 42-8-0. **6.** 30*s.*; 15*s.*; 10*s.*
7. 30·23 sq. cm.; 27207 sq. yds. **8.** Rs. 1,076-8-11.
9. 1884. **10.** 20 per cent. **11.** 878¾ yds.
12. (i) $60\left(\frac{1}{m}-\frac{1}{n}\right)$ miles, (ii) $\frac{3mn}{n-m}$ min.; (iii) 3 hrs. 22 min.

1920.

1. (i) 66·9 per cent.; (ii) Rs. 12,65,368. **2.** 7½ min.
3. Rs. 115-8-0. **4.** 175 ft. **5.** Rs. 240; Rs. 360. **6.** 3 miles per hour, 6 p.m. **7.** P to S 31½ miles; A's rate 3 miles an hour, A reaches S at 6 p.m.; they are together for the 3rd time at 2.30 p.m. and at a place 10½ miles from S. **8.** 37°, 35 sec. or 56 sec. **9.** 38 men. **10.** 3 in. **11.** 3000 c.c.
12. 6·8 per cent.; 1936, £2,516-16-0.

1921.

1. 6076 ft. nearly. **2.** $c = 1{\cdot}9''$, $a = 2{\cdot}1''$, $b = 1{\cdot}5''$, $\angle A = 79°$, $\angle B = 41°$, $\angle C = 60°$. **3.** 14·96 sq. in. **4.** Re. 0-8-0.
5. Rs. 45-4-10. **6.** 75 per cent. mathematical books. **7.** 34·7 per cent.
8. 14½ days. **9.** 12667 sq. ft. **10.** Re. 1. **11.** $\frac{8}{13}$; 294.
12. (i) 10 ft.; (ii) 10 ft. from one bank and 8 ft. from the other bank.

1922.

1. (i) Rs. 17,040, (ii) Rs. 24,538, (iii) 7·1 per cent. **2.** 250 yds.
3. 1·6 in., 3·9 in.; 7·1 cm.; 11·4 cm. **4.** 24 performances.
5. Rs. 95,575. **6.** 22·8 in., 14·3 in. **7.** 91 ounces.
8. Ans. is 50 per cent. **9.** Rs. 72,765. **10.** 34·40 tons.
11. Asia 12°·4, Europe 27°·4, Africa 22°·1, Australia 21°·1 North America 57°·6 and South America 49°·0.
12. $CD = 2\sqrt{ab}$, CD = 3·9 in.

1923.

1. Rs. 80,87,462; Rs. 81,89,960, 3·64 per cent., 152·5 per cent.
2. (i) 15·573 kg., (ii) 35 and 32. **3.** 17$\frac{3}{16}$ per cent. **4.** 4 as.

5. Rs. 192. **6.** 26. **7.** July 1st 1916, August 1st 1920.
8. $59\frac{7}{12}$ miles per hour. **9.** $13\frac{1}{3}$ ft. **10.** $\alpha = 46{\cdot}6$, $\beta = 41{\cdot}6$.
11. 25000 c. ft. **12.** 656 sq. in.

1924.

1. (i) G = 1798, North America = 2047 ; Tanjore = 2324 ; Trichy 1901, R = 1722, Tin. = 1901 ; Hindus = 10,379, Mah. = 6 Ch. = 647, (ii) G = 7·3 per cent., T = 5·6 per cent., Tin. 3·3 per cent., Tm. = 5·7 per cent. G has the highest percenta
2. (i) 62 lbs. 6·8 oz. (ii) Rs. 7-5-0. **3.** (*a*) Rs. 2,000 ; (*b*) 13 gal. wine and 19 gal. of water. **4.** Cost of cup = Rs. 41-8 cost of each medal Rs. 3-7-4. **5.** 54·5. **6.** 98.
7. 6 hrs. 12 min. ; 12 hrs. 40 min. **8.** 27 : 9 yds. ; 28 yds.
9. 31·8 ac. **10.** $a = {\cdot}3$, $b = {\cdot}8$; 2·125 lbs. **11.** 120 sq. ft ; 7,920,00
12. 17 per cent.

1925.

1. + 2·7 per cent., + 2·2 per cent., — 1·4 per cent., + ·9 per cent.
2. A Rs. 15-12-0, B Rs. 28-14-0., C Rs. 20-4-0. **3.** £28-16-0.
4. 40 years. **5.** Rs. 958-13-0 or 12 as. **6.** 5·7 miles.
7. £24-13-4. **8.** 40 cms. **9.** 53°. **10.** 2 ft. 30°.
11. Rs. 135 and Rs. 30. **12.** 4·9 acres. **13.** (*a*) 2·45 gal. (*b*) 9 as. 11 ps.

R.B.A.N.M's

Hr. Sec. School Library

DATE LABEL

CL. No. BN 26; 1 Accn. No. ~~2351~~ 2286

This book ought to be returned on or before the date last marked below, or fines will be incurred.

Due Date	Ticket No.	Inls.	Due Date	Ticket No.	Inls.

R.B.A.N.M's

Hr. Sec. School Library

BOOK CARD 2286

CL. No. BN 26; 1 Acc. No. ~~851~~

Author Venkata Subbayya (V)

Title A text book of elementary mathematics

Issue Date	Return Date	Ticket No.	Issue Date	Return Date